

Section 1.7

Parametric Equations

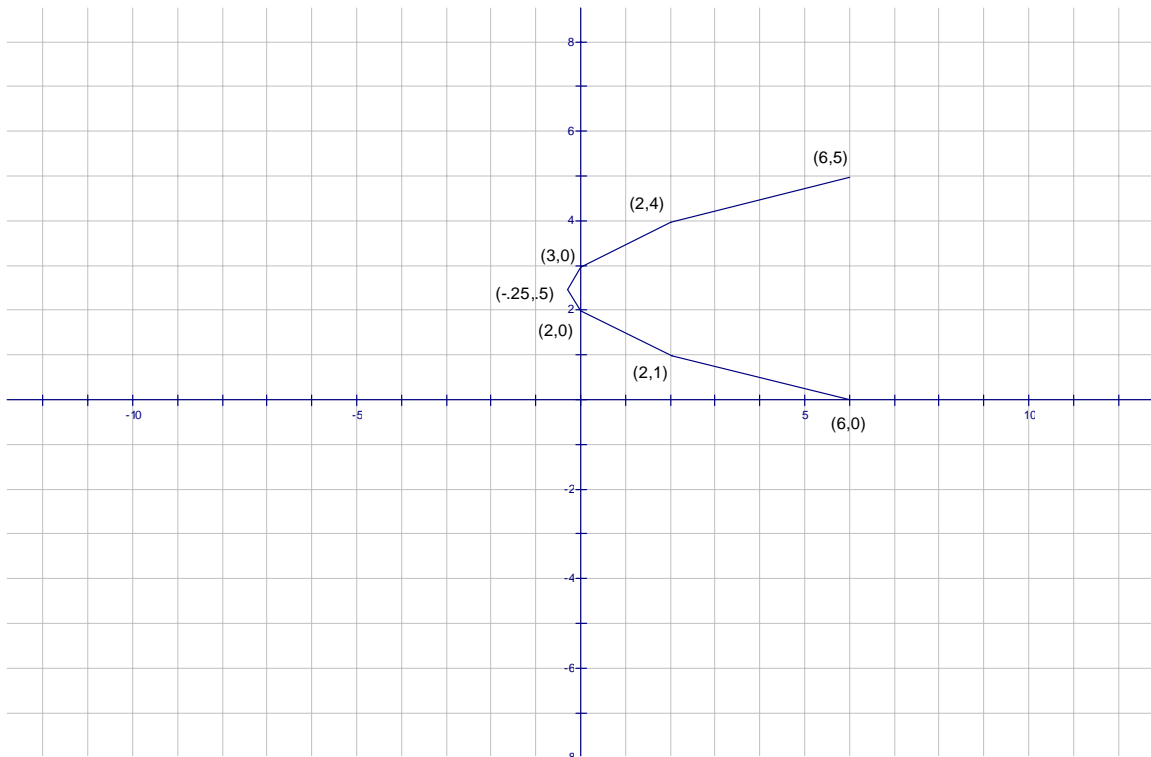
Parametric equations are by a set of variables x and y that are expressed as functions of a third variable called the parameter usually represented by t .

$$x = f(t); y = g(t)$$

Example 1 Sketch the following curve by using parametric equations to plot points.

$$x = t^2 - t \quad y = t + 2$$

t	$x = t^2 - t$	$y = t + 2$
-2	$x = (-2)^2 - (-2) = 4 + 2 = 6$	$y = -2 + 2 = 0$
-1	$x = (-1)^2 - (-1) = 1 + 1 = 2$	$y = -1 + 2 = 1$
0	$x = 0^2 - 0 = 0 - 0 = 0$	$y = 0 + 2 = 2$
.5	$x = .5^2 - .5 = .25 - .5 = -.25$	$y = .5 + 2 = 2.5$
1	$x = 1^2 - 1 = 1 - 1 = 0$	$y = 1 + 2 = 3$
2	$x = 2^2 - (2) = 4 - 2 = 2$	$y = 2 + 2 = 4$
3	$x = 3^2 - (3) = 9 - 3 = 6$	$y = 3 + 2 = 5$

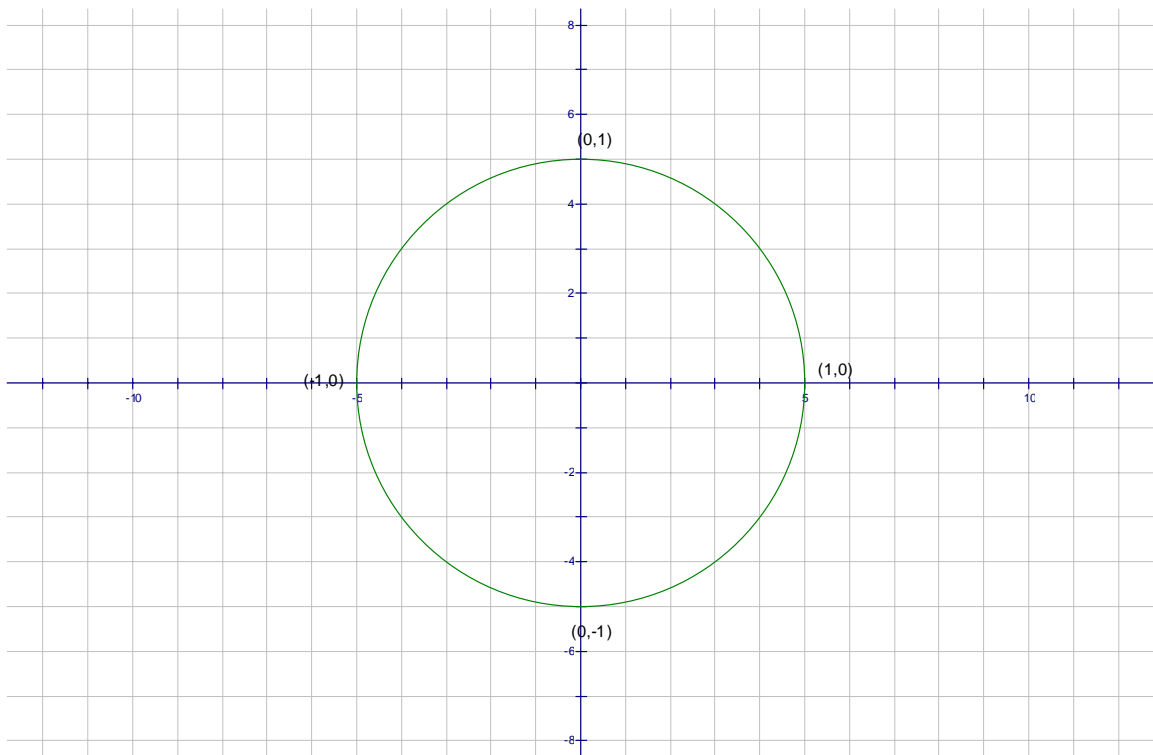


Example 2

Sketch the following curve by using parametric equations to plot points.

$$y = \sin(t) \quad x = \cos(t)$$

t	$x = \cos(t)$	$y = \sin(t)$
0	$x = \cos(0) = 1$	$y = \sin(0) = 0$
$\frac{\pi}{4}$	$x = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	$y = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
$\frac{\pi}{2}$	$x = \cos\left(\frac{\pi}{2}\right) = 0$	$y = \sin\left(\frac{\pi}{2}\right) = 1$
π	$x = \cos(\pi) = -1$	$y = \sin(\pi) = 0$
$\frac{3\pi}{2}$	$x = \cos\left(\frac{3\pi}{2}\right) = 0$	$y = \sin\left(\frac{3\pi}{2}\right) = -1$
2π	$x = \cos(2\pi) = 1$	$y = \sin(2\pi) = 0$

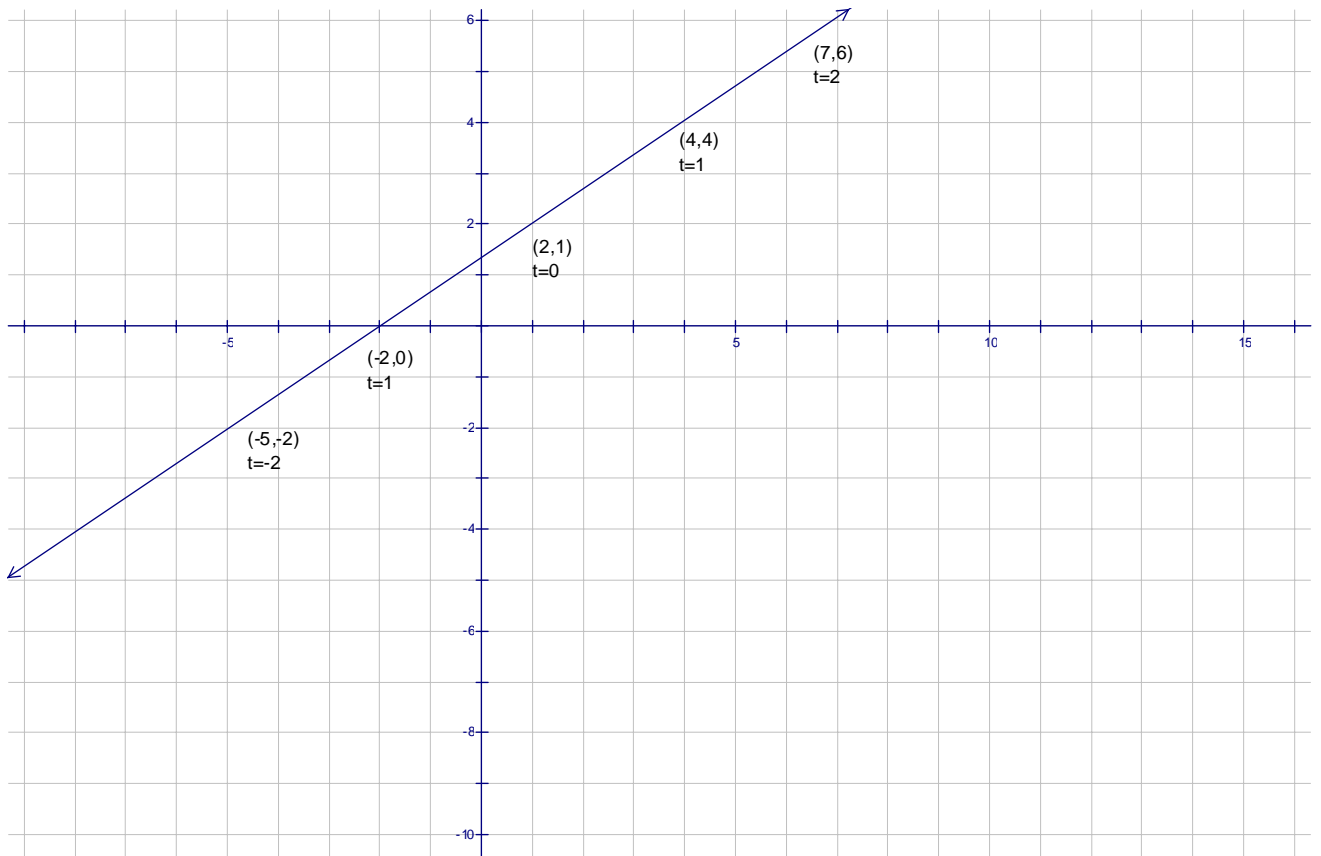


Example 3

Sketch the following curve by using parametric equations to plot points.

$$x = 3t + 1 \quad y = 2t + 2$$

t	$x = 3t + 1$	$y = 2t + 2$
-2	$x = 3(-2) + 1 = -5$	$y = 2(-2) + 2 = -2$
-1	$x = 3(-1) + 1 = -2$	$y = 2(-1) + 2 = 0$
0	$x = 3(0) + 1 = 1$	$y = 2(0) + 2 = 2$
1	$x = 3(1) + 1 = 4$	$y = 2(1) + 2 = 4$
2	$x = 3(2) + 1 = 7$	$y = 2(2) + 2 = 6$



Slope and Intercept

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept

$$y = mx + b$$

$$m = \text{slope}$$

$$b = y\text{-int.}$$

Point-slope

$$y - y_1 = m(x - x_1)$$

Example 1

Find the slope of line passing through the points (2,3) and (5,7)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{5 - 2} = \frac{4}{3}$$

Example 2

Find the equation of a line that has slope of $m = \frac{2}{3}$ and passes through the point (-1,4).

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{3}(x - (-1))$$

$$y - 4 = \frac{2}{3}(x + 1)$$

$$y - 4 = \frac{2}{3} + \frac{2}{3}$$

$$y = \frac{2}{3}x - \frac{10}{3}$$

Example 3

Find the equation of a line passing through the points (1,2) and (4,1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{4 - 1} = -\frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{3}(x - 1)$$

$$y - 2 = -\frac{1}{3}(x - 1)$$

$$y - 2 = -\frac{1}{3}x + \frac{1}{3}$$

$$y = -\frac{1}{3}x - \frac{5}{3}$$