

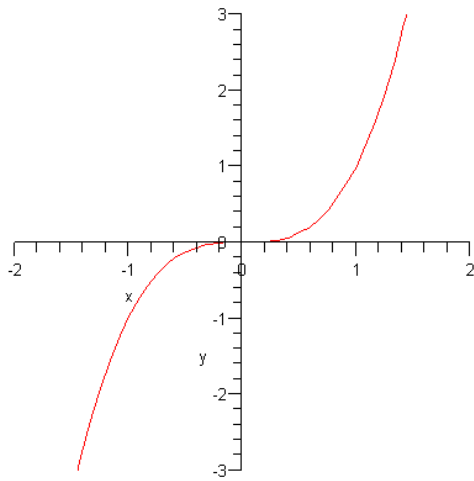
Section 1.6 Inverse Functions and Logarithms

One-to-one functions

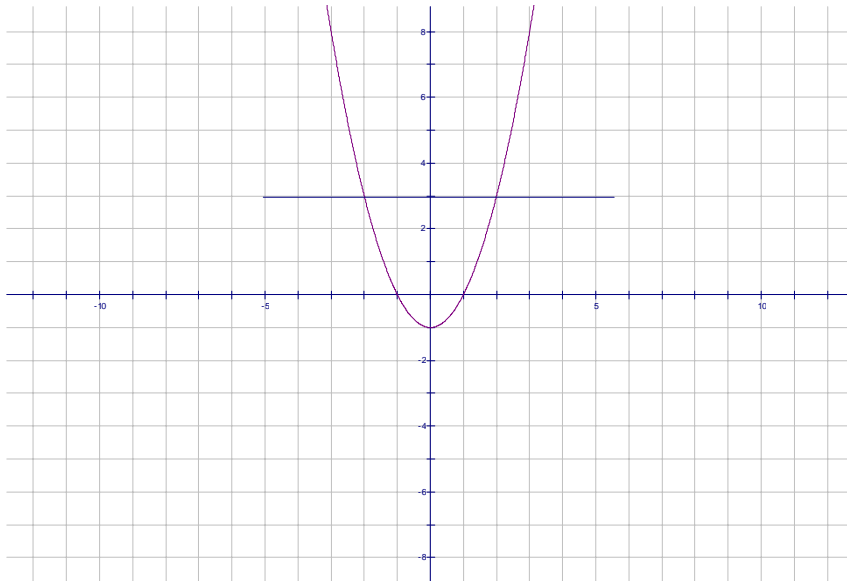
Definition: A function f is an one-to-one function if it never take one the same value twice. *If $f(x_1) = f(x_2)$, then $x_2 = x_1$*

Horizontal Line Test: A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Graph of a one-to-one function



Graph of a function that is not one-to-one



Inverse functions

$$f^{-1}(x) = y \Leftrightarrow f(x) = y$$

domain of f^{-1} = range of f

range of f^{-1} = domain of f

The relationship between the inverse function and the composition

$$f(f^{-1}(x)) = x \text{ and } f^{-1}(f(x)) = x$$

Example 1

Find the inverse of the function and show that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ given $f(x) = 2x - 3$

First, find the inverse.

$$f(x) = 2x - 3$$

$$y = 2x - 3$$

$$x = 2y - 3$$

$$x + 3 = 2y$$

$$\frac{x+3}{2} = y \Rightarrow f^{-1}(x) = \frac{x+3}{2}$$

$$f(f^{-1}(x)) = f\left(\frac{x+3}{2}\right) = 2\left(\frac{x+3}{2}\right) - 3 = x + 3 - 3 = x$$

$$f^{-1}(f(x)) = f^{-1}(2x - 3) = \frac{2x - 3 + 3}{2} = \frac{2x}{2} = x$$

Example 2

Find the inverse of the function and show that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ given

$$f(x) = x^3 - 3$$

$$f(x) = x^3 - 3$$

$$y = x^3 - 3$$

$$x = y^3 - 3$$

$$x + 3 = y^3$$

$$\sqrt[3]{x+3} = \sqrt[3]{y^3}$$

$$\sqrt[3]{x+3} = y$$

$$f^{-1}(x) = \sqrt[3]{x+3}$$

$$f(f^{-1}(x)) = f(\sqrt[3]{x+3}) = (\sqrt[3]{x+3})^3 - 3 = x + 3 - 3 = x$$

$$f^{-1}(f(x)) = f^{-1}(x^3 - 3) = \sqrt[3]{x^3 - 3 + 3} = \sqrt[3]{x^3} = x$$

Basic Logarithms

Definition of a logarithm

$$\log_a x = y \Leftrightarrow a^y = x$$

Example 3

a) Write $\log_4 64 = 3$ as an exponential expression.

$$\log_4 64 = 3 \Rightarrow 4^3 = 64$$

b) Write $\log_5 \left(\frac{1}{25} \right) = -2$

$$\log_5 \left(\frac{1}{25} \right) = -2 \Rightarrow 5^{-2} = \frac{1}{25}$$

Rules for Logarithms

$$1) \log_a(xy) = \log_a x + \log_a y$$

$$2) \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$3) \log_a(x^r) = r \log_a x \quad (\text{where } r \text{ is any real number})$$

Example 4

Simplify $\log_2 20 - \log_2 5$

$$\log_2 20 - \log_2 5 = \log_2\left(\frac{20}{5}\right) = \log_2 4 = 2$$

Example 5

Simplify $\log_4 8 + \log_4 8$

$$\log_4 8 + \log_4 8 = \log_4(8 \cdot 8) = \log_4 64 = 3$$

Change of base formula.

$$\log_b a = \frac{\log a}{\log b}$$

Example 6

$$\text{Evaluate } \log_6 40 = \frac{\log 40}{\log 6} = \frac{1.6021}{.7882} = 2.033$$

The Natural Logarithm

$$\log_e x = y \Rightarrow e^y = x$$

Basic natural logarithm rules

$$1) \ln x + \ln y = \ln(xy)$$

$$2) \ln x - \ln y = \ln\left(\frac{x}{y}\right)$$

$$3) \ln(x^a) = a \ln x$$

$$4) e^{\ln x} = x \text{ and } \ln(e^x) = x$$

Example 7

Simplify $\ln 4 + 3 \ln 2$

$$\ln 4 + 3 \ln 2 = \ln 4 + \ln 2^3 = \ln 4 + \ln 8 = \ln(4 \cdot 8) = \ln 32$$

Note: $\log(100)$ is understood to be $\log_{10}(100)$

Therefore, you solve this logarithm as follows:

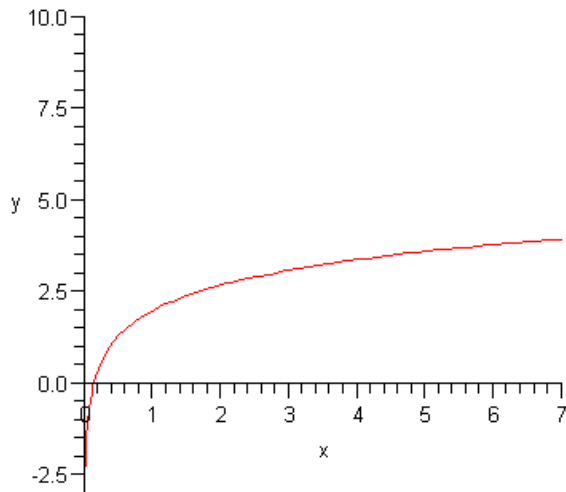
$$\log_{10}(100) = x \Rightarrow 10^x = 100 \Rightarrow 10^x = 10^2 \Rightarrow x = 2$$

Graphs of the natural log and log functions

Example 8

Graph $f(x) = \log(x) + 2$

| x | $f(x) = \log(x) + 2$ |
|-----|--|
| 1 | $f(1) = \log(1) + 2 = 0 + 2 = 2$ |
| 10 | $f(10) = \log(10) + 2 = 1 + 2 = 3$ |
| 20 | $f(20) = \log(20) + 2 = 1.3 + 2 = 3.3$ |
| 30 | $f(30) = \log(30) + 2 = 1.5 + 2 = 3.5$ |
| 100 | $f(100) = \log(100) + 2 = 3 + 2 = 5$ |

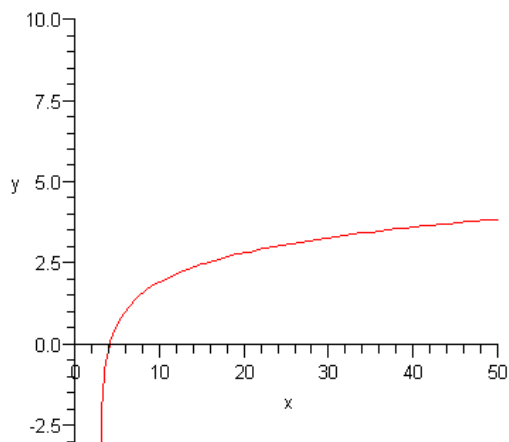


Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Example 9Graph $f(x) = \log(x - 3)$

| x | $f(x) = \log(x - 3)$ |
|-----|--|
| 4 | $f(4) = \log(4 - 3) = \log(1) = 0$ |
| 10 | $f(10) = \log(10 - 3) = \log(7) = .85$ |
| 20 | $f(20) = \log(20 - 3) = \log(17) = 1.23$ |
| 30 | $f(30) = \log(30 - 3) = \log(27) = 1.43$ |
| 100 | $f(100) = \log(100 - 3) = \log(97) = 1.99$ |

Domain: $(3, \infty)$ Range: $(-\infty, \infty)$

Example 9

Find the inverse of $f(x) = e^x - 3$

$$f(x) = e^x - 3$$

$$y = e^x - 3$$

$$x = e^y - 3$$

$$x + 3 = e^y$$

$$\ln(x + 3) = \ln e^y$$

$$\ln(x + 3) = y$$

$$f^{-1}(x) = \ln(x + 3)$$

Example 10

Find the inverse of $f(x) = \ln(x) + 3$

$$f(x) = \ln(x) + 3$$

$$y = \ln(x) + 3$$

$$x = \ln(y) + 3$$

$$x - 3 = \ln(y)$$

$$e^{x-3} = e^{\ln(y)}$$

$$e^{x-3} = y$$

$$f^{-1}(x) = e^{x-3}$$

Example 11

Solve $e^{x-2} + 3 = 5$

$$e^{x-2} + 3 = 5$$

$$e^{x-2} = 5 - 3$$

$$e^{x-2} = 2$$

$$\ln e^{x-2} = \ln(2)$$

$$(x - 2) \ln e = \ln(2)$$

$$x - 2 = \ln(2)$$

$$x = \ln(2) + 2$$

Example 12

Solve $3\ln(x+1) = 6$

$$3\ln(x+1) = 6$$

$$\ln(x+1) = 2$$

$$e^{\ln(x+1)} = e^2$$

$$x + 1 = e^2$$

$$x = e^2 - 1$$
