

Section 1.5 (Limits)

Definition of a Limit

$$\lim_{x \rightarrow a} f(x) = L$$

The limit of $f(x)$, as x approaches a , equals L

We can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a (on either side of a) but not equal to a .

Evaluating limits using the graph of a function

Approximating Limits

$$\lim_{x \rightarrow 1} x^2 + 1$$

| x | .9 | .99 | .999 | 1.1 | 1.01 | 1.001 |
|--------|--------------|---------------|----------------|---------------|----------------|-----------------|
| $f(x)$ | $f(.9)$ | $f(.9)$ | $f(.9)$ | $f(.9)$ | $f(1.01)$ | $f(1.001)$ |
| | $= .9^2 + 1$ | $= .99^2 + 1$ | $= .999^2 + 1$ | $= 1.1^2 + 1$ | $= 1.01^2 + 1$ | $= 1.001^2 + 1$ |
| | $= 1.81$ | $= 1.98$ | $= 1.998$ | $= 2.21$ | $= 2.02$ | $= 2.002$ |

The limit as x approaches from the right or “Right hand limit”

$$\lim_{x \rightarrow 1^+} x^2 + 1 = 2$$

The limit as x approaches from the left or “Left hand limit”

$$\lim_{x \rightarrow 1^-} x^2 + 1 = 2$$

The regular limit

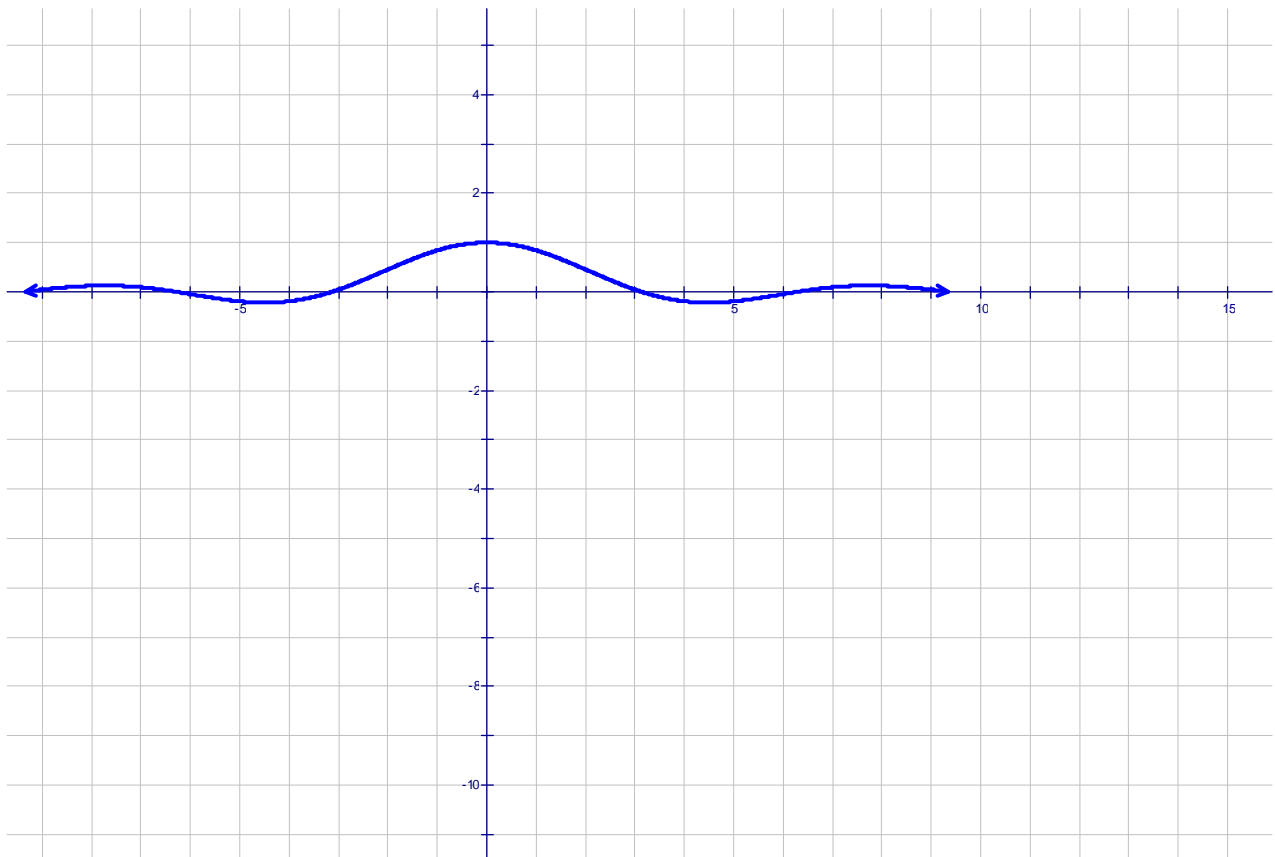
$$\lim_{x \rightarrow 1} x^2 + 1 = 2$$

Example 1

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

| | | | | | | |
|--------|---|--|--|---|--|--|
| x | $-.1$ | $-.01$ | $-.001$ | 1.1 | 1.01 | 1.001 |
| $f(x)$ | $f(-.1)$ $= \frac{\sin(-.1)}{-.1}$ $= .988$ | $f(-.01)$ $= \frac{\sin(-.01)}{-.01}$ $= .999$ | $f(-.001)$ $= \frac{\sin(-.001)}{-.001}$ $= .9999$ | $f(0.1)$ $= \frac{\sin(0.1)}{0.1}$ $= .988$ | $f(0.01)$ $= \frac{\sin(0.01)}{0.01}$ $= .999$ | $f(0.001)$ $= \frac{\sin(0.001)}{0.001}$ $= .9999$ |

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

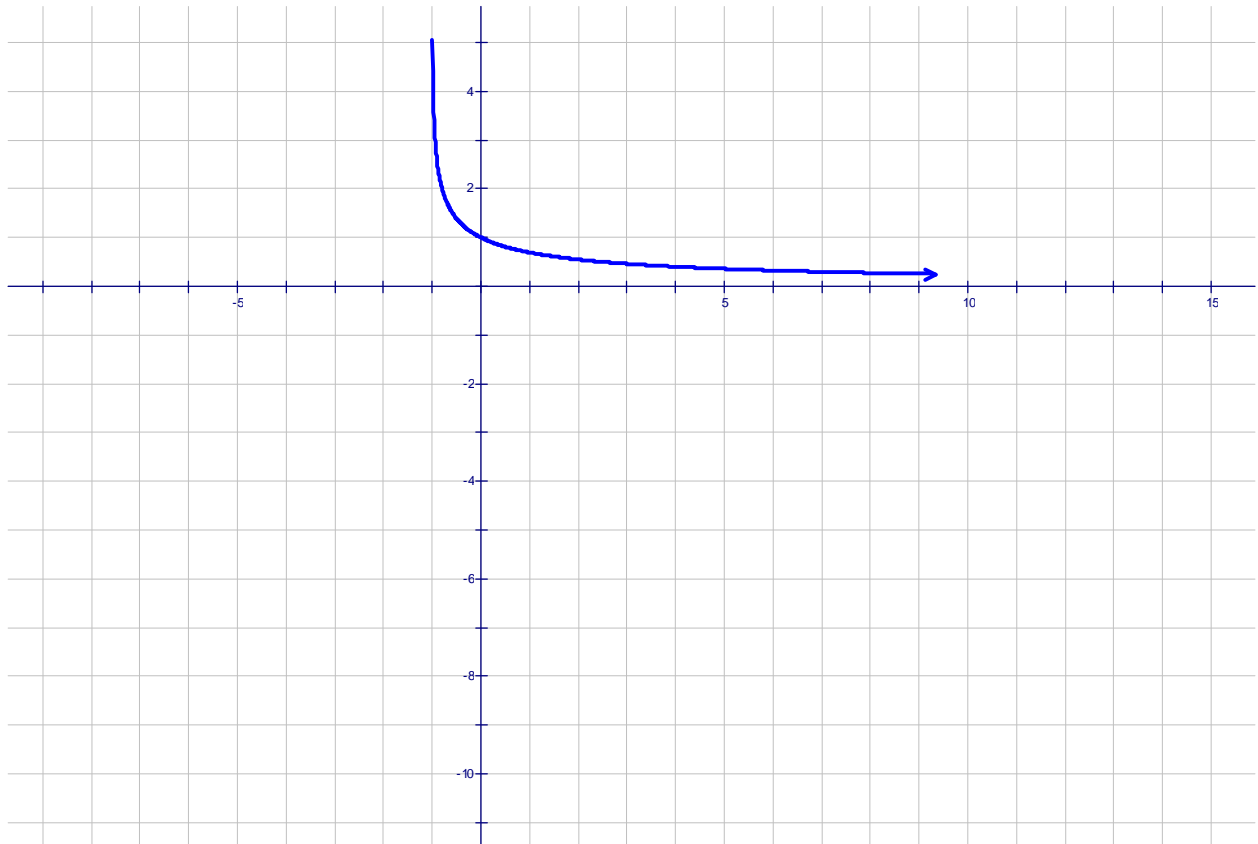


Example 2

$$\lim_{x \rightarrow 0} \frac{\ln(x+1)}{x}$$

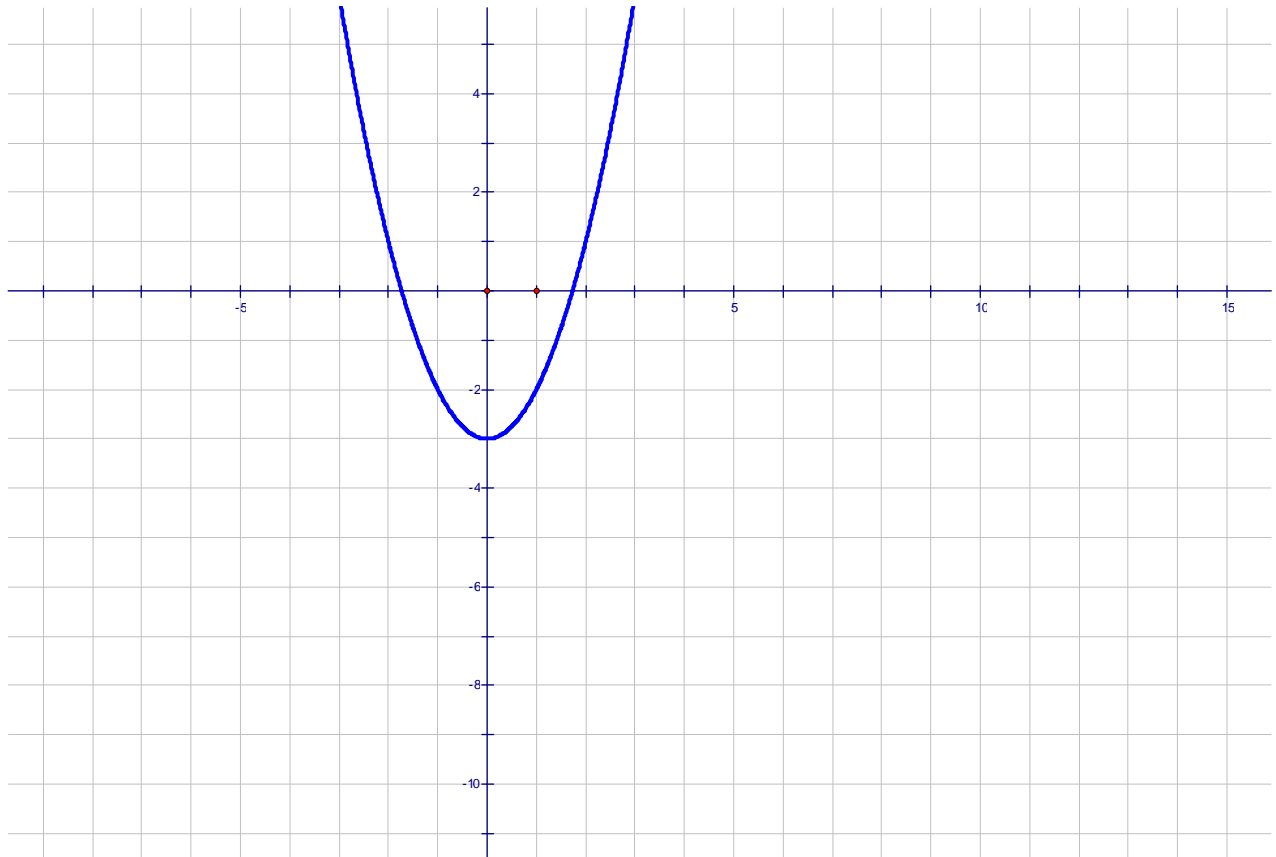
| x | .1 | .01 | .001 | -.1 | -.01 | -.001 |
|--------|---|--|--|--|--|--|
| $f(x)$ | $f(.1)$ $= \frac{\ln(.1+1)}{.1}$ $= .953$ | $f(.01)$ $= \frac{\ln(.01+1)}{.01}$ $= .995$ | $f(.001)$ $= \frac{\ln(.001+1)}{.001}$ $= .9995$ | $f(-.1)$ $= \frac{\ln(-.1+1)}{-.1}$ $= 1.05$ | $f(-.01)$ $= \frac{\ln(-.01+1)}{-.01}$ $= 1.005$ | $f(-.001)$ $= \frac{\ln(-.001+1)}{-.001}$ $= 1.0005$ |

$$\lim_{x \rightarrow 0} \frac{\ln(x+1)}{x} = 1$$



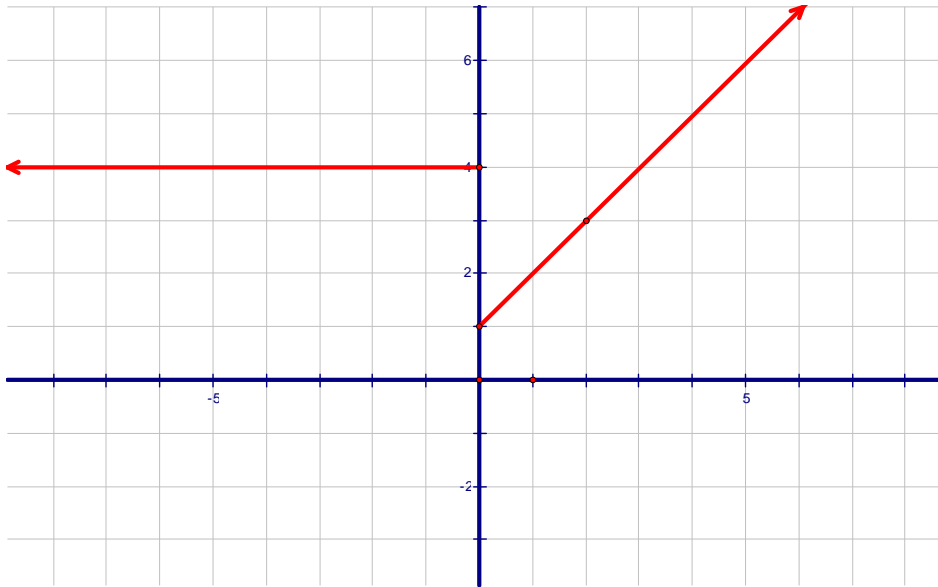
Example 3

$$\lim_{x \rightarrow 1} x^2 - 3$$



$$\lim_{x \rightarrow 1} x^2 - 3 = 2$$

Example 4



a) Find $\lim_{x \rightarrow 2} f(x)$

Solution: $\lim_{x \rightarrow 2} f(x) = 4$

b) Find $\lim_{x \rightarrow -3} f(x)$

Solution: $\lim_{x \rightarrow -3} f(x) = 1$

c) Find $\lim_{x \rightarrow 0^+} f(x)$

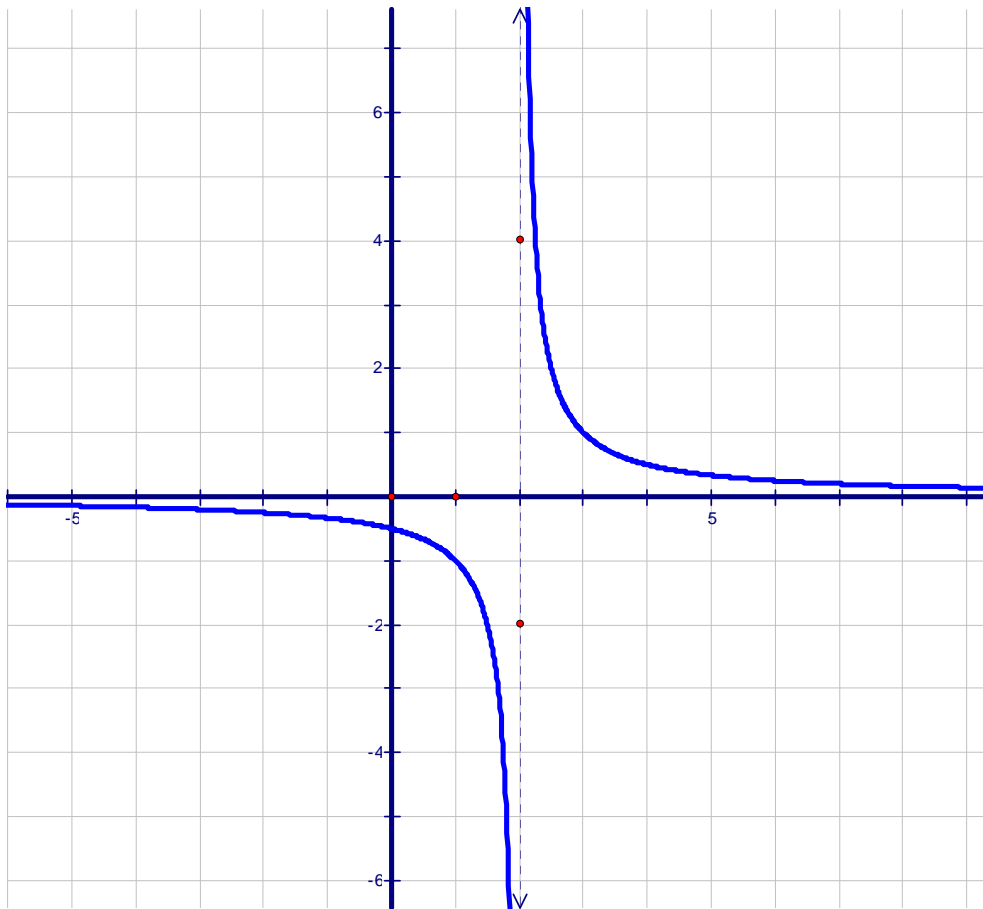
Solution $\lim_{x \rightarrow 0^+} f(x) = 1$ **Left hand limit (The y value x approaches from the left)**

d) Find $\lim_{x \rightarrow 0^-} f(x)$

Solution: $\lim_{x \rightarrow 0^-} f(x) = 4$ **Right hand limit (The y value x approaches from the right)**

e) Find $\lim_{x \rightarrow 0} f(x)$: *Does Not Exist* **The limit doesn't exist since the right hand and the left hand limits are different.**

Example 5



a) Find $\lim_{x \rightarrow 2} f(x)$:

Solution: No Limit (Limit does not exist)

b) Find $\lim_{x \rightarrow 0} f(x)$:

Solution: $\lim_{x \rightarrow 0} f(x) = \frac{1}{2}$

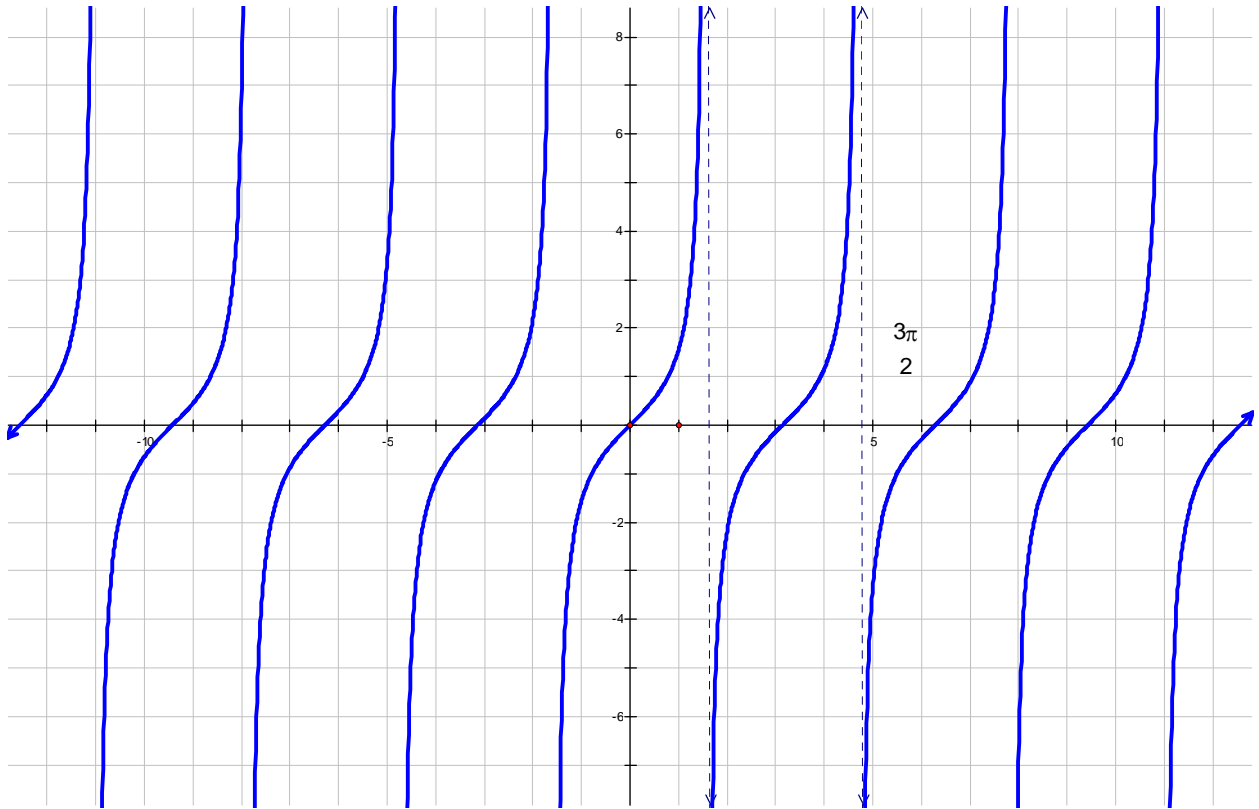
c) Find $\lim_{x \rightarrow 3} f(x)$

Solution: $\lim_{x \rightarrow 3} f(x) = 1$

Example 6

Find the limit using the graph of the function

$$\lim_{x \rightarrow \frac{3\pi}{2}} \tan x$$



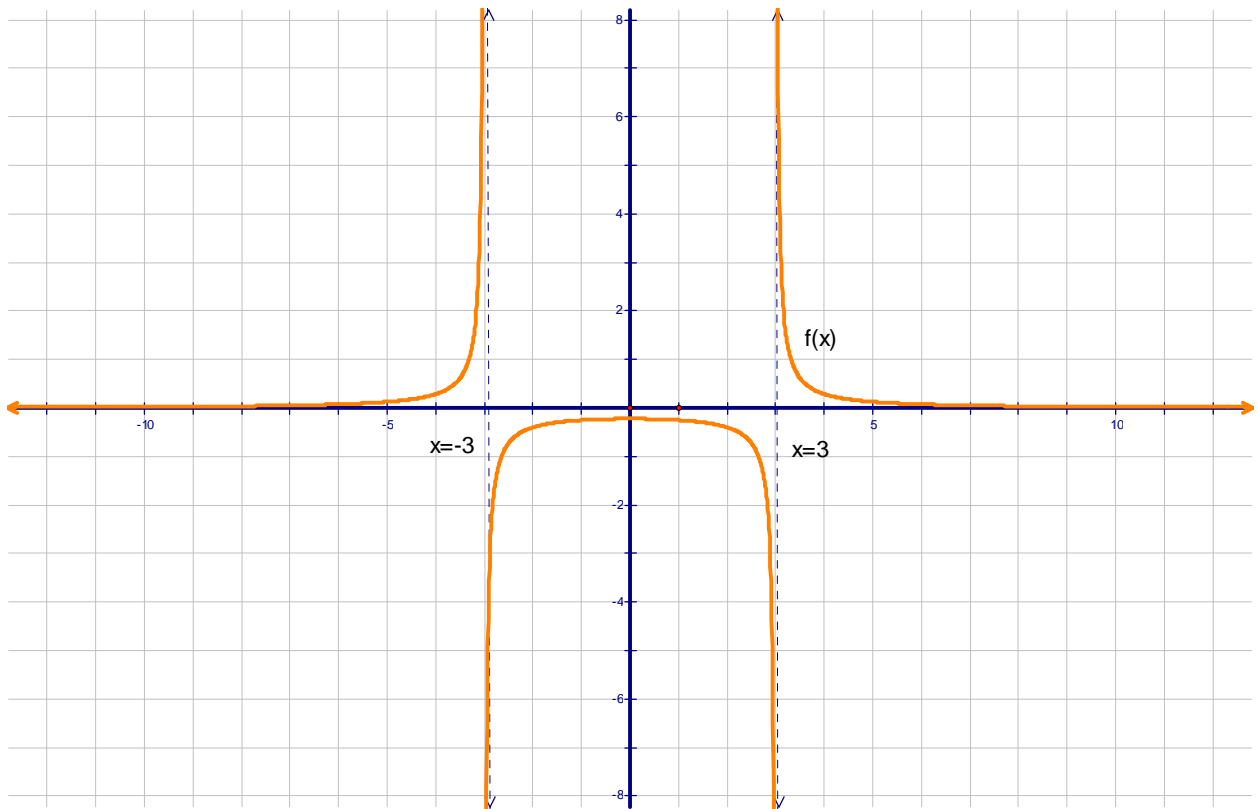
$$\lim_{x \rightarrow \frac{3\pi}{2}} \tan x \text{ (Does not exist)}$$

Example 7

Find each limit.

a) $\lim_{x \rightarrow -3} f(x) = \underline{\hspace{2cm}}$

b) $\lim_{x \rightarrow 3} f(x) = \underline{\hspace{2cm}}$



Solution:

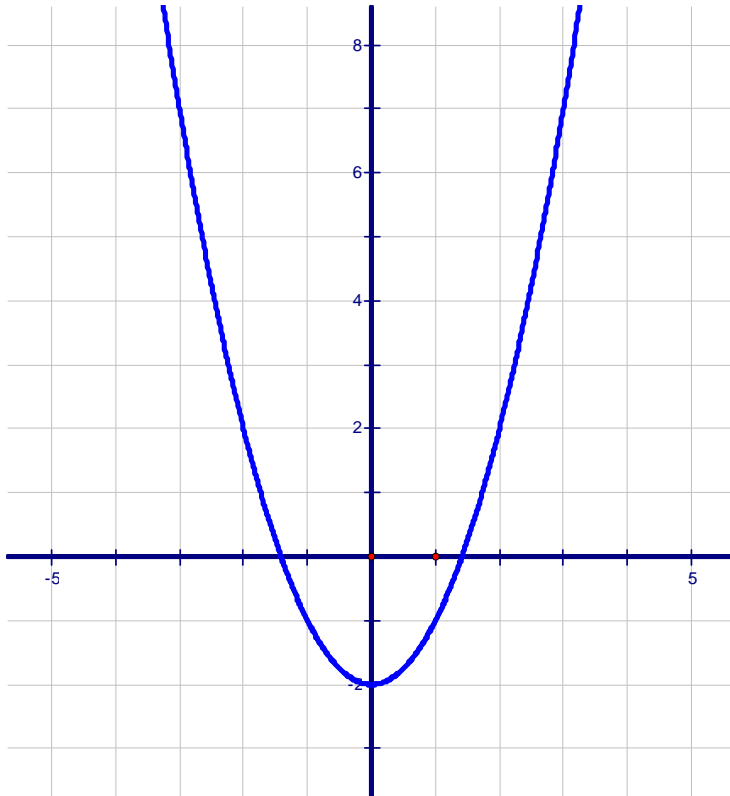
$$\lim_{x \rightarrow -3} f(x) \text{ Does not exist}$$

$$\lim_{x \rightarrow 3} f(x) \text{ Does not exist}$$

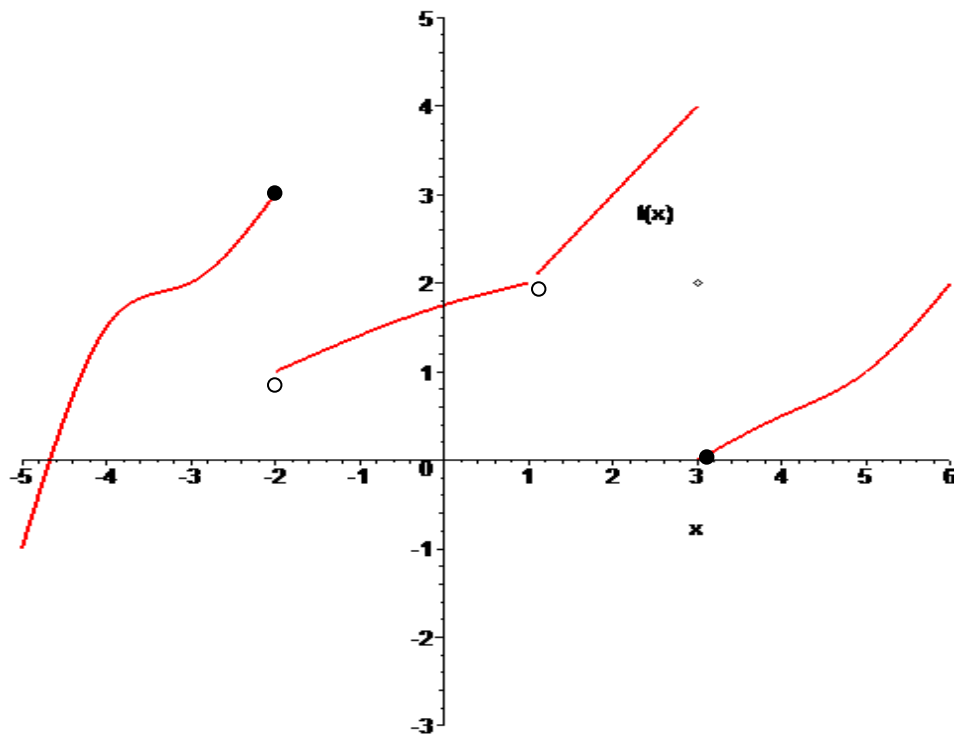
Example 7

Using the graph below evaluate each limit.

- a) Find $\lim_{x \rightarrow 2} f(x)$: **Solution** $\lim_{x \rightarrow 2} f(x) = 2$
b) Find $\lim_{x \rightarrow 0} f(x)$: **Solution** $\lim_{x \rightarrow 0} f(x) = -2$
c) Find $\lim_{x \rightarrow -2} f(x)$: **Solution** $\lim_{x \rightarrow -2} f(x) = 2$



Example 8



- a) Find $\lim_{x \rightarrow -2^-} f(x)$: **Solution** $\lim_{x \rightarrow -2^-} f(x) = 3$
- b) Find $\lim_{x \rightarrow -2^+} f(x)$: **Solution** $\lim_{x \rightarrow -2^+} f(x) = 1$
- c) Find $\lim_{x \rightarrow -2} f(x)$: **Solution** $\lim_{x \rightarrow -2} f(x)$ DNE
- d) Find $\lim_{x \rightarrow 1^-} f(x)$: **Solution** $\lim_{x \rightarrow 1^-} f(x) = 2$
- e) Find $\lim_{x \rightarrow 1^+} f(x)$: **Solution** $\lim_{x \rightarrow 1^+} f(x) = 2$
- f) Find $\lim_{x \rightarrow 3^-} f(x)$: **Solution** $\lim_{x \rightarrow 3^-} f(x) = 4$
- g) Find $\lim_{x \rightarrow 3^+} f(x)$: **Solution** $\lim_{x \rightarrow 3^+} f(x) = 0$

h) Find $\lim_{x \rightarrow 3} f(x)$: **Solution** $\lim_{x \rightarrow 3^+} f(x) DNE$

Example 9

Using the graph below evaluate each limit.

a) Find $\lim_{x \rightarrow 5} f(x)$: **Solution** $\lim_{x \rightarrow 5} f(x) = \text{Does Not Exist}$

b) Find $\lim_{x \rightarrow -7} f(x)$: **Solution** $\lim_{x \rightarrow -7} f(x) = -4$

c) Find $\lim_{x \rightarrow 0} f(x)$: **Solution** $\lim_{x \rightarrow 0} f(x) = 2$

