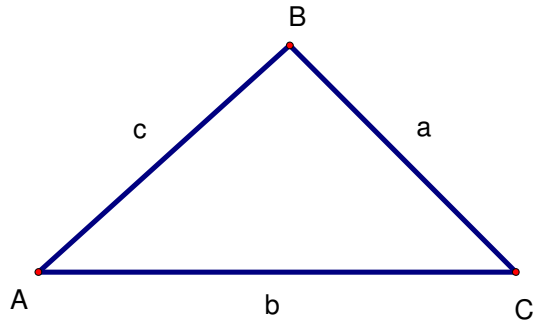
Section 6.4

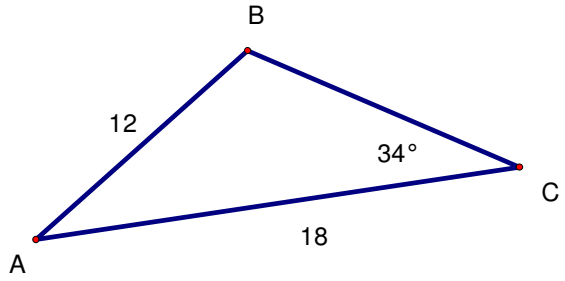
Law of Sines



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1

Find the measure of angle B



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 34}{12} = \frac{\sin B}{18}$$

$$12\sin B = 18(\sin 34)$$

$$12\sin B = 18(.559)$$

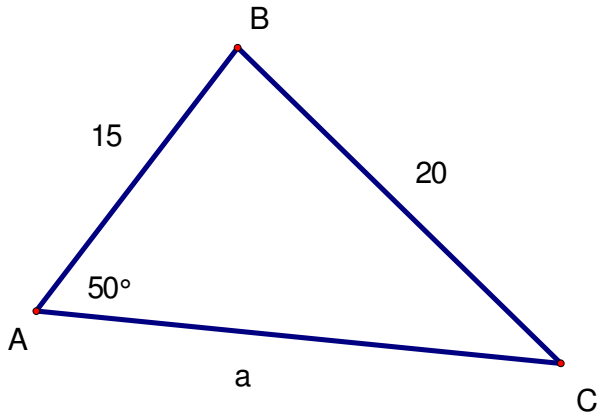
$$12\sin B = 10.1$$

$$\sin B = \frac{10.1}{12}$$

$$\sin B = .838 \Rightarrow \angle B = 57°$$

Example 2

Solve the following triangle



$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 50}{20} = \frac{\sin C}{15}$$

$$20\sin C = 15(\sin 50)$$

$$20\sin C = 15(.776)$$

$$20\sin C = 11.49$$

$$\sin C = \frac{11.49}{20}$$

$$\sin C = .575$$

$$\angle C = 36°$$

$$\angle B = 180° - (50° + 36°) = 180° - 86° = 94°$$

$$\frac{\sin 50}{20} = \frac{\sin 94}{c}$$

$$c(\sin 50) = 20(\sin 94)$$

$$c(.776) = 20(.998)$$

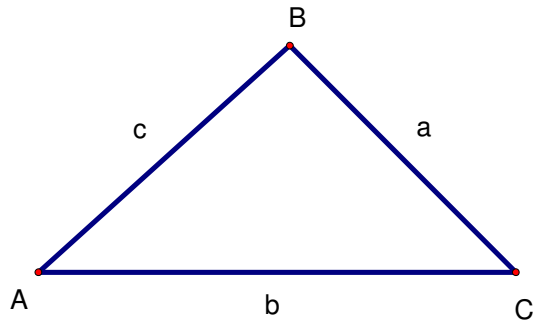
$$.776c = 19.96$$

$$c = 26$$

$$a = 20, b = 15, c = 26$$

$$\angle A = 50°, \angle B = 94°, \angle C = 36°$$

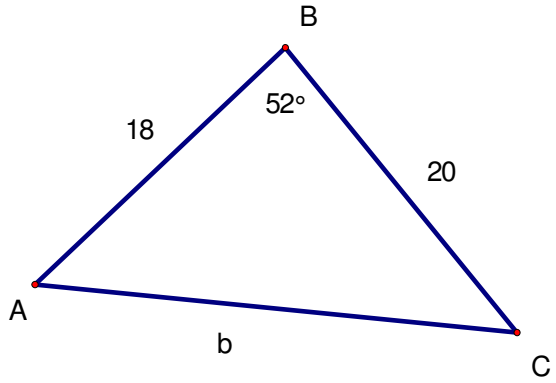
Law of Cosines



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

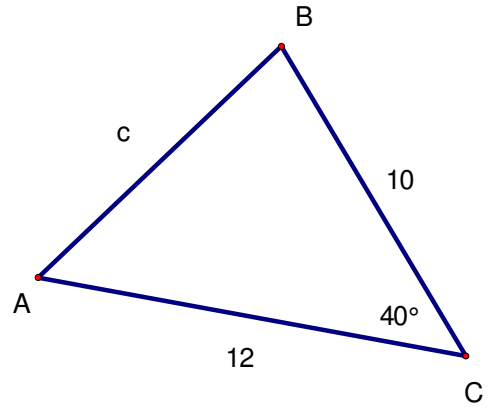
Example 3

Find the measure of b



$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 20^2 + 18^2 - 2(20)(18)\cos(52)$$
$$b^2 = 400 + 324 - 720\cos(52)$$
$$b^2 = 724 - 432$$
$$b^2 = 277$$
$$b = \sqrt{277} \approx 16.6$$

Example 4: Solve the following triangle



Find c

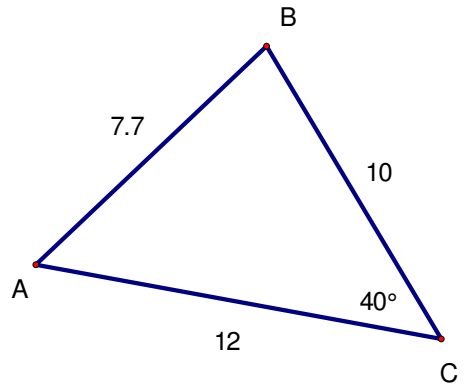
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 10^2 + 12^2 - 2(10)(12)\cos 40$

$c^2 = 100 + 144 - 240(.766)$

$c^2 = 244 - 184$

$c^2 = 60 \Rightarrow c \approx 7.7$



$$\frac{\sin 40}{7.7} = \frac{\sin A}{10}$$

$10\sin 40 = 7.7\sin A$

$7.66 = 7.7\sin A$

$$\sin A = \frac{7.66}{7.7}$$

$\sin A = .994 \Rightarrow \angle A = 84°$

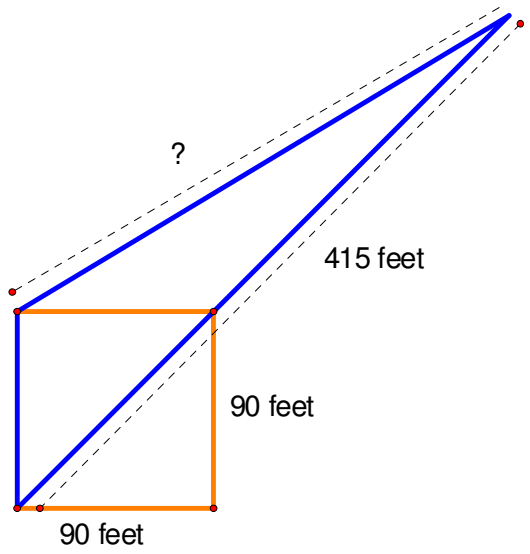
$\angle B = 180° - (84° + 40°) = 180° - 124° = 56°$

$a = 10, b = 12, c = 7.7$

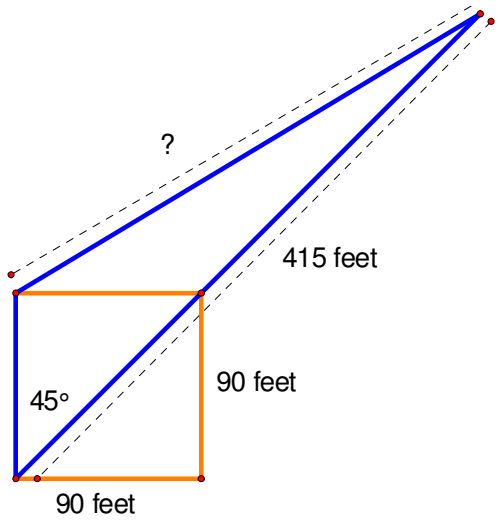
$\angle A = 84°, \angle B = 56°, \angle C = 40°$

Example 5

The baseball player catches the ball in dead center field in direct line of sight with home plate which is approximately 415 feet away. If he wants to throw a runner out at third plate, how far must he throw the baseball? **Hint:** The distance between each plate is 90 feet.



Solution:

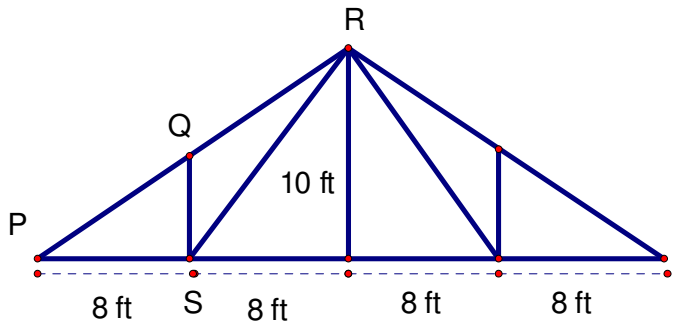


Use the law of cosines to find the solution.

$$b^2 = 90^2 + 415^2 - 2(90)(415)\cos 45$$
$$c^2 = 8100 + 172225 - 74700(.707)$$
$$c^2 = 180325 - 52813$$
$$c^2 = 127512 \Rightarrow c \approx 357 \ \ feet$$

Example 6

If Q is the midpoint of line segment $\overline{PR}$, find the lengths of the line segments $\overline{PQ}$, $\overline{QS}$, and $\overline{RS}$ on the truss rafter shown in the follow figure.



Find $\overline{PR}$ using the Pythagorean Theorem

$c^2 = 16^2 + 10^2$

$c^2 = 256 + 100$

$c^2 = 356$

$c \approx 18.9$

$\overline{PQ}$ is half the distance of $\overline{PR}$: $PQ = \frac{1}{2}PR = \frac{1}{2}(18.9) = 9.5\ ft$

Find $\overline{QS}$ using the Pythagorean Theorem

$9.5^2 = a^2 + 8^2$

$90.3 = a^2 + 64$

$a^2 = 90.3 - 64$

$a^2 = 26.3$

$a \approx 5.1$

Find $\overline{RS}$ using the Pythagorean Theorem

$c^2 = 8^2 + 10^2$

$c^2 = 64 + 100$

$c^2 = 164$

$c \approx 12.8$