

Section 5.2

Logarithms

Common Logarithms

$$\log_b a = x \Leftrightarrow b^x = a$$

$$\ln a = x \Leftrightarrow e^x = a$$

$$\log_{10} a = x \Leftrightarrow 10^x = a$$

Example 1

Write each expression as a logarithm expression

1) $3^4 = 81$

$$3^4 = 81$$

$$\log_3 81 = 4$$

2) $e^2 = 7.39$

$$e^2 = 7.39$$

$$\ln(7.39) = 2$$

3) $10^4 = 10000$

$$10^4 = 10000$$

$$\log 10000 = 4$$

Example 2

Write each expression as a exponential expression

1) $\log_2 128 = 7$

$$\log_2 128 = 7 \Rightarrow 2^7 = 128$$

$$2) \ln 2.71 = 1$$

$$\ln 2.71 = 1 \Rightarrow e^1 = 2.71$$

$$3) \log 1000 = 3$$

$$\log 1000 = 3 \Rightarrow 10^3 = 1000$$

Example 3

Solve for x by writing the equation in the exponential form.

$$a) \log_2 x = 16$$

Solution:

$$\log_4 x = 3$$

$$4^3 = x$$

$$x = 4^3$$

$$x = 64$$

$$b) \log_5(2x - 5) = 3$$

Solution:

$$\log_5(2x - 5) = 3$$

$$2x - 5 = 5^3$$

$$2x - 5 = 125$$

$$2x - 5 + 5 = 125 + 5$$

$$2x = 130$$

$$x = 65$$

$$c) \log_8 x = \frac{1}{3}$$

Solution:

$$\log_8 x = \frac{1}{3} \Rightarrow x = 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

Rule 1: $\ln e^x = x$

Rule 2: $e^{\ln x} = x$

Example 5

Show that the following functions are inverses of each other

$$f(x) = \ln x$$

$$g(x) = e^x$$

$$h(g(x)) = h(e^x) = \ln(e^x) = x$$

$$g(h(x)) = g(\ln x) = e^{\ln x} = x$$

Example 6

Show that the following functions are inverses of each other

$$f(x) = \ln x + 3$$

$$g(x) = e^{x-3}$$

$$h(g(x)) = h(e^{x-3}) = \ln(e^{x-3}) + 3 = x - 3 + 3 = x$$

$$g(h(x)) = g(\ln x - 3) = e^{\ln x - 3 + 3} = \ln e^x = x$$

Logarithm Laws

$$1) \ln AB = \ln A + \ln B \text{ or } \log(AB) = \log A + \log B$$

$$2) \ln\left(\frac{A}{B}\right) = \ln A - \ln B \text{ or } \log\left(\frac{A}{B}\right) = \log A - \log B$$

$$3) \ln(x^n) = n \ln x \text{ or } \log(x^n) = n \log x$$

Example 7

Write each expression as a single logarithm.

$$1) \ln(3x + 5) - \ln(3x - 5)$$

$$\ln(3x + 5) - \ln(3x - 5) = \ln\left(\frac{3x + 5}{3x - 5}\right)$$

$$2) 3 \ln x + 2 \ln y - 2 \ln z$$

$$3 \ln x + 2 \ln y - 2 \ln z = \ln x^3 + \ln y^2 - \ln z^2 = \ln(x^3 y^2) - \ln z^2 = \ln\left(\frac{x^3 y^2}{z^2}\right)$$

$$3) 2 \ln x + \ln(x + 4) - \ln(x + 2)$$

$$2 \ln x + \ln(x + 4) - \ln(x + 2) = \ln x^2 + \ln(x + 4) - \ln(x + 2) = \ln(x^2(x + 4)) - \ln(x + 2) = \ln\left(\frac{x^2(x + 4)}{x + 2}\right)$$

Example 8

Solve $e^{x+4} = 6$

$$e^{x+4} = 6$$

$$\ln e^{x+4} = \ln 6$$

$$(x+4)\ln e = \ln 6$$

$$x+4 = \ln 6$$

$$x = \ln 6 - 4 \approx -2.21$$

Example 9

Solve $3^x = 12$

$$3^x = 12$$

$$\ln 3^x = \ln 12$$

$$x \ln 3 = \ln 12$$

$$x = \frac{\ln 12}{\ln 3}$$

$$x \approx 2.26$$

Example 10

Solve $6^x = 88$

$$6^x = 88$$

$$\ln 6^x = \ln 88$$

$$x \ln 6 = \ln 88$$

$$x = \frac{\ln 88}{\ln 6}$$

$$x \approx 2.499$$

Example 11

Solve $e^{\ln x} = 4$

$$e^{\ln x} = 4$$

$$x = 4$$

Example 12

Find the inverse of $f(x) = e^x + 4$

$$f(x) = e^x + 4$$

$$y = e^x + 4$$

$$x = e^y + 4$$

$$x - 4 = e^y + 4 - 4$$

$$\ln(x - 4) = \ln(e^y)$$

$$\ln(x - 4) = y$$

$$f^{-1}(x) = \ln(x - 4)$$

Example 13

Find the inverse of $f(x) = \ln(x) + 3$

$$f(x) = \ln(x) + 3$$

$$y = \ln(x) + 3$$

$$x = \ln(y) + 3$$

$$x - 3 = \ln(y)$$

$$e^{x-3} = e^{\ln(y)}$$

$$e^{x-3} = y$$

$$f^{-1}(x) = e^{x-3}$$

Example 14

Find the inverse of $f(x) = e^{x+5}$

$$f(x) = e^{x+5}$$

$$y = e^{x+5}$$

$$x = e^{y+5}$$

$$\ln(x) = \ln(e^{y+5})$$

$$\ln(x) = y + 5$$

$$\ln(x) - 5 = y + 5 - 5$$

$$\ln(x) - 5 = y$$

$$f^{-1}(x) = \ln(x) - 5$$