

Math 126 Section 4.3

Derivatives of Exponential functions

The derivative of e^x

The definition of the number e

e is the number such that $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$

The derivative of the exponential function

$$f(x) = e^x$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x e^h - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x (e^h - 1)}{h} \\ &= \lim_{h \rightarrow 0} e^x \left(\frac{e^h - 1}{h} \right) = e^x \left[\lim_{h \rightarrow 0} \left(\frac{e^h - 1}{h} \right) \right] = e^x (1) = e^x \end{aligned}$$

Definition: The derivative of $f(x) = e^x$ is $f'(x) = e^x$

Example 1

Find the derivative of $y = 2e^x$

$$y = 2e^x$$

$$y' = 2e^x$$

Example 2

Find the derivative of $y = e^{2x^2}$

Use the chain rule to find the derivative

$$y = e^{2x^2}$$

$$\text{Let } u = 2x^2 \Rightarrow du = 4x$$

$$y = e^u$$

$$y' = e^u du$$

$$y' = 4xe^{2x^2}$$

Example 3

Find the derivative of $y = e^{7x}$

Use the chain rule to find the derivative

$$y = e^{7x}$$

$$y = e^u$$

$$u = 7x \Rightarrow du = 7$$

$$y' = e^u du$$

$$y' = 7e^{7x}$$

Example 4 Find the derivative of $y = 3xe^{x^2}$

Use the product with the chain rule:

$$y = 3xe^{x^2}$$

$$y' = \frac{d}{dx}(3x)e^{x^2} + \frac{d}{dx}(e^{x^2})(3x)$$

$$y' = 3e^{x^2} + 2xe^{x^2}(3x) = 3e^{x^2} + 6x^2e^{x^2}$$

Example 5 Find the derivative of $y = (x^2 + 2x + 4)e^{4x^2}$

Use the product with the chain rule:

$$y = (x^2 + 2x + 4)e^{4x^2}$$
$$y' = (x^2 + 2x + 4)e^{4x^2} + (e^{4x^2})(x^2 + 2x + 4)'$$
$$y' = (2x + 2)e^{4x^2} + 8xe^{4x^2}(x^2 + 2x + 4)$$

Example 6

Find the derivative of $f(x) = \frac{x^2}{e^{3x}}$

Use the quotient rule with the chain rule

$$f(x) = \frac{x^2}{e^{3x}}$$
$$f'(x) = \frac{(e^{3x})(x^2)' - (x^2)(e^{3x})'}{(e^{3x})^2}$$
$$f'(x) = \frac{e^{3x}(2x) - (x^2)(3e^{3x})}{e^{6x}}$$
$$f'(x) = \frac{2xe^{3x} - 3x^2e^{3x}}{e^{6x}}$$

Example 7

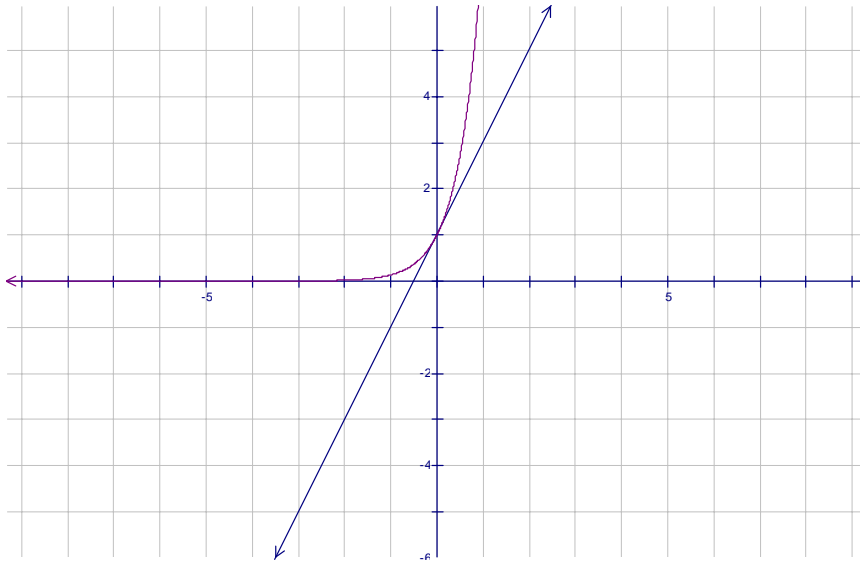
Find the derivative of $f(x) = \frac{5x}{e^{x^2}}$

$$f'(x) = \frac{(5x)(e^{x^2})' - (e^{x^2})(5x)'}{(e^{x^2})^2} = \frac{(5x)(2xe^{x^2}) - (e^{x^2})(5)}{e^{2x^2}} = \frac{10x^2e^{3x} - 5e^{x^2}}{e^{2x^2}}$$

Example 8

Find the slope of the tangent line to through the point (0,1)

$$f(x) = e^{2x}$$



Solution:

$$f(x) = e^{2x}$$

$$f'(x) = 2e^{2x}$$

$$m = f'(0) = 2e^{2(0)} = 2e^0 = 2(1) = 2$$



Example 9

Find the equation of tangent line to the function $f(x) = e^{x^2+2x}$ at the point (0,1)

$$f(x) = e^{x^2+2x}$$

$$\text{let } u = x^2 + 2x \Rightarrow du = 2x + 2$$

$$f(x) = e^u$$

$$f'(x) = e^u du = (2x + 2)e^{x^2+2x}$$

$$m = f'(0) = (2(0) + 2)e^{0^2+2(0)} = 2e^0 = 2(1) = 2$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x - 0)$$

$$y - 1 = 2x$$

$$y = 2x + 1$$

Example 10 Compound Interest

The compound interest of an account is given by the function $A = 6000e^{.04t}$ where t is measured in years? Find the rate of change for 1 year and 10 years?

Find the derivative and substitute in for t

$$A = 6000e^{.04t}$$

$$A' = 6000(.04)e^{.04t} = 240e^{.04t}$$

$$t = 1: A' = 240e^{.04(1)} = 240e^{.04} = 249.79$$

$$t = 10: A' = 240e^{.04(10)} = 240e^{.4} = 358.04$$