

## Section 1.5

### Limits

Example of limits

Find the limit:  $\lim_{x \rightarrow 1} x^2 + 1$

Find the limit of the function as x approaches 1

x	.9	.99	.999	1.1	1.01	1.001
f(x)	1.810	1.980	1.998	2.210	2.020	2.002

The values in the table seem to approach 2 as x approaches 1. There the limit of function as x approaches 1 is 2. This is written symbolically as  $\lim_{x \rightarrow 1} x^2 + 1 = 2$

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**Definition 1:**  $\lim_{x \rightarrow c} f(x) = f(c)$

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Thus the limit in the above example can be found by using definition 1,

$$\lim_{x \rightarrow 1} f(x) = f(1) = 1^2 + 1 = 1 + 1 = 2$$

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### Example 1

Find  $\lim_{x \rightarrow 3} x^3$

**Solution:**  $\lim_{x \rightarrow 3} x^3 = 3^3 = 27$

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### Example 2

Find  $\lim_{x \rightarrow -4} 3x^2 + 8x$

**Solution:**  $\lim_{x \rightarrow -4} 3x^2 + 8x = 3(-4)^2 + 8(-4) = 3(16) - 32 = 48 - 32 = 16$

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**Example 3**

Find  $\lim_{x \rightarrow 1} \frac{3}{x-1}$

**Solution:**  $\lim_{x \rightarrow 1} \frac{3}{x-1} = \frac{3}{1-1} = \frac{3}{0}$  *Undefined*

**Limit does not exist**

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**Example 4**

Find  $\lim_{x \rightarrow 5} \sqrt{x-7}$

**Solution:**  $\lim_{x \rightarrow 5} \sqrt{x-7} = \sqrt{5-7} = \sqrt{-2}$  *Undefined*

**Limit does not exist**

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**Example 5**

Find  $\lim_{x \rightarrow 3} \frac{x-3}{x^2-9}$

**Solution:**  $\lim_{x \rightarrow 3} \frac{x-3}{x^2-9} = \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(x+3)} = \lim_{x \rightarrow 3} \frac{1}{x+3} = \frac{1}{3+3} = \frac{1}{6}$

**In example 5, if you substitute 3 in for x before you reduce the polynomial down,**

**you will get an undetermined form.  $\frac{0}{0}$**

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**Example 6**

$$\text{Find } \lim_{x \rightarrow 2} \frac{2-x}{x^2-4}$$

$$\text{Solution: } \lim_{x \rightarrow 2} \frac{2-x}{x^2-4} = \lim_{x \rightarrow 3} \frac{-(x-2)}{(x-2)(x+2)} = \lim_{x \rightarrow 3} \frac{-1}{x+2} = \frac{-1}{2+2} = -\frac{1}{4}$$

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**Example 7**

$$\text{Find } \lim_{t \rightarrow 1} \frac{t^2+t-2}{t^2-1}$$

$$\text{Solution: } \lim_{t \rightarrow 1} \frac{t^2+t-2}{t^2-1} = \lim_{t \rightarrow 1} \frac{(t+2)(t-1)}{(t-1)(t+1)} = \lim_{t \rightarrow 3} \frac{t+2}{t+1} = \frac{1+2}{1+1} = \frac{3}{2}$$

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**Example 8**

$$\text{Find } \lim_{x \rightarrow -1} \frac{4x-5}{3-x}$$

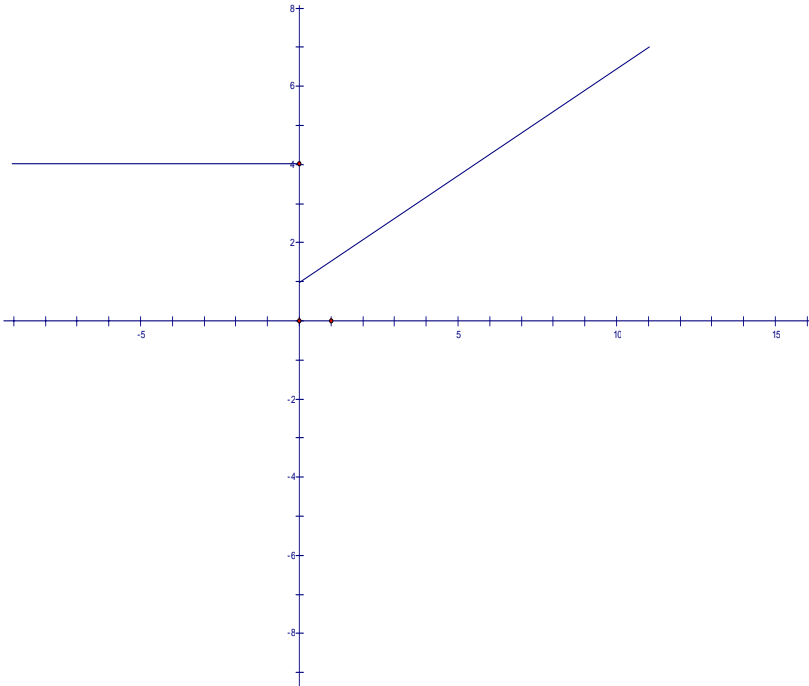
$$\text{Solution: } \lim_{x \rightarrow -1} \frac{4x-5}{3-x} = \frac{4(-1)-5}{3-(-1)} = \frac{-4-5}{4} = -\frac{9}{4}$$

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## Evaluating a limit from a graph

### Example 9



a) Find  $\lim_{x \rightarrow 2} f(x)$      **Solution:**  $\lim_{x \rightarrow 2} f(x) = 2$

b) Find  $\lim_{x \rightarrow -3} f(x)$      **Solution:**  $\lim_{x \rightarrow -3} f(x) = 4$

c) Find  $\lim_{x \rightarrow 0^+} f(x)$

**Solution**  $\lim_{x \rightarrow 0^+} f(x) = 1$  **Left hand limit (The y value x approaches from the left)**

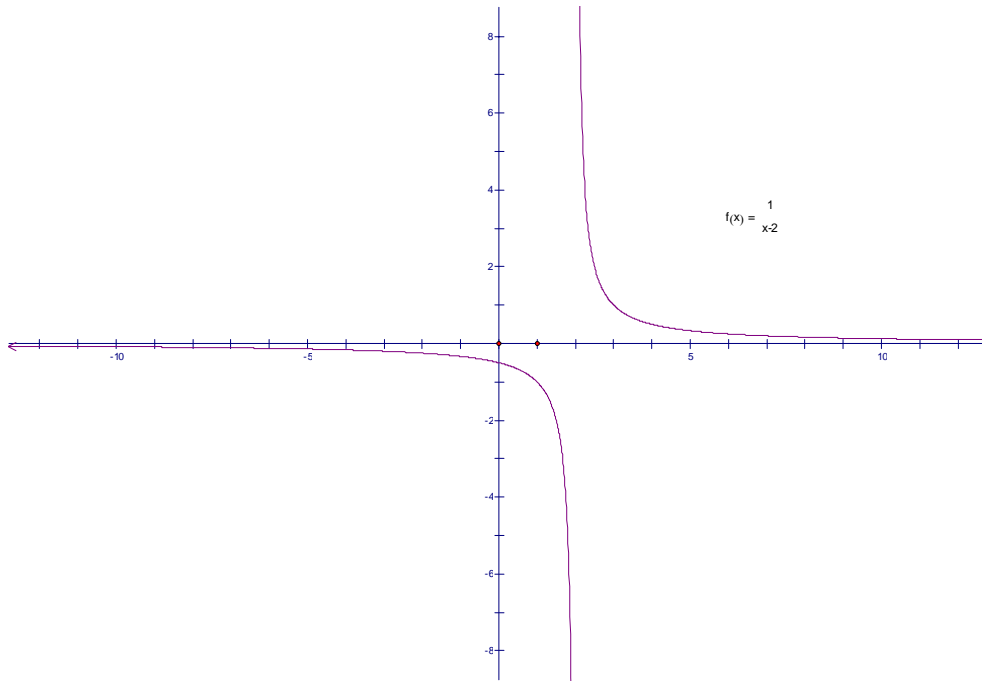
d) Find  $\lim_{x \rightarrow 0^-} f(x)$

**Solution:**  $\lim_{x \rightarrow 0^-} f(x) = 4$  **Right hand limit (The y value x approaches from the right)**

e) Find  $\lim_{x \rightarrow 0} f(x)$ : *Does Not Exist* **The limit doesn't exist since the right hand and the left hand limits are different.**

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### Example 10



a) Find  $\lim_{x \rightarrow 2} f(x)$ :

**Solution:** No Limit (Limit does not exist)

b) Find  $\lim_{x \rightarrow 0} f(x)$ :

**Solution:**  $\lim_{x \rightarrow 0} f(x) = \frac{1}{2}$

c) Find  $\lim_{x \rightarrow 3} f(x)$

**Solution:**  $\lim_{x \rightarrow 3} f(x) = 1$