

Math 126
Exponent Notes

Exponent Laws

Properties for Exponents

Rule 1 $a^0 = 1$ example $4^0 = 1$

Rule 2 $a^{-n} = \frac{1}{a^n}$ example $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Rule 3 $a^{\frac{1}{n}} = \sqrt[n]{a}$ example $5^{\frac{1}{3}} = \sqrt[3]{5}$

Rule 4 $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ example $4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

Examples

1) Simplify x^{-5}

Solution: $x^{-5} = \frac{1}{x^5}$

2) Simplify $4^{\frac{1}{2}}$

Solution: $4^{\frac{1}{2}} = \sqrt[2]{4} = \sqrt{4} = 2$

3) Simplify $8^{\frac{1}{3}}$

Solution: $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

4) Simplify $8^{\frac{2}{3}}$

Solution: $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = (2)^2 = 4$

5) Simplify 6^0

Solution: $6^0 = 1$

Operations with Exponents

Rule 5 $a^n a^m = a^{n+m}$ example $x^4 x^7 = x^{4+7} = x^{11}$

Rule 6 $\frac{a^n}{a^m} = a^{n-m}$ example $\frac{x^9}{x^4} = x^5$

Rule 7 $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ example $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

Rule 8 $(ab)^n = a^n b^n$ example $(xy)^3 = x^3 y^3$

Rule 9 $(a^n)^m = a^{nm}$ example $(x^3)^4 = x^{12}$

Examples

1) Simplify $x^{13} x^5$

Solution: $x^{13} x^5 = x^{13+5} = x^{18}$

2) Simplify $(x^5)^3$

Solution: $(x^5)^3 = x^{5 \cdot 3} = x^{15}$

3) Simplify $\frac{y^{12}}{y^5}$

Solution: $\frac{y^{12}}{y^5} = y^{12-5} = y^7$

4) Simplify $(2x^3)(12x^{12})$

Solution: $(2x^3)(12x^{12}) = 2 \cdot 12x^{3+12} = 24x^{15}$

5) Simplify $(2x^2 y^3)^3$

Solution: $(2x^2 y^3)^3 = 2^3 (x^2)^3 (y^3)^3 = 8x^6 y^9$

Basic Factoring

Factoring out a Greatest Common Factor

In each example of this type of factoring, the objective is to factor out the largest factor or greatest common factor from the given expression.

Examples

1) Factor $x^2 - 4x$

Solution: In this example the largest expression that can be divided out of both terms is x . If you factor out an x , you will get the following expression.

$$x^2 - 4x$$

$$x(x - 4)$$

2) Factor $m^3 - 4m^2 + 6m$

Solution:

$$m^3 - 4m^2 + 6m$$

$$m(m^2 - 4m + 6)$$

3) Factor $5x^4 + 10x^3 + 25x^2$

Solution:

$$5x^4 + 10x^3 + 25x^2$$

$$5x^2(x^2 + 2x - 5)$$

Examples

1) Factor $5m^3 - 15m^2$

Solution: $5m^3 - 15m^2 = 5m^2(m - 3)$

2) Factor $x^3 + 2x^2 + 5x$

Solution: $x^3 + 2x^2 + 5x = x(x^2 + 2x + 5)$

How to factor a trinomial as two binomials

Factoring a trinomial is the same as working the **FOIL** process

So, here is a short review of **FOIL**

$$\begin{aligned}(x+3)(x+4) \\ x \cdot x + 4x + 3x + 3 \cdot 4 \\ x^2 + 7x + 12\end{aligned}$$

Example of factoring a trinomial as two binomials

Factor $x^2 + 10x + 25$

In this example you want to find two numbers that multiply to get 25 and add to get 10.

By using x as the first entry in each binomial you get:

$$x^2 + 10x + 25 = (x+5)(x+5)$$

Here are some similar examples

1) **Factor** $x^2 - 10x + 24$

Solution: $x^2 - 10x + 24 = (x-6)(x-4)$

2) **Factor** $x^2 - 2x - 35$

Solution: $x^2 - 2x - 35 = (x-7)(x+5)$

3) **Factor** $x^2 + 6x + 9$

Solution: $x^2 + 6x + 9 = (x+3)(x+3)$

How to factor a difference of two squares

General form of a difference of two squares

$$a^2 - b^2 = (a + b)(a - b)$$

Examples

1) Factor $m^2 - 36$

Answer: $(m - 6)(m + 6)$

2) Factor $b^2 - 16$

Answer: $(b - 4)(b + 4)$

1) Factor $d^2 - 144$

Solution: $d^2 - 144 = (d - 12)(d + 12)$

2) Factor $m^2 - 4$

Solution: $m^2 - 4 = (m - 2)(m + 2)$

3) Factor $25x^2 - 16y^2$

Solution: $25x^2 - 16y^2 = (5x)^2 - (4y)^2 = (5x - 4y)(5x + 4y)$

Sum and Difference of two cubes

General form of a difference of two cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

General form of a sum of two cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Examples

1) Factor $x^3 - 27$

Answer: $x^3 - 27 = x^3 - 3^3 = (x - 3)(x^2 + 3x + 3^2) = (x - 3)(x^2 + 3x + 9)$

2) Factor $m^3 - 8$

Answer: $m^3 + 8 = m^3 + 2^3 = (m + 2)(m^2 - 2m + 2^2) = (m + 2)(m^2 - 2m + 4)$

3) Factor $8a^3 - 27b^3$

Answer:

$$8a^3 - 27b^3 = (2a)^3 - (3b)^3 = (2a - 3b)((2a)^2 + (2a)(3b) + (3b)^2) = (2a - 3b)(4a^2 + 6ab + 9b^2)$$

Examples

4) Factor $m^3 - 64$

Solution: $m^3 - 64 = m^3 - 4^3 = (m - 4)(m^2 + 4m + 4^2) = (m^2 - 4m + 16)$

5) Factor $a^3 - 27b^3$

Solution:

$$a^3 - 27b^3$$

$$a^3 - (3b)^3$$

$$(a - 3b)(a^2 + (a)(3b) + (3b)^2)$$

$$(a - 3b)(a^2 + 3ab + 9b^2)$$