

Section 4.2
Natural Exponential Functions

The exponential function

Limit definition of e^x

$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

x	.1	.01	.001
$f(x) = (1+x)^{\frac{1}{x}}$	$f(.1) = (1+.1)^{\frac{1}{.1}}$	$f(.01) = (1+.01)^{\frac{1}{.01}}$	$f(.001) = (1+.001)^{\frac{1}{.001}}$
	$f(.1) = (1.1)^{10}$	$f(.01) = (1.01)^{100}$	$f(.001) = (1.001)^{1000}$
	$f(.1) = 2.59$	$f(.01) = 2.70$	$f(.001) = 2.717$

x	-.1	-.01	-.001
$f(x) = (1+x)^{\frac{1}{x}}$	$f(-.1) = (1-.1)^{\frac{1}{-.1}}$	$f(-.01) = (1-.01)^{\frac{1}{-.01}}$	$f(-.001) = (1-.001)^{\frac{1}{-.001}}$
	$f(-.1) = (.9)^{-10}$	$f(-.01) = (.99)^{-100}$	$f(-.001) = (.999)^{-1000}$
	$f(-.1) = 2.86$	$f(-.01) = 2.73$	$f(-.001) = 2.719$

Actual approximation for $e \approx 2.718$

Example 1

Evaluate each expression

a) e^2

$$e^2 \approx 2.718$$

b) $e^2 e^4$

$$e^2 e^4 = e^6$$

c) $\frac{e^4}{e^3}$

$$\frac{e^4}{e^3} = e$$

$$\text{d) } \frac{e^4}{e^{-3}}$$

$$\frac{e^4}{e^{-3}} = e^{4-(-3)} = e^7$$

Solving exponential equations

Example 2

Solve $e^{2x} = e^9$

$$e^{2x} = e^9$$

$$2x = 9$$

$$\frac{2x}{2} = \frac{9}{2}$$

$$x = \frac{9}{2}$$

Example 3

Solve $e^{\sqrt{x}} = e^2$

$$e^{\sqrt{x}} = e^2$$

$$\sqrt{x} = 2$$

$$(\sqrt{x})^2 = 2^2$$

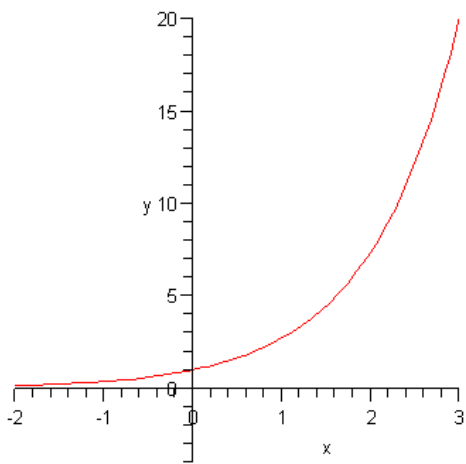
$$x = 4$$

Graphing the exponential function

Example 4

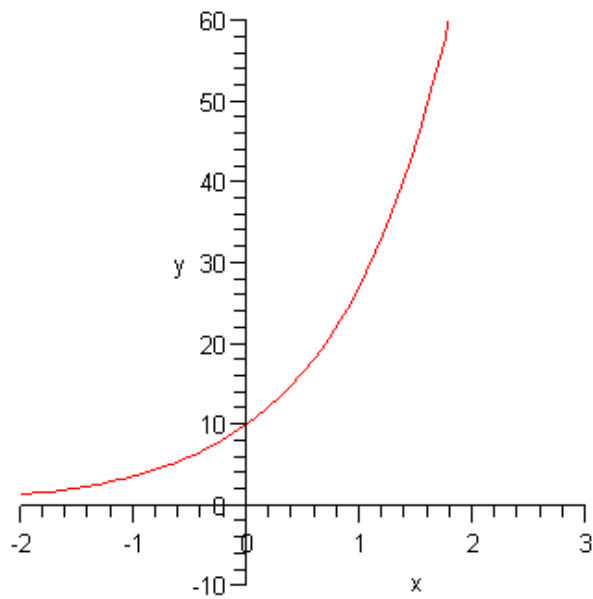
Graph $y = e^x$

x	y
-2	$y = e^{-2} = .14$
-1	$y = e^{-1} = .37$
0	$y = e^0 = 1$
1	$y = e^1 = 2.7$
2	$y = e^2 = 7.4$



Example 4Graph $y = 10e^{-2x}$

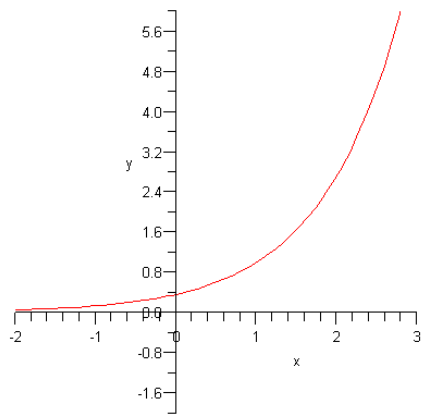
x	y
-2	$y = 10e^{-2(-2)} = 10e^{-4} = 6.7$
-1	$y = 10e^{-2(-1)} = 10e^{-2} = 8.2$
0	$y = 10e^{-2(0)} = 10e^0 = 10$
1	$y = 10e^{-2(1)} = 10e^{-2} = 12.2$
2	$y = 10e^{-2(2)} = 10e^4 = 14.9$



Other graph of the exponential function

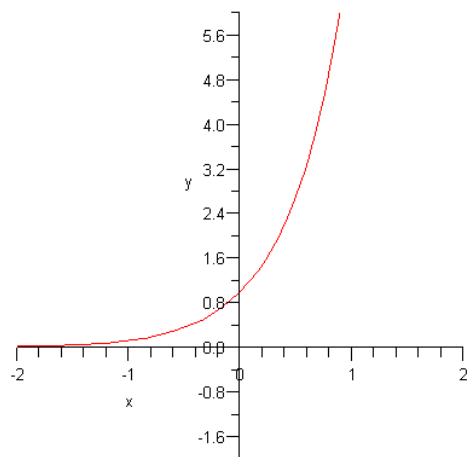
Example 5

Graph $y = e^{x-1}$



Example 6

Graph $y = e^{2x}$



Continuous interest

$A =$ accumulated Balance, $P =$ principle, $r =$ rate, $t =$ time

$$A = Pe^{rt}$$

Example 7

Suppose you deposit \$10,000 in a saving bond that compounds interest continuously at a rate of 5 % per month. Find out the balance after 10 years.

$$A = ?$$

$$P = \$10,000$$

$$r = .05$$

$$t = 10$$

$$A = 10,000e^{.05(10)} = 10,000e^{.5} = \$16487.21$$

Compound Interest Formula

$P = \text{principle}$

$R = \text{rate}$

$T = \text{time}$

$n = \text{number of times interest is computed}$

$A = \text{Accumulated Balance}$

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Example 8

Mike invests \$5000 in a special saving bond that compounds interest quarterly at a rate of 5% per year. How much money would Mike have in this saving bond after 10 years?

$$P = \$5000, T = 10, R = .05, n = 4 \text{ (quarterly)}$$

$$A = \$5000 \left(1 + \frac{.05}{4} \right)^{4 \cdot 10} = \$5000(1 + .0125)^{40} = \$5000(1.0125)^{40} = \$5000(1.6436) = \$8218.09$$

Present Value

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^t}$$

Example 9

How much should be deposited in an account paying 5% interest compound monthly in order to have a new balance of \$22,000 in 5 years?

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^t} = \frac{22000}{\left(1 + \frac{.05}{12}\right)^5} = \frac{22000}{(1.0416667)^5} = \frac{22000}{1.21007} = 21,547.34$$

Example 10 (See page 271 #32)

n	1	2	4	12	356	Continuous Compounding
A						

$$P = \$3500, \quad r = 5\%, \quad t = 10 \text{ years}$$

$$n = 1$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{1}\right)^{1(10)} = 3500(1.05)^{10} = \$5701.13$$

$$n = 2$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{2}\right)^{2(10)} = 3500(1.025)^{20} = \$5735.15$$

$$n = 4$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{4}\right)^{4(10)} = 3500(1.0125)^{40} = \$5752.66$$

$$n = 12$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3500\left(1 + \frac{.05}{12}\right)^{12(10)} = 3500(1.00416667)^{120} = \$5764.53$$

$$n = 365$$

$$A = P \left(1 + \frac{r}{n} \right)^{nt} = 3500 \left(1 + \frac{.05}{365} \right)^{365(10)} = 3500(1.000137)^{3650} = \$5770.32$$

Continuous Interest

$$A = Pe^{rt} = \$3500e^{(.05)(10)} = \$3500e^{.5} = \$3770.52$$
