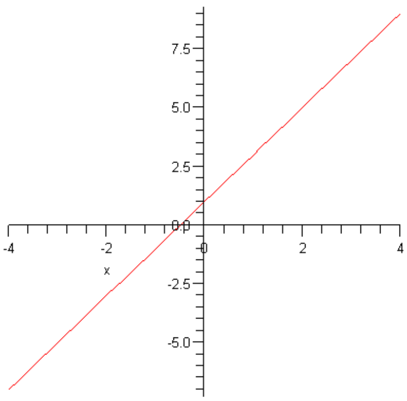
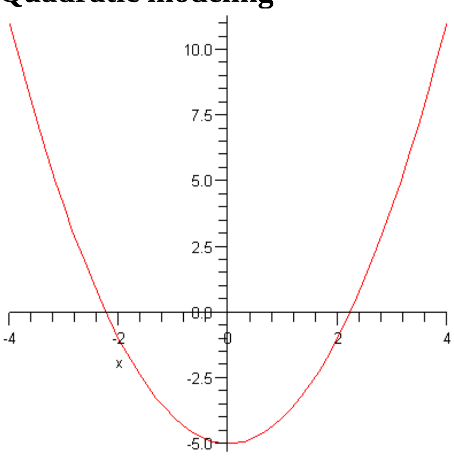
Chapter 9 Modeling

Types of models

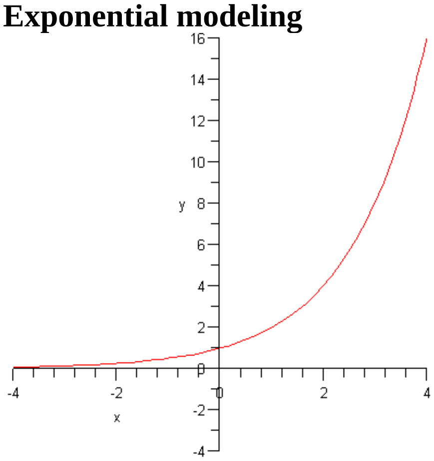
Linear Modeling



Quadratic modeling



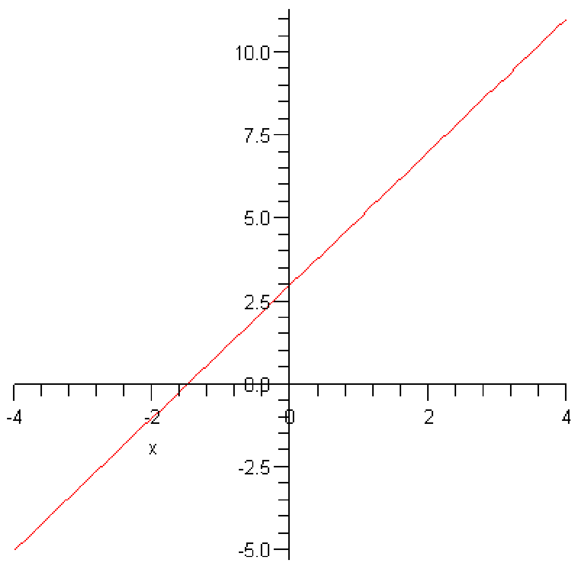
Exponential modeling



Section 9.1

Linear Models

Standard linear model



Slope-intercept formula

$$y = mx + b$$
$$m = slope$$
$$b = y - \text{int}ercept$$

Slope

$$Slope = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x}$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Graphing a linear function

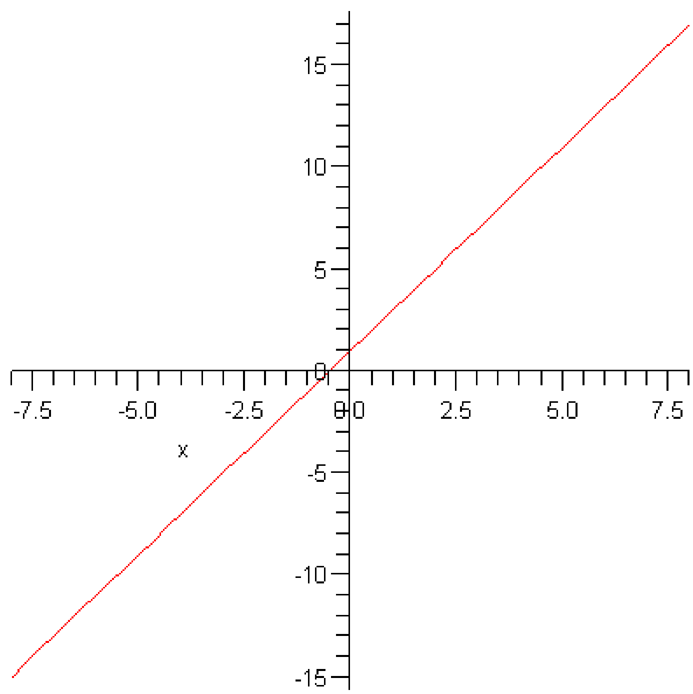
Example 1

Graph $y = 2x + 1$

Step1 make a table of values from the equation $y = 2x + 1$

x	y
2	$y = 2(2) + 1 = 4 + 1 = 5$
4	$y = 2(4) + 1 = 8 + 1 = 9$
6	$y = 2(6) + 1 = 12 + 1 = 13$
8	$y = 2(8) + 1 = 16 + 1 = 17$

Make a scatter or point plot of the points above



Example 2 Graph $2x+3y=6$

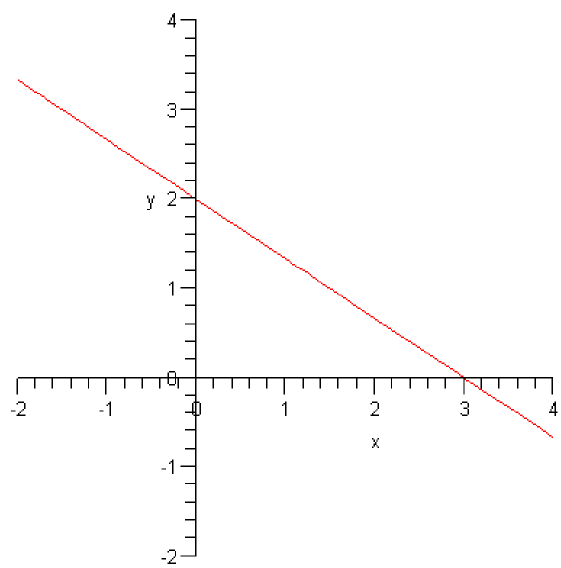
Solve for y to put the equation in the correct form

$$2x+3y=6$$
$$2x-2x+3y=-2x+6$$
$$3y=-2x+6$$
$$\frac{3y}{3}=\frac{-2x}{3}+\frac{6}{3}$$
$$y=-\frac{2}{3}x+2$$

Make a table of values

x	Y
1	$y=-\frac{2}{3}(1)+2=-.7+2=1.3$
2	$y=-\frac{2}{3}(2)=-1.3+2=.7$
3	$y=-\frac{2}{3}(3)+2=-2+2=0$
4	$y=-\frac{2}{3}(4)+2=-2.7+2=-.7$

Make a graph using the values in the above table



Example 3

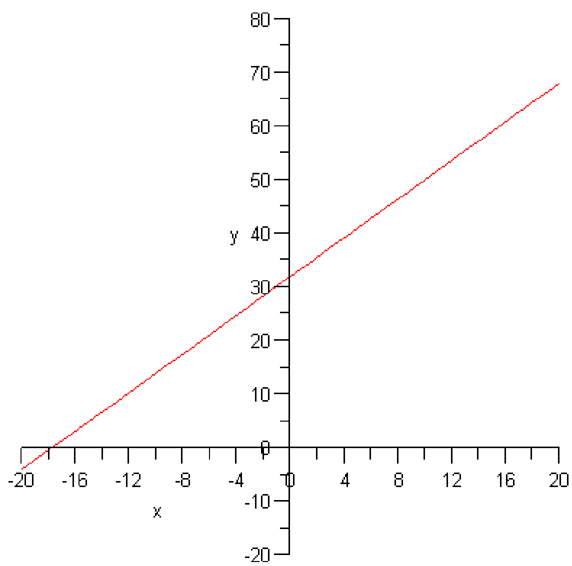
Temperature conversion

The following linear model converts temperature in Fahrenheit to Celsius. $F = \frac{9}{5}C + 32$

Use the formula to answer the following questions

a) Sketch a graph of $F = \frac{9}{5}C + 32$

F	$F = \frac{9}{5}C + 32$
10	$F = \frac{9}{5}(10) + 32 = 9(2) + 32 = 18 + 32 = 50$
20	$F = \frac{9}{5}(20) + 32 = 9(4) + 32 = 36 + 32 = 68$
30	$F = \frac{9}{5}(30) + 32 = 9(6) + 32 = 54 + 32 = 86$
40	$F = \frac{9}{5}(40) + 32 = 9(8) + 32 = 72 + 32 = 104$



b) Use the model to convert 120 degrees Celsius to degrees Fahrenheit.

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(120) + 32$$

$$F = 216 + 32$$

$$F = 248$$

c) Use the model to convert 212 degrees Fahrenheit to Celsius.

$$F = \frac{9}{5}C + 32$$

$$212 = \frac{9}{5}C + 32$$

$$212 - 32 = \frac{9}{5}C + 32 - 32$$

$$180 = \frac{9}{5}C$$

$$\frac{5}{9}(180) = \frac{5}{9} \cdot \frac{9}{5}C$$

20

$$C = 100^0 C$$

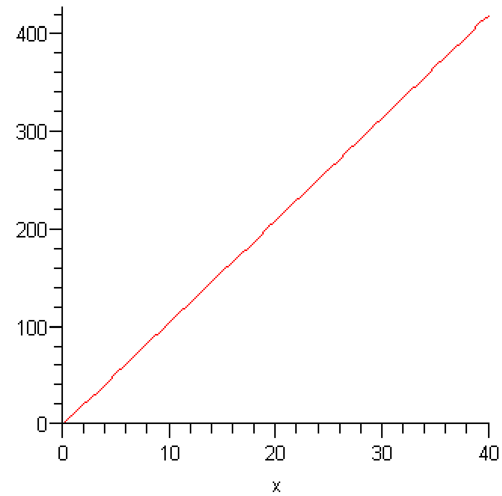
Example 4

The revenue of a company that makes Pokémon figures is given by the formula $R = 10.50x$ where x represents the number of Pokémon sold.

a) Graph the linear model $R = 10.50x$

x	$R = 10.50x$
10	$R = 10.50(10) = 105$
20	$R = 10.50(20) = 210$
30	$R = 10.50(30) = 315$
40	$R = 10.50(40) = 420$

Graph of $R = 10.50x$



b) Use the model to calculate the revenue for selling 50 Pokémon figures

$x = 50$

$R = 10.50x = 10.5(50) = \$525.00$

c) What is the slope

$m = \$10.50$

d) What is the meaning of the slope?

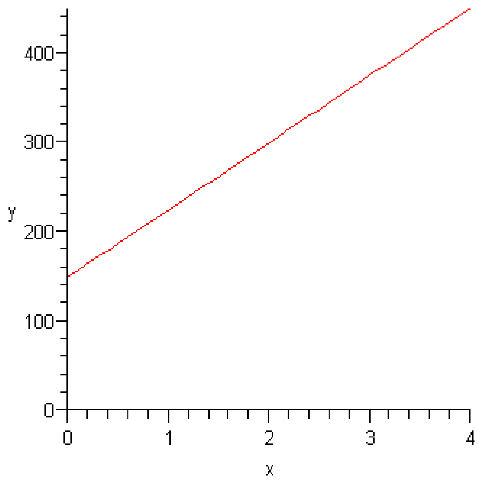
Cost per unit sold
Revenue made per Pokémon figure solid

Example 5

A salesperson is paid \$150 plus \$75 per sale each week. The model $S = 75x + 150$ is used to calculate the salesperson's weekly salary where x is the number of sales per week.

a) Graph $S = 75x + 150$

X	$S = 75x + 150$
1	$S = 75(1) + 150 = 75 + 150 = 225$
2	$S = 75(2) + 150 = 150 + 150 = 300$
3	$S = 75(3) + 150 = 225 + 150 = 375$
4	$S = 75(4) + 150 = 300 + 150 = 450$



b) Use the model to calculate the salespersons weekly salary if he/she makes 10 sales.

$S = 75(10) + 150 = 750 + 150 = \$900.00$

c) What is the slope of the equation

$$m = 75 \frac{\$}{sale}$$

d) What is the meaning of the slope

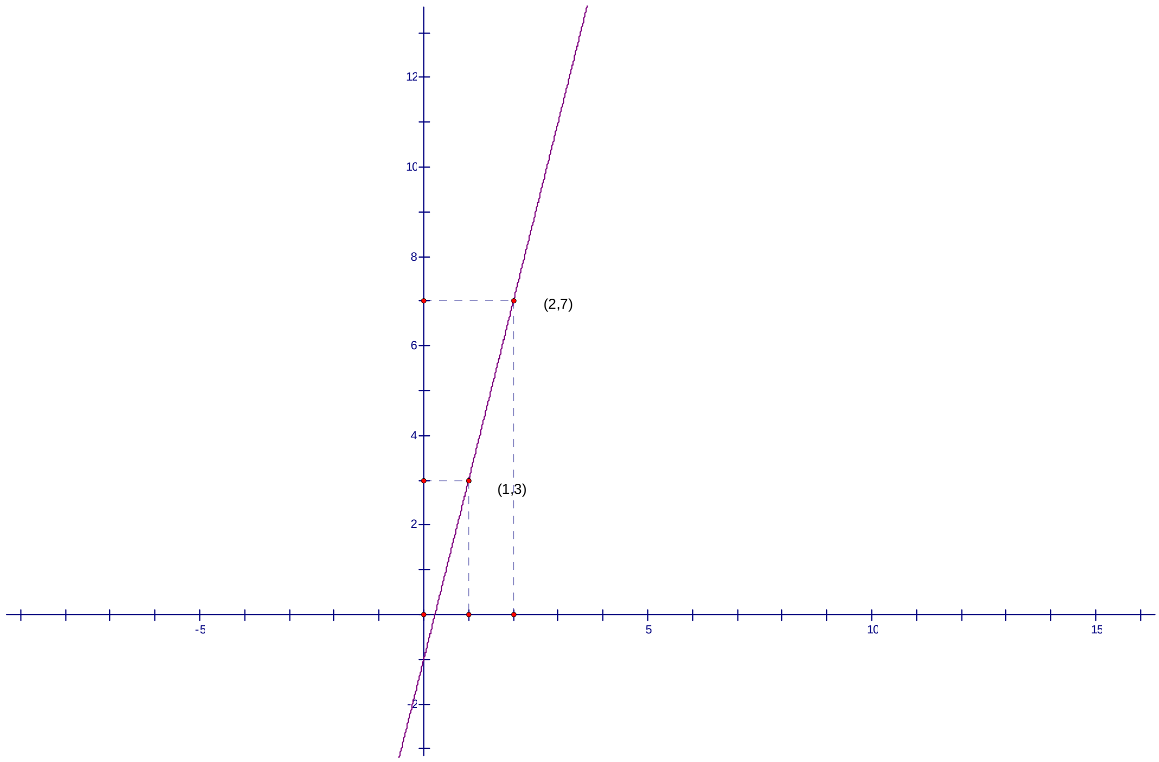
Dollars earned per each sale

Example 6

Given the following data sketch a graph

Time	Temperature
1 min	$3^0 C$
2 min	$7^0 C$
3 min	$11^0 C$
4 min	$14^0 C$

Sketch a graph of the given data and then compute the slope of the resulting line.



Use the points (1,3) and (2,7) in the above graph to compute the slope

$$m = \frac{7-3}{2-1} = \frac{4}{1} = 4$$

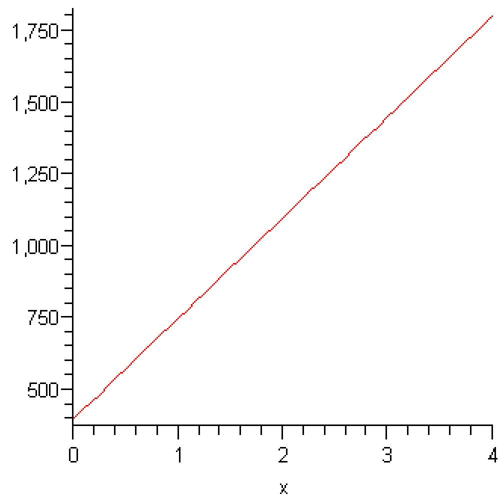
Example 7

The profit (P) of a small business is given by the following function $P = 350t + 400$ where (t) is the time in months. Sketch a graph (P) where $1 \le t \le 12$

Make a table of values

x	y
1	$P = 350(1) + 400 = 350 + 400 = 750$
2	$P = 350(2) + 400 = 700 + 400 = 1100$
3	$P = 350(3) + 400 = 1050 + 400 = 1450$
4	$P = 350(4) + 400 = 1400 + 400 = 1800$

Sketch $P = 350t + 400$



Assignment
Problem Set 1
Math 116

1) The revenue of a company that makes DVD players is given by the formula $R = 35x$ where x represents the number of DVD players sold.

a) Graph the linear model $R = 35x$
b) Use the model to calculate the revenue for selling 100 DVD players
c) What is the slope?
d) What is the meaning of the slope?

2) A salesperson is paid $120 plus $50 per sale each week. The model $S = 50x + 120$ is used to calculate the salesperson's weekly salary where x is the number of sales per week.

a) Graph $S = 50x + 120$
b) Use the model to calculate the salespersons weekly salary if he/she makes 10 sales.
c) What is the slope of the equation?
d) What is the meaning of the slope?

3) Graph $y = 3x + 4$
4) Graph $2x + 2y = 4$

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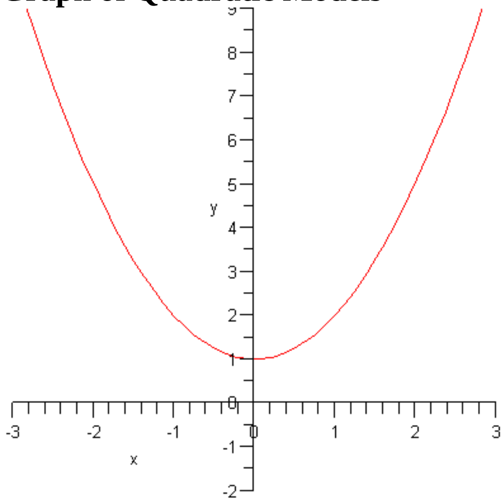
Quadratic Models

Graph of Quadratic Models



The parabola

A **quadratic function** is a function where the graph is a parabola and an equation of the form: $y = ax^2 + bx + c$ where $a \neq 0$

The x coordinate vertex is given by the equation: $x = -\dfrac{b}{2a}$

Example 1

Find the vertex and x-intercepts, and then make a sketch of the parabola.

$$y = x^2 - 2x$$
$$a = 1, b = -2$$
$$x = -\frac{-2}{2(1)} = \frac{2}{2} = 1$$

x-intercepts

$x^2 - 2x = 0$

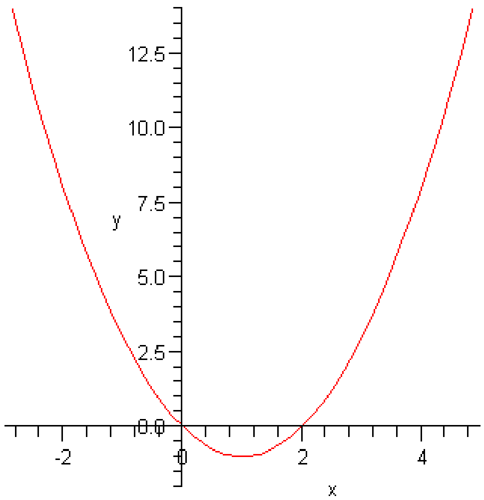
$x(x-2) = 0$

$x = 0 \;\; or \;\; x - 2 = 0$

$x = 0 \; or \;\; x = 2$

$(0,0) \; and \; (2,0)$

Graph



Example 2

$y = x^2 - 3x$

Vertex

$$x = -\frac{-3}{2(1)} = \frac{3}{2}$$

x-intercepts

$x^2 - 3x = 0$

$x(x-3) = 0$

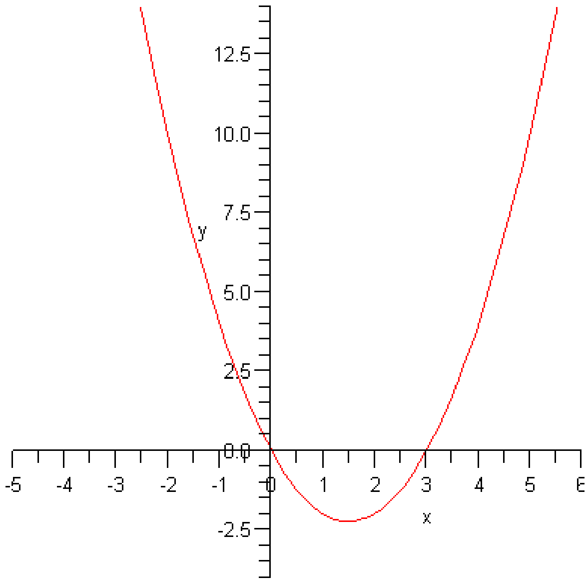
$x = 0 \; or \; x - 3 = 0$

$x = 0 \quad x - 3 = 0$

$\qquad\qquad x = 3$

$(0,0) \;\; and \; (3,0)$

Graph of the function



Example 3

$vertex:$

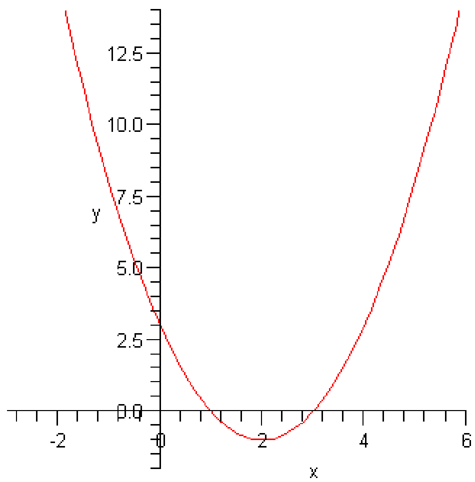
$$x = -\frac{-(-4)}{2(1)} = 2$$

$y - coordinate: \; y = 2^2 - 4(2) + 3 = -1$

x-intercepts

$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x - 3 = 0 \quad or \quad x - 1 = 0$$
$$x - 3 + 3 = 0 + 3 \quad x - 1 + 1 = 0 + 1$$
$$x = 3 \qquad\qquad x = 1$$
$$(1,0) \; and \; (3,0)$$

Graph



Example 4

$y = x^2 - 3$

$a = 1, c = -3$

$x = -\frac{0}{2(1)} = -\frac{0}{2} = 0$

x-intercepts

$x^2 - 3 = 0$

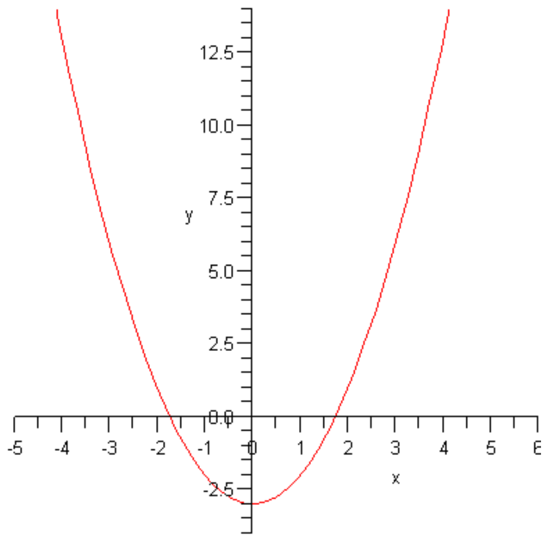
$x^2 = 3$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

$(\sqrt{3},0)$ *and* $(-\sqrt{3},0)$

Graph



Example 5

Graph $y = x^2 + 2x - 3 = 0$

$$vertex: \; x = -\frac{2}{2(1)} = -\frac{2}{2} = -1$$

$$y = (-1)^2 + 2(-1) - 3 = 3 - 3 = 0$$

$$Vertex: (-1,0)$$

x-intercepts

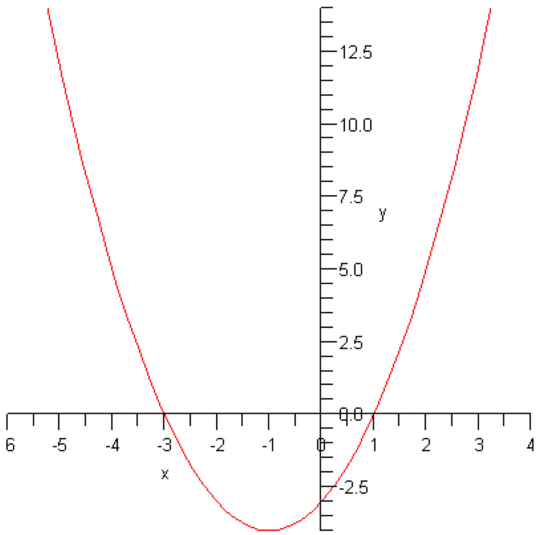
$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x + 3 = 0 \quad or \quad x - 1 = 0$$

$$x = -3 \quad or \quad x = 1$$

$$(-3,0),(1,0)$$



Using the quadratic formula to solve an equation

The Quadratic Formula

The solution to the equation $y = ax^2 + bx + c$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 6

Solve $x^2 + 5x - 7 = 0$

$a = 1$
$b = 5$
$c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 28}}{2} = \frac{-5 \pm \sqrt{53}}{2}$$

Example 7

Solve $x^2 + 7x - 9 = 0$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-9)}}{2(7)} = \frac{-7 \pm \sqrt{49 + 36}}{2(7)} = \frac{-7 \pm \sqrt{85}}{14}$$

Problem Set 2

Find the vertex and x-intercepts, and then make a sketch of the parabola

1) $y = x^2 - 5x + 6$
2) $y = x^2 + 3x - 4$
3) $y = x^2 - 5x$
4) $y = x^2 - 9$
5) $y = x^2 + 7x + 6$

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8) Find the vertex, graph, and x intercepts of each parabola

$y = -x^2 + 6x - 5.5$
$a = -1$
$b = 6$
$c = -5.5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-5.5)}}{2(-1)} = \frac{-6 \pm \sqrt{36 - 22}}{-2} = \frac{-6 \pm \sqrt{14}}{-2}$$

x-intercepts

$(-6 + \sqrt{14}, 0)$ *and* $(-6 - \sqrt{14}, 0)$

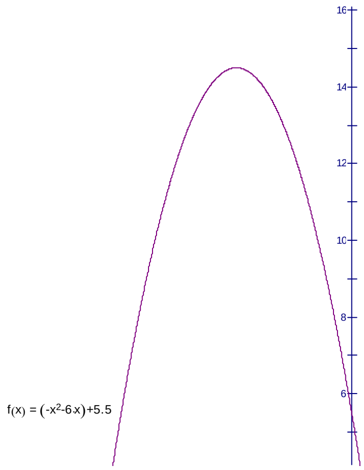
Vertex

$$x = -\frac{6}{2(-1)} = 3$$

$y - coordinate$

$y = -3^2 + 6(3) - 5.5 = -9 + 18 - 5.5 = 4.5$

$(3, 4.5)$



12) At a local frog jumping contest. Rivet's jump can be approximated by the equation $y = -\frac{1}{6}x^2 + 2x$ and Croak's jump can be approximate by $y = -\frac{1}{2}x^2 + 4x$, where x = the length of jump in feet and y = the height of the jump in feet.

a) Which frog can jump higher

Rivet's vertex: $x = -\frac{2}{2\left(-\frac{1}{6}\right)} = -\frac{2}{-\frac{1}{3}} = 6$ Height: $y = -\frac{1}{6}(6)^2 + 2(6) = -6 + 12 = 6\ ft$

Croak's vertex: $x = -\frac{4}{2\left(-\frac{1}{2}\right)} = \frac{-4}{-1} = 4$ Height: $y = -\frac{1}{2}(4)^2 + 4(4) = -8 + 16 = 8\ ft$

Croak can jump higher at 8 feet

b) Which frog can jump farther

Rivet's can jump farther at 2(6 ft) = 12 feet

Example 8

The path of a ball thrown by a boy is given in yards by the equation $y = -.04x^2 + 1.5x$ where x is the horizontal distance the ball travels and y is the height of the ball. Find the maximum height of the ball in yards.

Find the vertex of the ball

$$x = -\frac{1.5}{2(-.04)} = \frac{1.5}{.08} = 18.75$$

$$y = -.04(18.75)^2 + 1.5(18.75) = -14.1 + 28.1 = 14 \ \textit{yards}$$

Example 9

The path of a cannon ball is given in feet by the equation $y = -.1x^2 + 6.0x$ where x is the horizontal distance the ball travels and y is the height of the cannon ball. Find the maximum height of the cannon ball in feet.

Find the vertex of the cannon ball.

$$x = -\frac{6.0}{2(-.1)} = -\frac{6.0}{-.2} = 30$$

$$y = -.1(30)^2 + 6(30) = -90 + 180 = 90 \ \textit{feet}$$

Problem Set 3

1) Find the vertex and x-intercepts of the given parabola, and then make a sketch of the parabola. $y = 2x^2 - 4x$

2) The path of a ball thrown by a baseball player is given by the equation $y = -.02x^2 + 1.6x$ where x is the horizontal distance the ball travels and y is the height of the ball. Find the maximum height of the ball in yards.

3) The path of a ball thrown by a boy is given by the equation $y = -.06x^2 + 1.8x$ where x is the horizontal distance the ball travels and y is the height of the ball. Find the maximum height of the ball in yards.

4) The path of a cannon ball is given by the equation $y = -.05x^2 + 6.0x$ where x is the horizontal distance the ball travels and y is the height of the cannon ball. Find the maximum height of the cannon ball in feet.

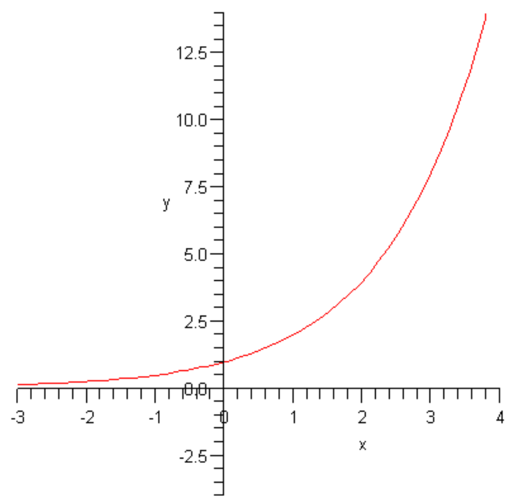
5) The path of a cannon ball is given by the equation $y = -.1x^2 + 8.0x$ where x is the horizontal distance the ball travels and y is the height of the cannon ball. Find the maximum height of the cannon ball in feet.

Exponential Models

Example of an exponential model

1) Graph $y = 2^x$

x	y
-2	$y = (2)^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$y = (2)^{-1} = \frac{1}{2^1} = \frac{1}{2}$
0	$y = 2^0 = 1$
1	$y = 2^1 = 2$
2	$y = 2^2 = 4$



The Euler number

$e \approx 2.718.......$

Example 1

Expressions with the exponential function

1) Simplify $e^2 \approx 7.389$

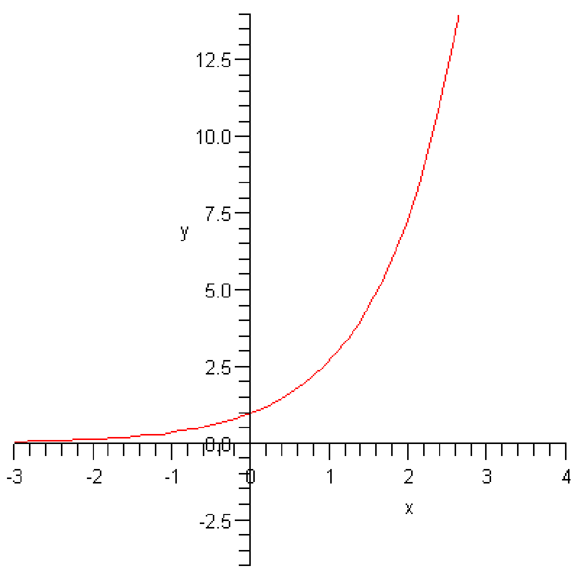
2) Simplify $e^{-3} \approx .049$

Graphs of the exponential functions

Example 2

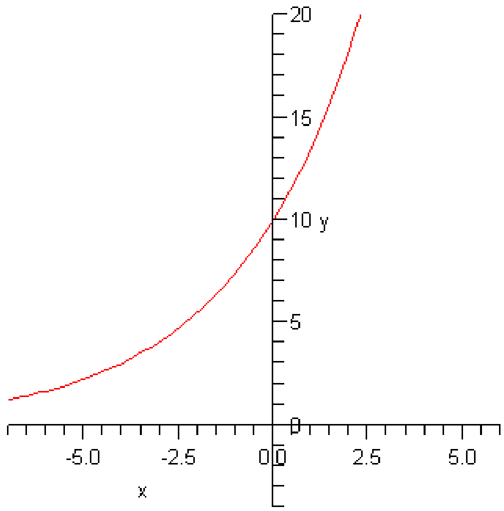
Graph $y = e^x$

x	y
-2	$y = e^{-2} = .14$
-1	$y = e^{-1} = .37$
0	$y = e^0 = 1$
1	$y = e^1 = 2.7$
2	$y = e^2 = 7.3$



Example 3

Graph $y = 10e^{.3x}$



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14) Suppose Parker Brothers determines that the profit P for a board game that is on the market for t years is given by the following equation.

$$P = 6000 + 20000(3)^{-0.2t}$$

b) What is the profit after 25 years?

$$P = 6000 + 20000(3)^{-0.2t}$$
$$P = 6000 + 20000(3)^{-0.2(25)}$$
$$P = 6000 + 20000(3)^{-5}$$
$$P = 6000 + 20000(.004115)$$
$$P = 6000 + 82.30$$
$$P = \$6082.30$$

Exponential Models

Exponential Growth

$P = P_0(1+r)^t$

$P = New\ Value$

$P_0 = Original\ Value$

$r = rate$

$t = time$

Example 1

The population of the United States is 290 million. What would be the population of the U. S. be in 20 years, if its population would growth at a steady rate of 1.2 % for 20 years?

$P = P_0(1+r)^t$

$P_0 = 290{,}000{,}000$

$r = .7\% = .012$

$t = 20$

$P = 290000000(1+.012)^{20} = 290000000(1.012)^{20} = 368135965$

Example 2

The population of Blacksburg, Virginia is 41,000. What would be the population in 10 years, if Blacksburg’s population would grow at a rate of 1.1 % per year?

$P = P_0(1+r)^t$

$P_0 = 41000$

$r = 1.1\% = .011$

$t = 10$

$P = 41000(1+.011)^{10} = 41000(1.011)^{10} = 45740$

Example 3

In 1995 the United States had greenhouse emissions of about 1400 million tons, where as China had greenhouse emissions of about 850 million tons. If in the next 25 years China greenhouse emission grew by 4 percent and the U. S. greenhouse emission grew by 1.3 percent, what would the emissions in tons for both countries in 2020?

$$U.S.\ Emissions\ in\ 2020$$
$$P = P_0(1+r)^t$$
$$P_0 = 1400\ million$$
$$r = 1.3\% = .013$$
$$t = 25$$
$$P = 1400(1+.013)^{25} = 1400(1.013)^{25} = 1934\ million\ tons$$

$$China's\ Emissions\ in\ 2020$$
$$P = P_0(1+r)^t$$
$$P_0 = 850\ million$$
$$r = 4.0\% = .04$$
$$t = 25$$
$$P = 850(1+.04)^{25} = 850(1.04)^{25} = 2266\ million\ tons$$

Example 4

Using the exponential growth formula, find the amount of money that you would have in a bank account if you deposited $3,000 in the account for 15 years at 1.1 % interest rate?

$$P = P_0(1+r)^t$$
$$P_0 = 3000$$
$$r = 1.1\% = .011$$
$$t = 15$$
$$P = 3000(1+.011)^{15} = 3000(1.011)^{15} = \$3482.91$$

Exponential decay

Exponential decay models are use to measure radioactive decay, decreasing populations, Half-life, and other elements that fit an exponential model. Again, the one variable in an exponential decay models in found in the exponent.

Exponential Decay Formula

$$P = P_0(1-r)^t$$
$$P = New\ Value$$
$$P_0 = Original\ Value$$
$$r = rate$$
$$t = time$$

Example 5

A certain population of black bears in the eastern United States has been decreasing by 3.1 percent per year. If this trend keeps up, what will be the population of bears in 20 years if there are currently 1000 bears?

$$P = P_0(1-r)^t$$
$$P_0 = 1000$$
$$r = 3.1\% = .031$$
$$t = 20$$
$$P = 1000(1-.031)^{20} = 1000(.969)^{20} = 533$$

Example 6

A certain isotope decreases at a rate of 5% per year. It there is currently 340 grams of the isotope, how many grams of the isotope will there be in 20 years?

$$P = P_0(1-r)^t$$
$$P_0 = 340$$
$$r = 5\% = .05$$
$$t = 25$$
$$P = 340(1-.05)^{20} = 340(.95)^{20} = 122\ grams$$

Section 9.4

Basic Logarithms

Definition of a Logarithm

$\log_b a = x \quad \Leftrightarrow \quad b^x = a$

Example 1

i) Write $3^5 = 243$ as a logarithmic expression.

$3^5 = 243 \Rightarrow \log_3 243 = 5$

ii) Write $5^4 = 625$ as a logarithmic expression.

$5^4 = 625 \Rightarrow \log_5 625 = 4$

Example 2

i) Write $\log_4 16 = 2$ as exponential expression.

$\log_4 16 = 2 \Rightarrow 4^2 = 16$

ii) Write $\log_{10} 10{,}000 = 4$ as an exponential expression.

$\log_{10} 10{,}000 = 4 \Rightarrow 10^4 = 10{,}000$

Log base ten

Another way of writing $\log_{10} 1000$ is $\log 1000$.

The way we find the answer to $\log 1000$ is to ask the question of 10 raised to what power gives you 1000? Since we know that $10^4 = 1000$, the answer is 4.

Example 3

i) Find log 100,000

Since $10^5 = 100{,}000$, $\log 100{,}000 = 5$

ii) Find log 100

Since $10^2 = 100$, $\log 100 = 2$

Example 4

Use a scientific calculator to evaluate the following logarithms

i) log 567

Answer: log 567 = 2.754

ii) log 30890

Answer: log 30890 = 4.490

iii) log 456782

Answer: log 456782 = 5.660

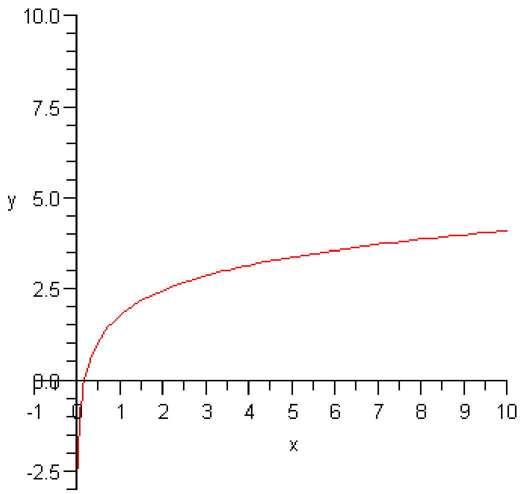
Graph of basic logarithms

Example 5

Graph $y = \log 6x$

x	y
2	$y = \log(6(2)) = \log(12) = 1.07$
10	$y = \log(6(10)) = \log(60) = 1.8$
20	$y = \log(6(20)) = \log(120) = 2.1$
40	$y = \log(6(40)) = \log(240) = 2.4$

Plot the given values from the table gives the following graph

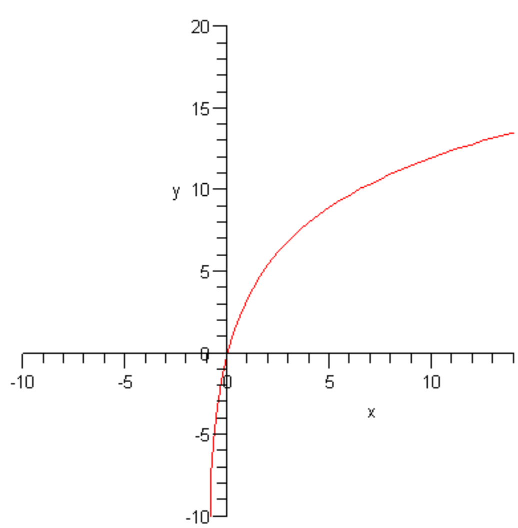


Example 6

Graph $y = 5\log(x+1)$

x	y
2	$y = 5\log(2+1) = 5\log(3) = 2.4$
10	$y = 5\log(10+1) = 5\log(11) = 5.2$
20	$y = 5\log(20+1) = 5\log(21) = 6.6$
40	$y = 5\log(40+1) = 5\log(41) = 8.1$

Plot the given values from the table gives the following graph



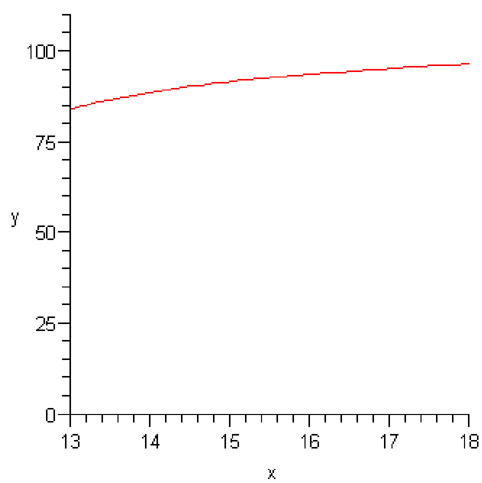
Example 7 (Using logarithmic models to model height)

A logarithmic model to approximate the percentage P of an adult height a male has reached at an age A form 13 and 18 is $P = 16\log(A-12)+84$

1) Sketch a graph of this function.

A	P
13	$P = 16\log(13-12)+84 = 16\log(1)+84 = 0+84 = 84$
14	$P = 16\log(14-12)+84 = 16\log(2)+84 = 4.8+84 = 88.8$
15	$P = 16\log(15-12)+82 = 16\log(3)+84 = 7.6+84 = 90.6$
18	$P = 16\log(18-12)+84 = 16\log(6)+84 = 12.5+84 = 96.5$

Plot the given values from the table gives the following graph



2) What does the graph tell you about the height of male after age of 18?

Usually males stop growing after age 18

3) Use the model to compute the average height of a 16 year old male.

$P = 16\log(16-12)+84 = 16.\log(4)+84 = 9.6+84 = 93.6$

93.6%

Example 8

Use the following model for $1000 invested in saving account given by the formula $n = -694.2 + 231.4\log(A)$, to find the amount of time (n) for the amount of money A to grow to $100,000.

$n = -694.2 + 231.4\log 100000$

$n = -694.2 + 231.4(5)$

$n = -694.2 + 1157$

$n = 462.8$