Section 3.1

Logic Statement

A **statement** is a declarative sentence that is either true or false, but not true and false.

Example 1

Determine if each sentence is a statement.

1) Ohio is a state in the United States.

Statement: This sentence can be classified as true or false

2) Elephants are pink.

Statement: This sentence can be classified as true or false

3) How are you?

Not a Statement: Questions are not statements

4) 3 + 4 = 7

Statement: This sentence can be classified as true or false

Compound Statements

A compound statement is a statement formed by joining two or more statement together.

Connectors

Connector	Symbol	Type of connector
Or	$\vee$	Disjunctive
And	$\wedge$	Conjunctive
If-then	$\rightarrow$	Conditional
If and only if	$\leftrightarrow$	Biconditional
Negation	~	Negation

Negating Statements

The negation of a statement has the opposite meaning of the statement.

Example 2

Find the negation of the following statements

1) The flower is red

Negation: The flower is not red

2) The number 12 is a prime number

Negation: The number 12 is not a prime number

3) The Highlands won the basketball game.

Negation: The highlands didn't win the basketball game.

4) Some elephants are grey

Negation: All elephants are not grey

Example 3

Consider the following statements

$p: The\ game\ is\ in\ Charlottesvillle$
$a: The\ game\ is\ shown\ on\ ABC$
$b: The\ game\ is\ shown\ on\ ESPN$
$c: The\ Hokies\ are\ favored\ to\ win$

Write the following symbolic statements in words.

1) $p \wedge b$

The game is shown on ESPN and the game is in Charlottesville.

2) $\sim p \rightarrow c$

If the game is not in Charlottesville, then the Hokies are favored to win.

Example 4

Use the symbols from example 3 to write the statements in symbolic form.

1) The game is in Charlottesville and it’s on ABC.

$p \wedge a$

2) The game is in Charlottesville and the Hokies are not favored to win.

$p \wedge \sim c$

Section 3.2/3.3

Truth Tables

Review of the connectors

Connector	Symbol
Or	$\vee$
And	$\wedge$
If – then (Conditional)	$\rightarrow$
Negation	~

Using the connectors in a truth table

Basic Truth tables

Example 1 Construct a truth table for $p \vee q$

p	Q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Think of the statement "Either you like apple or you like oranges"

This statement is true unless "you don't like oranges" and "you don't like apples" (See red row in the truth table)

Example 2 Construct a truth table for $p \wedge q$

p	Q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Using the statement "You like apples and oranges", it turns out that this statement is true only if you both like apples and oranges. (See blue) In the last three cases (rows) the statement is false. (See red)

Example 3

Construct a truth table for $p \to q$

p	Q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Using the example "If you study for the test, you will pass the test", it turns out that this is all true accept when the hypothesis "If you study for the test" is true, and the conclusion "you will pass the test" is false (See red)

Other examples of Truth Tables

Example 4

Construct a truth table for $(p \wedge q) \to q$

p	Q	$p \wedge q$	$(p \wedge q) \to q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

When a compound statement results with all true statements in the last column it is called a **tautology** (True in all cases)

Example 5

Construct a truth table for $(p \wedge q) \vee p$

p	Q	$p \wedge q$	$(p \wedge q) \vee p$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

Example 6

Construct a truth table for $(p \wedge q) \vee (\sim p)$

p	Q	~p	$p \wedge q$	$(p \wedge q) \vee (\sim p)$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Example 7

Construct a truth table for $\sim (p \vee q)$

p	Q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Example 8

Construct a truth table for

$(\sim (p \vee q)) \rightarrow q$

p	Q	$p \vee q$	$\sim (p \vee q)$	$(\sim (p \vee q)) \rightarrow q$
T	T	T	F	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

Example 9

Construct a truth table for $(p \wedge q) \rightarrow (p \vee q)$

p	Q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

This is a tautology

Example 10

Construct a truth table for $p \vee (\sim r \wedge q)$

p	Q	R	~r	$(\sim r \wedge q)$	$p \vee (\sim r \wedge q)$
T	T	T	F	F	T
T	T	F	T	F	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

Example 11

Construct a truth table for $(p \wedge q) \rightarrow r$

p	Q	r	$(p \wedge q)$	$(p \wedge q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Equivalent Statements

Equivalent Statements will have the same result in the last column of their truth tables.

Example 12

Compare the truth tables for $\sim (p \vee q)$ and $\sim p \wedge \sim q$

Truth table for $\sim (p \vee q)$

P	q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Truth table for $\sim p \wedge \sim q$

P	q	~p	~q	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Notice that the last columns of each table are identical. Thus, the arguments are equivalent.

Example 13

Compare the truth tables for $\sim(p \wedge q)$ and $\sim p \vee \sim q$

Truth table for $\sim(p \wedge q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Truth table for $\sim p \vee \sim q$

p	q	~p	~q	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Again the truth tables have the same last column. Thus, the statements are equivalent.

De Morgan's Laws

$\sim(p \vee q) = \sim p \wedge \sim q$

$\sim(p \wedge q) = \sim p \vee \sim q$

Section 3.5 Validity

Write arguments in symbolic form and valid arguments

Writing an argument in symbolic form

I have a college degree (p)
I am lazy (q)

If I have a college degree, then I am not lazy
I don't have a college degree
Therefore, I am lazy

Symbolic form:

If I have a college degree, then I am not lazy $(p \rightarrow \sim q)$
I don't have a college degree $(\sim p)$
Therefore, I am lazy q

Hypothesis: $((p \rightarrow \sim q) \wedge \sim p)$
Conclusion: q

Argument in symbolic form: $((p \rightarrow \sim q) \wedge \sim p) \rightarrow q$

To test to see if the argument is valid, we take the argument in symbolic form and construct a truth table. If the last column in the truth table results in all true's, then the argument is valid

p	q	$\sim p$	$\sim q$	$(p \rightarrow \sim q)$	$((p \rightarrow \sim q) \wedge \sim p)$	$((p \rightarrow \sim q) \wedge \sim p) \rightarrow q$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

Therefore, this argument is invalid because the last column has a false item.

Example 1

Symbolize the argument, construct a truth table, and determine if the argument is valid.

If I pass the exam, then I will graduate.
I graduated
Therefore, I passed the exam

p = pass the exam
g = I will graduate

If I pass the exam, then I will graduate. ($p \to g$)
I graduated (g)
Therefore, I passed the exam (p)

Argument: $((p \to g) \wedge g) \to p$

P	g	$(p \to g)$	$((p \to g) \wedge g)$	$((p \to g) \wedge g) \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

This argument is invalid

Example 2

Symbolize the argument, construct a truth table, and determine if the argument is valid.

Jen and Bill will be at the party
Bill was at the party.
Therefore, Jen was at the party

J = Jen will be at the party
B = Bill will be at the party

Jen and Bill will be at the party $J \wedge B$
Bill was at the party. B
Therefore, Jen was at the party J

Argument in symbolic form: $((J \wedge B) \wedge B) \to J$

J	B	$J \wedge B$	$(J \wedge B) \wedge B$	$((J \wedge B) \wedge B) \to J$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

Since the last column is all true, the argument is valid

Example 3

Symbolize the argument, construct a truth table, and determine if the argument is valid.

It will be sunny or cloudy today
<u>It isn't sunny</u>
Therefore, it will be cloudy

S = It will be sunny
C = It will be cloudy

It will be sunny or cloudy today $S \vee C$
<u>It isn't sunny ~S</u>
Therefore, it will be cloudy C

Hypothesis: $(S \vee C) \wedge \sim S$
Conclusion: C

S	C	~S	$S \vee C$	$(S \vee C) \wedge \sim S$	$((S \vee C) \wedge \sim S) \to C$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

This is a valid argument

Example 4

Write in symbolic form

p: The senator supports new taxes.
q: The senator is reelected

The senator is not reelected if she supports new taxes
The senator does not support new taxes
Therefore, the senator is reelected

Symbolic form:

The senator is not reelected if she supports new taxes $p \to \sim q$
The senator does not support new taxes $\sim p$
Therefore, the senator is reelected q

Hypothesis: $(p \to \sim q) \wedge \sim p$
Conclusion: q

Argument: $((p \to \sim q) \wedge \sim p) \to q$

Determine if the argument in problem 6 above is valid

p	q	~q	~p	$p \to \sim q$	$(p \to \sim q) \wedge \sim p$	$((p \to \sim q) \wedge \sim p) \to q$
T	T	F	F	F	F	T
T	F	T	F	T	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	F

Since the last row results in false, the argument is invalid

Example 5

If you complete your computer training, you can be eligible to work in the computer lab. You did not finish your computer training. Therefore, you can not work in the computer lab.

$p: \textit{You finished your computer training}$

$q: \textit{You can work in the computer lab}$

Argument in symbolic: $((p \to q) \wedge \sim p) \to \sim q$

p	q	$\sim p$	$\sim q$	$p \to q$	$((p \to q) \wedge \sim p)$	$((p \to q) \wedge \sim p) \to \sim q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Invalid Arguments