Math 121
Semester Exam Study Guide Solutions
Radford University

1) Factor $x^2 - 3x - 4$

$x^2 - 3x - 4$
$(x-4)(x+1)$

2) Factor $x^2 - 36$

$x^2 - 36$
$(x-6)(x+6)$

3) Factor $x^3 - 8$

$m^3 - 8$
$m^3 - 2^3$
$(m-2)(m^2 + 2m + 2^2)$
$(m-2)(m^2 + 2m + 4)$

4) Factor $2x^3 - 3x^2 - 2x + 3$

$2x^3 - 3x^2 - 2x + 3$
$x^2(2x-3) - 1(2x-3)$
$(x^2 - 1)(2x-3)$
$(x+1)(x-1)(2x-3)$

5) Solve $2x + 3 = 7$

$2x + 3 = 7$
$2x + 3 - 3 = 7 - 3$
$2x = 4$
$\frac{2x}{2} = \frac{4}{2}$
$x = 2$

6) Solve $4(x+3)=x+27$

$4(x+3)=x+27$
$4x+12=x+27$
$4x-x+12=x-x+27$
$3x+12=27$
$3x+12-12=27-12$
$3x=15$
$\frac{3x}{3}=\frac{15}{3}$
$x=5$

7) Solve $x^2-10x+25=0$

$x^2-10x+25=0$
$(x-5)(x-5)=0$
$x-5=0$ *or* $x-5=0$
$x=5$ $\quad\quad$ $x=5$

8) Solve $x^2-49=0$

$x^2-49=0$
$(x-7)(x+7)=0$
$x-7=0$ *or* $x+7=0$
$x=7$ $\quad\quad$ $x=-7$

9) Find the inverse of $f(x) = x^3 - 2$ and graph f and f $^{-1}$

$f(x) = x^3 - 2$

$y = x^3 - 2$

$x = y^3 - 2$ *switch* x *and* y

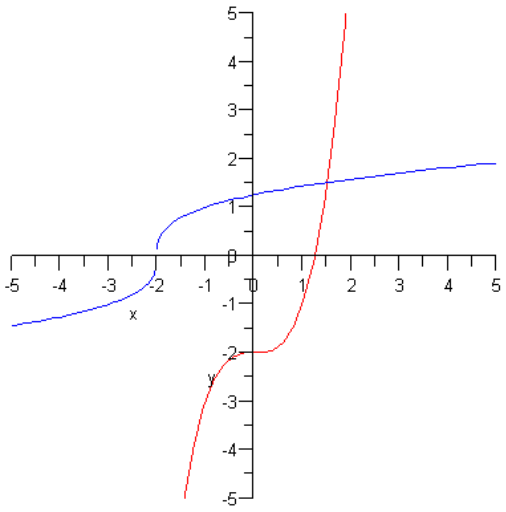
Solve for y

$x + 2 = y^3 - 2 + 2$

$x + 2 = y^3$

$\sqrt[3]{x+2} = \sqrt[3]{y^3}$

Thus, $y = \sqrt[3]{x+2}$ *or* $f^{-1}(x) = \sqrt[3]{x+2}$



10) Determine if the lines given by the equations are parallel or perpendicular.

$4x + 2y = 6$

$2x + y = 5$

Write each equation in slope – int form $y = mx + b$

$4x + 2y = 6$

$2y = -4x + 6$

$\frac{2y}{2} = \frac{-4x}{2} + \frac{6}{2}$

$y = -2x + 3$

Thus $m = -2$

$2x + y = 5$

$y = -2x + 5$

Thus $m = -2$

Since the slope are equal, the lines are parallel

12) Find the center and the radius of a circle with the given equation.

$x^2 + y^2 + 2x + 6y + 9 = 0$

$x^2 + 2x + y^2 + 6y = -9$

$x^2 + 2x + 1 + y^2 + 6y + 9 = -9 + 1 + 9$

$x^2 + 2x + 1 + y^2 + 6y + 9 = 1$

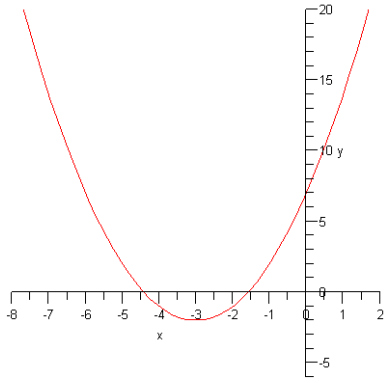
$(x+1)(x+1) + (y+3)(y+3) = 1$

$(x+1)^2 + (y+3)^2 = 1^2$

center $= (-1, -3)$

radius $= 1$

13) Graph $y = (x+3)^2 - 2$



14) Given $f(x) = 3x^4 + 2x^2 + 4x$, find $f'(x)$

$f'(x) = 4 \cdot 3x^{4-1} + 2 \cdot 2x^{2-1} + 4$

$f'(x) = 12x^3 + 4x + 4$

15) Find the slope of the tangent line to $f(x) = 2x^2 - 7$ passing through the point at (1,-5)

$f(x) = 2x^2 - 7$

$f'(x) = 4x$

$m = f'(1) = 4 \cdot 1 = 4$

16) Given the revenue function $R(x) = x^2 - 8x + 300$, find the marginal revenue at x =100.

$R(x) = x^2 - 8x + 300$

$R'(x) = 2x - 8$

$R'(100) = 2(100) - 8 = 192$

17) Evaluate $\lim_{x \to 4} \frac{x-4}{x^2 - 6x + 8}$

$$\lim_{x \to 4} \frac{x-4}{x^2 - 6x + 8} = \lim_{x \to 4} \frac{x-4}{(x-2)(x-4)} = \lim_{x \to 4} \frac{1}{x-2} = \frac{1}{4-2} = \frac{1}{2}$$

18) Evaluate $\lim_{x \to 3} x^2 + 3x + 8$

$$\lim_{x \to 3} x^2 + 3x + 8 = 3^2 + 3 \cdot 3 + 8 = 9 + 9 + 8 = 26$$

19) Evaluate $\lim_{x \to 4} \frac{6}{x-4}$

$\lim_{x \to 4} \frac{6}{x-4} = \frac{6}{4-4} = \frac{6}{0}$ The limit does not exist

20) Given $y = \left(x^2 + 2x\right)^4$, find the derivative of y.

$y = \left(x^2 + 2x\right)^4$

$Let\ u = x^2 + 2x\ where\ du = 2x + 2$

$y = u^4$

$y' = 4u^3 du$

$y' = 4(x^2 + 2x)^3 (2x + 2)$

21) Given $f(x) = \sqrt{x^3 + 4x}$ find $f'(x)$

$f(x) = \sqrt{x^3 + 4x}$

$f(x) = \left(x^3 + 4x\right)^{\frac{1}{2}}$

$Let\ u = x^3 + 4x\ where\ du = 3x^2 + 4$

$f(x) = u^{\frac{1}{2}}$

$f'(x) = \frac{1}{2} u^{\frac{1}{2}-1} du$

$f'(x) = \frac{1}{2} u^{-\frac{1}{2}} du$

$f'(x) = \frac{1}{2} (x^3 + 4x)^{-\frac{1}{2}} (3x^2 = 4)$

$f'(x) = \frac{3x^2 + 4}{2\sqrt{x^3 + 4x}}$

22) Find the first and second derivative of $f(x) = 3x^4 + 10x^3 + 6x$

$f(x) = 3x^4 + 10x^3 + 6x$

$f'(x) = 12x^3 + 30x^2 + 6$

$f''(x) = 36x^2 + 60x$

23) Given $f(x) = 3x^5 + 10x^3$, find $f'(x)$

$f(x) = 3x^5 + 10x^3$

$f'(x) = 15x^4 + 30x^2$

24) Given $f(x)=\frac{1}{x^3}$, find $f'(x)$

$f(x)=\frac{1}{x^3}$

$f(x)=x^{-3}$

$f'(x)=-3x^{-3-1}=-3x^{-4}=\frac{-3}{x^4}$

25) Discuss the continuity of $f(x)=x^3-5x$

Continuous on $(-\infty,\infty)$

26) Discuss the continuity of $f(x)=\frac{x}{x^2-1}$

$f(x)=\frac{x}{x^2-1}=\frac{x}{(x-1)(x+1)}\Rightarrow f$ *is undefined at* $x=1$ *and* $x=-1$*, so* f *is continuous on* $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

27) Find the distance between the points (1,1) and (3,5)

$d=\sqrt{(3-1)^2+(5-1)^2}=\sqrt{2^2+4^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$

28) Given $f(x)=3x^5+10x^3$, find f(3) and f(-1)

$f(3)=3(3)^5+10(3)^3=3(243)+10(27)=729+270=999$

$f(-1)=3(-1)^5+10(-1)^3=3(-1)+10(-1)=-13$

29) Given $f(x)=2x-3$ *and* $g(x)=x^2-x$,find $f(g(3))$

$f(g(x))=f(x^2-x)=2(x^2-x)-3=2x^2-2x-3$

30) Find the intervals where the function is increasing or decreasing $f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$

$x - 1 = 0$ *or* $x + 1 = 0$ *Critical Value*

$x = 1$ *or* $x = -1$

Interval	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
Test Value	x = -2	x = 0	x = 2
Sign of $f'(x)$	$f'(-2) = 3(-2)^2 - 3$ $= 12 - 3 = 9$ Positive	$f'(0) = 3(0)^2 - 3$ $= 0 - 3$ Negative	$f'(2) = 3(2)^2 - 3$ $= 12 - 3 = 9$ Positive
Conclusion	Increasing	Decreasing	Increasing

31) Find all extrema points of the function and discuss the concavity of the function.
$f(x) = x^3 - 15x^2$

$f'(x) = 3x^2 - 30x$

$3x^2 - 30x = 0$

$3x(x-10) = 0$

$3x = 0$ *or* $x - 10 = 0$

$x = 0$ *or* $x = 10$

Interval	$(-\infty,0)$	$(0,10)$	$(10,\infty)$
Test Value	x=-1	x=1	x=11
Sign of $f'(x)$	Positive	Negative	Positive
Conclusion	Increasing	Decreasing	Increasing

$f'(-1) = 3(-1)^2 - 30(-1) = 3(1) + 30 = 3 + 30 = 33$

$f'(1) = 3(1)^2 - 30(1) = 3(1) - 30 = 3 - 30 = -27$

$f'(11) = 3(11)^2 - 30(11) = 3(121) - 330 = 363 - 330 = 33$

Concavity Test

$f'(x) = 6x - 30$

$6x - 30 = 0$

$6x - 30 + 30 = 0 + 30$

$6x = 30$

$\frac{6x}{6} = \frac{30}{6}$

$x = 5$

Interval	$(-\infty,5)$	$(5,\infty)$
Test Value	x=4	x=6
Sign of $f''(x)$	Negative	Positive
Conclusion	Concave Down	Concave up

$f''(4) = 6(4) - 30 = -6$

$f''(6) = 6(6) - 30 = 6$

Extrema Points: Relative Maximum at x = 0 and a Relative Minimum x = 10

Page 129

23)

$f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$

$f'(x) = (3x^2 - 3)(2x^2 + 3x + 5) + (4x + 3)(x^3 - 3x)$

31)

$f(x) = \frac{3 - 2x - x^2}{x^2 - 1}$

$f'(x) = \frac{(x^2 - 1)(-2 - 2x) - (2x)(3 - 2x - x^2)}{(x^2 - 1)^2}$