Section 9.1

Linear Models

Standard linear model

$$y = mx + b$$
$$m = slope$$
$$b = y - \text{int}\,ercept$$

Slope

$$Slope = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

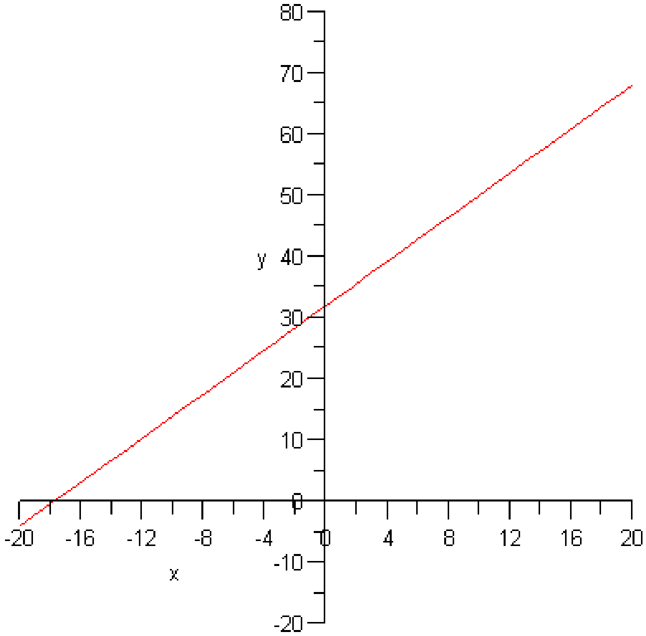
Example 1

Temperature conversion

$$F = \frac{9}{5}C + 32$$

a) Sketch a graph of $F = \frac{9}{5}C + 32$

C	$F = \frac{9}{5}C + 32$
10	$F = \frac{9}{5}(10) + 32 = 9(2) + 32 = 18 + 32 = 50$
20	$F = \frac{9}{5}(20) + 32 = 9(4) + 32 = 36 + 32 = 68$
30	$F = \frac{9}{5}(30) + 32 = 9(6) + 32 = 54 + 32 = 86$
40	$F = \frac{9}{5}(40) + 32 = 9(8) + 32 = 72 + 32 = 104$



b) Use the model to convert 120 degrees Celsius to degrees Fahrenheit.

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(120) + 32$$

$$F = 216 + 32$$

$$F = 248$$

c) Use the model to convert 212 degrees Fahrenheit to Celsius.

$$F = \frac{9}{5}C + 32$$

$$212 = \frac{9}{5}C + 32$$

$$212 - 32 = \frac{9}{5}C + 32 - 32$$

$$180 = \frac{9}{5}C$$

$$\frac{5}{9}(180) = \frac{5}{9} \cdot \frac{9}{5}C$$

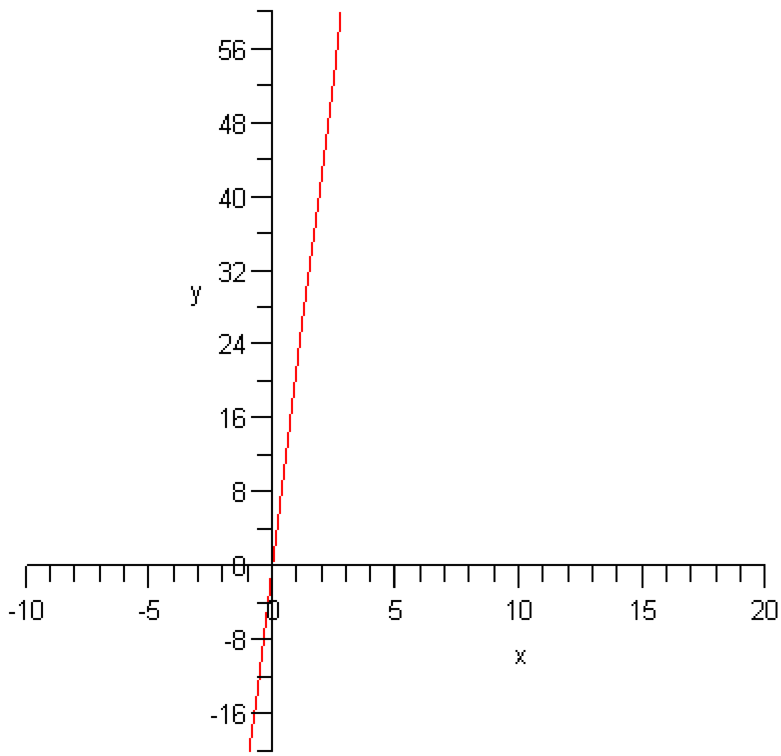
$$C = 100^0 C$$

Example 2

The revenue of a company that makes backpacks is given by the formula $R = 21.50x$ where x represents the number of backpacks sold.

a) Graph the linear model $R = 21.50x$

x	$R = 21.50x$
10	$R = 21.50(10) = 215$
20	$R = 21.50(20) = 430$
30	$R = 21.50(30) = 645$
40	$R = 21.50(40) = 860$



b) Use the model to calculate the revenue for selling 50 backpacks

$x = 50$

$R = 21.50x = 21.5(50) = \$1075.0$

c) What is the slope
$m = \$21.50$

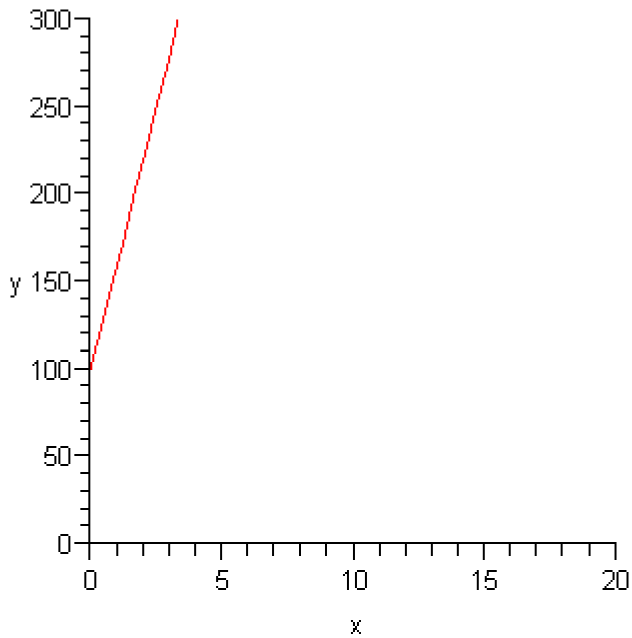
d) What is the meaning of the slope?
Cost per unit sold
Revenue made per backpack solid

Example 3

A salesperson is paid $100 plus $60 per sale each week. The model $S = 60x + 100$ is used to calculate the salesperson's weekly salary where x is the number of sales per week.

a) Graph $S = 60x + 100$

x	S
2	$S = 60(2) + 100120 + 100 = 220$
4	$S = 60(4) + 100 = 240 + 100 = 340$
6	$S = 60(6) + 100 = 360 + 100 = 460$
8	$S = 60(8) + 100 = 480 + 100 = 580$



b) Use the model to calculate the salespersons weekly salary if he/she makes 8 sales.

$$S = 60(8) + 100 = 480 + 100 = \$580.00$$

c) What is the slope of the equation

$$m = 60 \frac{\$}{sale}$$

d) What is the meaning of the slope

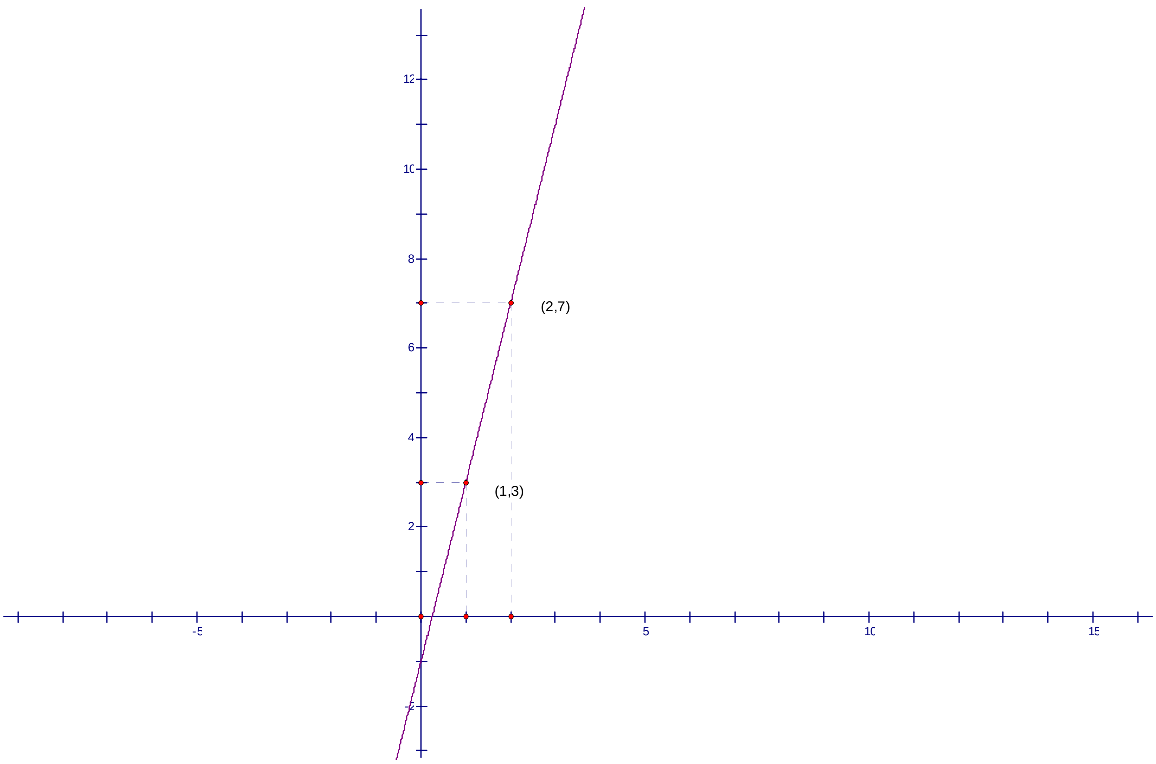
Dollars per each sale

Example 4

Given the following data sketch a graph

Time	Temperature
1 min	$3^0 C$
2 min	$7^0 C$
3 min	$11^0 C$
4 min	$14^0 C$

Sketch a graph of the given data and then compute the slope of the resulting line.

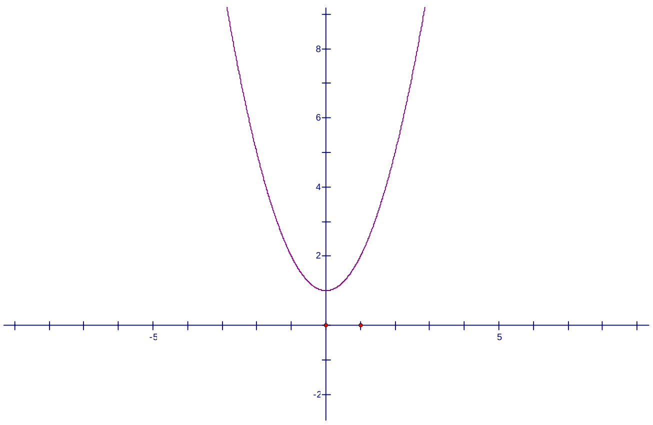


Use the points (1,3) and (2,7) in the above graph to compute the slope

$$m = \frac{7-3}{2-1} = \frac{4}{1} = 4$$

Quadratic Models

Graph of Quadratic Models



The parabola

A **quadratic function** is a function where the graph is a parabola and an equation of the form: $y = ax^2 + bx + c$ where $a \neq 0$

The x coordinate vertex is given by the equation: $x = -\dfrac{b}{2a}$

Examples

Find the vertex and x-intercepts, and then make a sketch of the parabola.

1)
$y = x^2 - 2x$

$a = 1, b = -2$

$x = -\dfrac{-2}{2(1)} = \dfrac{2}{2} = 1$

$x - \text{int}\,ercepts$

$x^2 - 2x = 0$

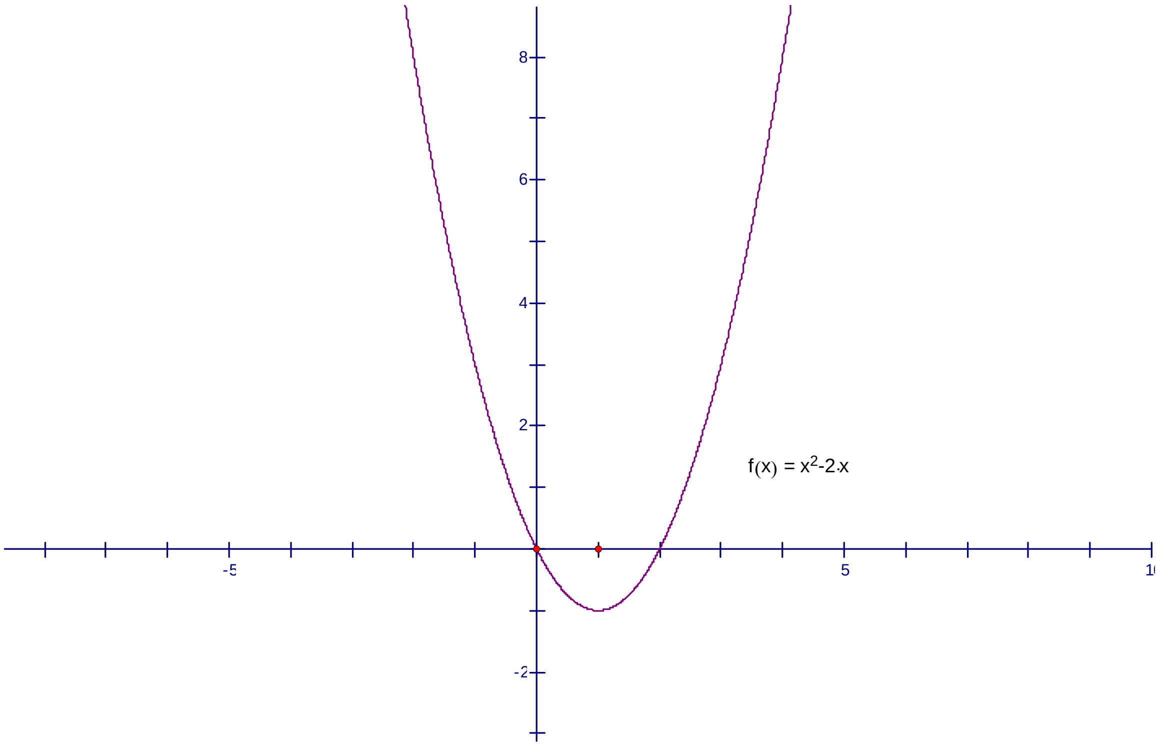
$x(x-2) = 0$

$x = 0 \;\; or \;\; x - 2 = 0$

$x = 0 \; or \;\; x = 2$

$(0,0) \; and \; (2,0)$

Graph



2)

$y = x^2 - 3x$

$Vertex$

$x = -\frac{-3}{2(1)} = \frac{3}{2}$

$x - \text{int}$

$x^2 - 3x = 0$

$x(x-3) = 0$

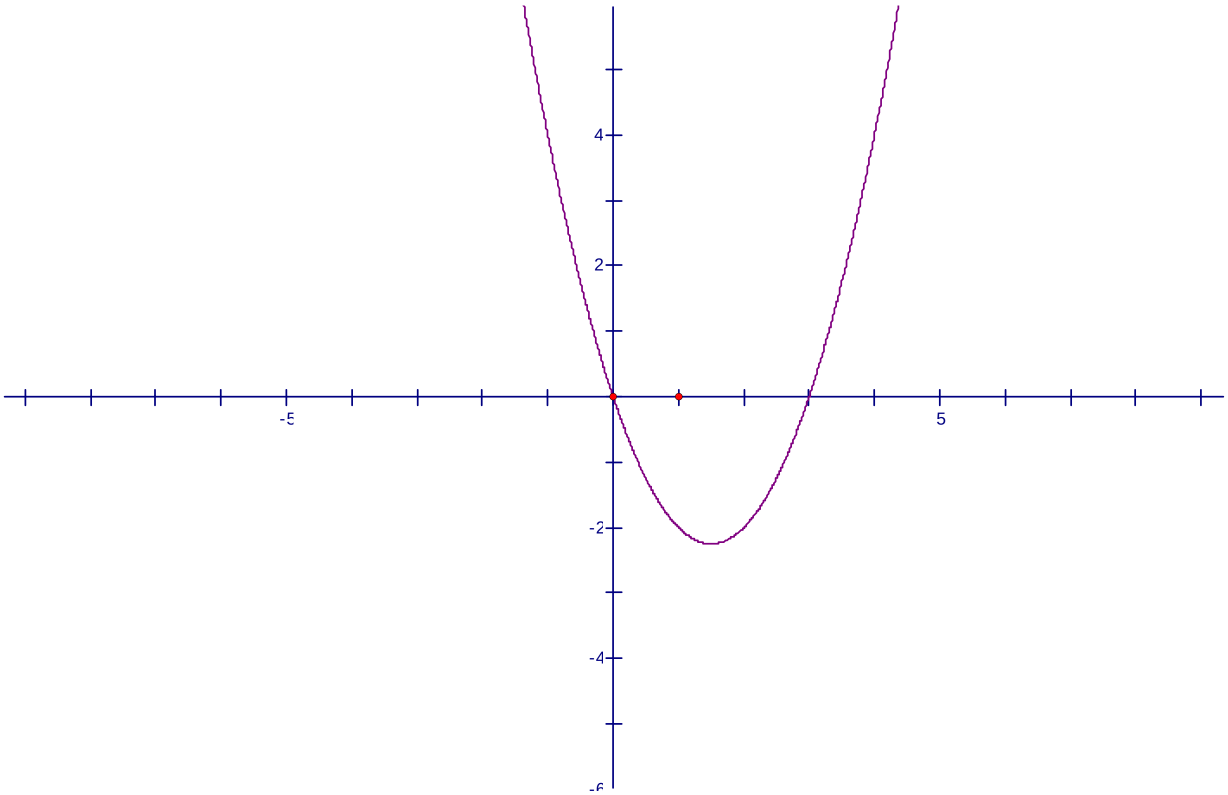
$x = 0 \; or \; x - 3 = 0$

$x = 0 \quad x - 3 = 0$

$x = 3$

$(0,0) \;\; and \;\; (3,0)$

Graph of the function



3) $y = x^2 - 4x + 3$

$$vertex: x = -\frac{-4}{2(1)} = 2$$

$y - coordinate: y = 2^2 - 4(2) + 3 = -1$

$x - \text{int}$

$x^2 - 3x + 4 = 0$

$(x-3)(x-1) = 0$

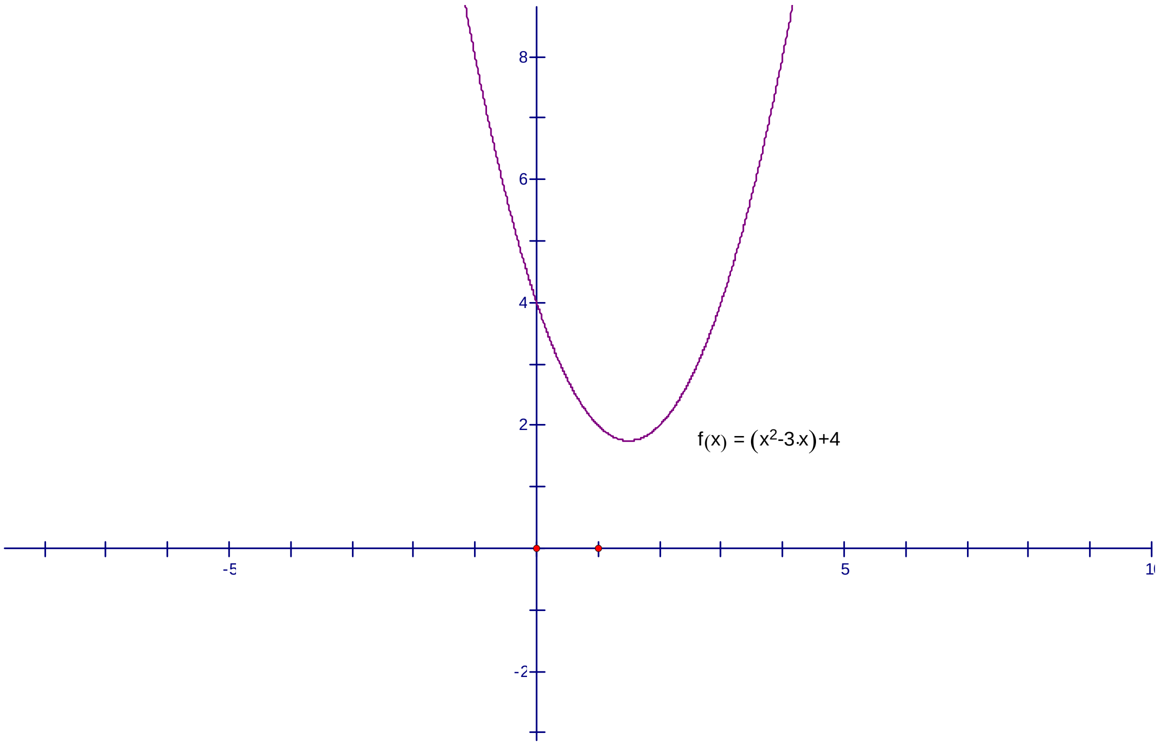
$x - 3 = 0 \quad or \quad x - 1 = 0$

$x - 3 + 3 = 0 + 3 \quad x - 1 + 1 = 0 + 1$

$x = 3 \qquad x = 1$

x-intercepts $(1,0)$ *and* $(3,0)$

Graph



4)

$y = x^2 - 3$

$a = 1, c = -3$

$x = -\frac{0}{2(1)} = -\frac{0}{2} = 0$

$x - \text{int}\,ercepts$

$x^2 - 3 = 0$

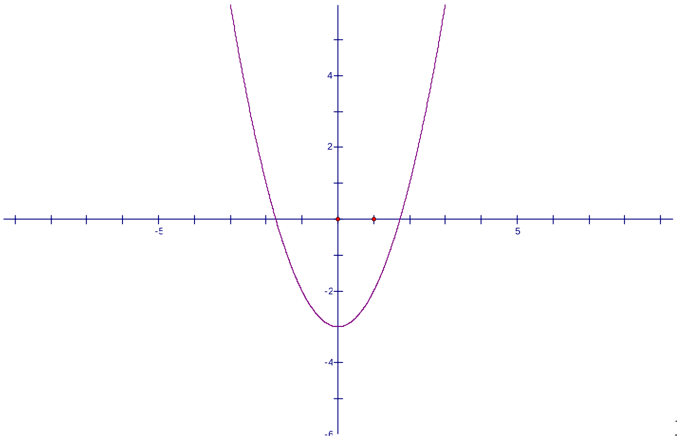
$x^2 = 3$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

$(\sqrt{3},0)$ *and* $(-\sqrt{3},0)$

Graph



x

5) $y = x^2 + 2x - 3 = 0$

$$vertex: \ x = -\frac{2}{2(1)} = -\frac{2}{2} = -1$$

$$y = (-1)^2 + 2(-1) - 3 = 3 - 3 = 0$$

$$Vertex: (-1,0)$$

x-intercepts

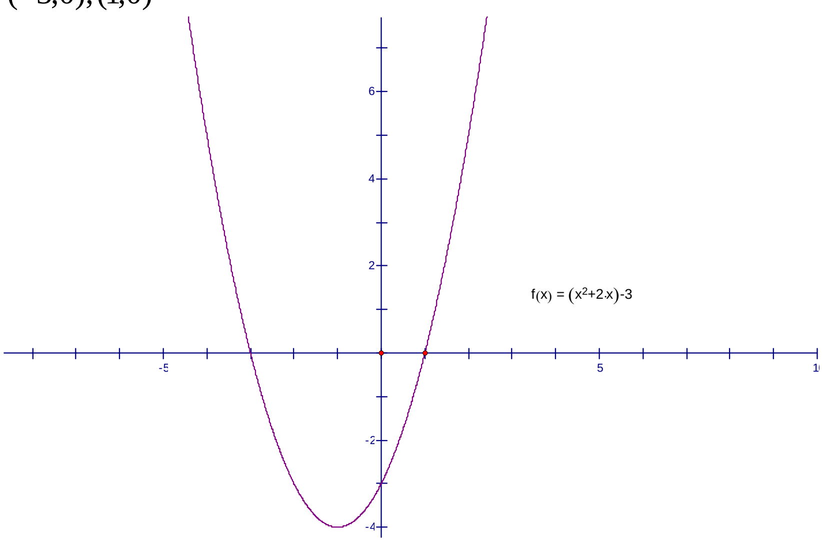
$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x + 3 = 0 \quad or \quad x - 1 = 0$$

$$x = -3 \quad or \quad x = 1$$

$$(-3,0), (1,0)$$



Using the quadratic formula to solve an equation

The Quadratic Formula

The solution to the equation $y = ax^2 + bx + c$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1) Solve $x^2 + 5x - 7 = 0$

$$a = 1$$
$$b = 5$$
$$c = -7$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 28}}{2} = \frac{-5 \pm \sqrt{53}}{2}$$

2) Solve $x^2 + 7x - 9 = 0$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-9)}}{2(7)} = \frac{-7 \pm \sqrt{49 + 36}}{2(7)} = \frac{-7 \pm \sqrt{85}}{14}$$

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8) Find the vertex, graph, and x intercepts of each parabola

$$y = -x^2 + 6x - 5.5$$
$$a = -1$$
$$b = 6$$
$$c = -5.5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-5.5)}}{2(-1)} = \frac{-6 \pm \sqrt{36 - 22}}{-2} = \frac{-6 \pm \sqrt{14}}{-2}$$

x-intercepts

$(-6 + \sqrt{14}, 0)$ *and* $(-6 - \sqrt{14}, 0)$

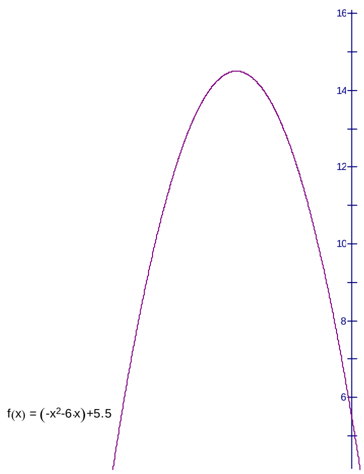
Vertex

$$x = -\frac{6}{2(-1)} = 3$$

$y - coordinate$

$$y = -3^2 + 6(3) - 5.5 = -9 + 18 - 5.5 = 4.5$$

(3,4.5)



12) At a local frog jumping contest. Rivet's jump can be approximated by the equation $y = -\frac{1}{6}x^2 + 2x$ and Croak's jump can be approximate by $y = -\frac{1}{2}x^2 + 4x$, where x = the length of jump in feet and y = the height of the jump in feet.

a) Which frog can jump higher

Rivet's vertex: $x = -\frac{2}{2\left(-\frac{1}{6}\right)} = -\frac{2}{-\frac{1}{3}} = 6$ Height: $y = -\frac{1}{6}(6)^2 + 2(6) = -6 + 12 = 6 \ ft$

Croak's vertex: $x = -\frac{4}{2\left(-\frac{1}{2}\right)} = \frac{-4}{-1} = 4$ Height: $y = -\frac{1}{2}(4)^2 + 4(4) = -8 + 16 = 8 \ ft$

Croak can jump higher at 8 feet

b) Which frog can jump farther

Rivet's can jump farther at 2(6 ft) = 12 feet

Examples

1) The path of a ball thrown by a boy is given in yards by the equation $y = -.04x^2 + 1.5x$ where x is the horizontal distance the ball travels and y is the height of the ball. Find the maximum height of the ball in yards.

   Find the vertex of the ball

   $$x = -\frac{1.5}{2(-.04)} = \frac{1.5}{.08} = 18.75$$

   $$y = -.04(18.75)^2 + 1.5(18.75) = -14.1 + 28.1 = 14 \ \textit{yards}$$

2) The path of a cannon ball is given in feet by the equation $y = -.1x^2 + 6.0x$ where x is the horizontal distance the ball travels and y is the height of the cannon ball. Find the maximum height of the cannon ball in feet.

   Find the vertex of the cannon ball.

   $$x = -\frac{6.0}{2(-.1)} = -\frac{6.0}{-.2} = 30$$

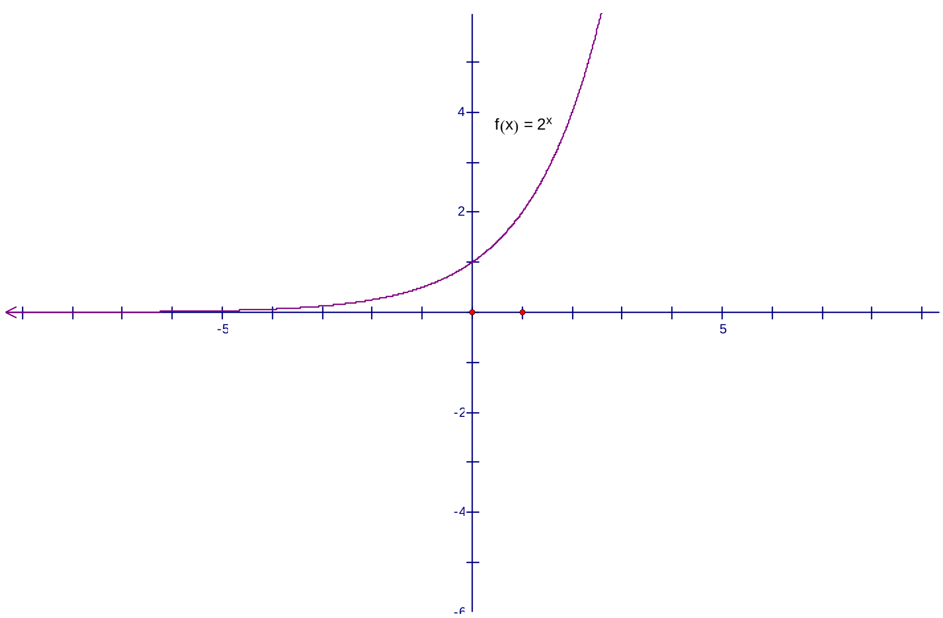
   $$y = -.1(30)^2 + 6(30) = -90 + 180 = 90 \ \textit{feet}$$

Exponential Models

Example of an exponential model

1) Graph $y = 2^x$

x	y
-2	$y = (2)^{-2} = \frac{1}{4}$
-1	$y = (2)^{-1} = \frac{1}{2}$
0	$y = 2^0 = 1$
1	$y = 2^1 = 2$
2	$y = 2^2 = 4$



The Euler number

$e \approx 2.718$

Example of expressions with the exponential function

1) Simplify $e^2 \approx 7.389$

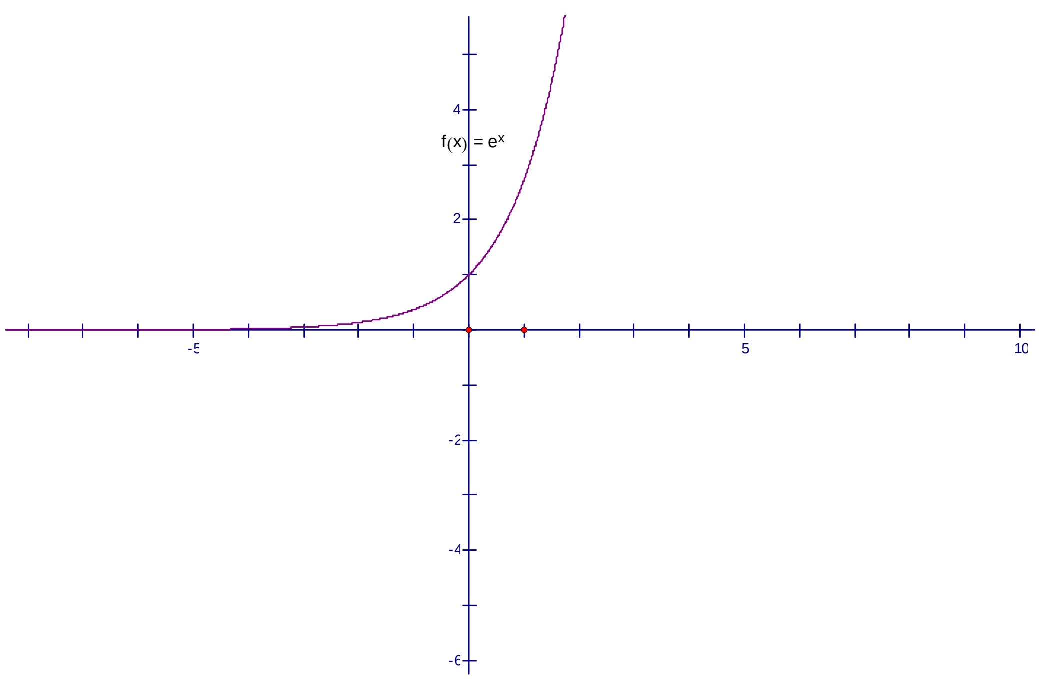
2) Simplify $e^4 \approx 54.60$

3) Simplify $e^{-3} \approx .049$

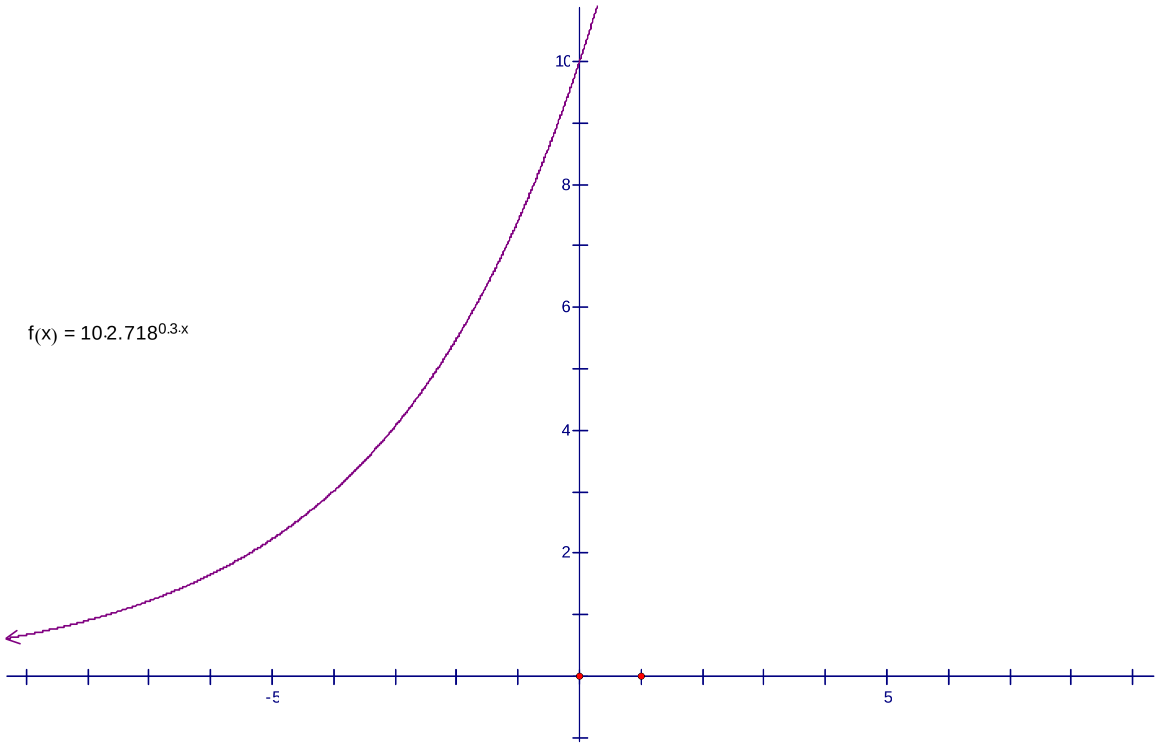
Graphs of the exponential functions

4) Graph $y = e^x$

x	y
-2	$y = e^{-2} = .14$
-1	$y = e^{-1} = .37$
0	$y = e^0 = 1$
1	$y = e^1 = 2.7$
2	$y = e^2 = 7.3$



5) Graph $y = 10e^{.3x}$



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14) Suppose Parker Brothers determines that the profit P for a board game that is on the market for t years is given by the following equation.

$$P = 6000 + 20000(3)^{-0.2t}$$

b) What is the profit after 25 years?

$$P = 6000 + 20000(3)^{-0.2t}$$
$$P = 6000 + 20000(3)^{-0.2(25)}$$
$$P = 6000 + 20000(3)^{-5}$$
$$P = 6000 + 20000(.004115)$$
$$P = 6000 + 82.30$$
$$P = \$6082.30$$

Exponential Models

Exponential Growth

$P = P_0(1+r)^t$

$P = New\ Value$

$P_0 = Original\ Value$

$r = rate$

$t = time$

Examples

1) The population of the United States is 290 million. What would be the population of the U. S. be in 20 years, if its population would growth at a steady rate of 1.2 % for 20 years?

$P = P_0(1+r)^t$

$P_0 = 290{,}000{,}000$

$r = .7\% = .012$

$t = 20$

$P = 290000000(1+.012)^{20} = 290000000(1.012)^{20} = 368135965$

2) The population of Blacksburg, Virginia is 41,000. What would be the population in 10 years, if Blacksburg's population would grow at a rate of 1.1 % per year?

$P = P_0(1+r)^t$

$P_0 = 41000$

$r = 1.1\% = .011$

$t = 10$

$P = 41000(1+.011)^{10} = 41000(1.011)^{10} = 45740$

3) In 1995 the United States had greenhouse emissions of about 1400 million tons, where as China had greenhouse emissions of about 850 million tons. If in the next 25 years China greenhouse emission grew by 4 percent and the U. S. greenhouse emission grew by 1.3 percent, what would the emissions in tons for both countries in 2020?

$U.S.\ Emissions\ in\ 2020$

$P = P_0(1+r)^t$

$P_0 = 1400\ million$

$r = 1.3\% = .013$

$t = 25$

$P = 1400(1+.013)^{25} = 1400(1.013)^{25} = 1934\ million\ tons$

$China's\ Emissions\ in\ 2020$

$P = P_0(1+r)^t$

$P_0 = 850\ million$

$r = 4.0\% = .04$

$t = 25$

$P = 850(1+.04)^{25} = 850(1.04)^{25} = 2266\ million\ tons$