

Unit 7
Logic
Math 116

Section 6.1

Logic and Statements

Statements

Definition of Statement: A group words or symbols that can be classified as true or false.

Examples of statements

Violets are blue
Five is a natural number
I like Algebra
 $3 + 7 = 10$

Examples of things that are not statements

Get out of here!
What's up!
Don't worry, be happy

Negations

Example 1

Negate the statement: $p =$ I like apples

Negation: $\sim p$ (I don't like apples)

Note: The negation of all is some and the negation of some is all.

Example 2

Negate the statement: All RU students love ice cream

Negation: Some RU students do not like ice cream.

Example 3

Negate the statement: Some students dislike geometry

Negation: All students like geometry

Example 4

Negate the statement: Everyone loves Raymond

Negation: Someone does love Raymond

Example 6

Write a sentence that is the negation of each statement

1) All candy promotes tooth decay.

Negation: Some candy does not promote tooth decay.

2) No lunch is free.

Negation: Some lunches are free.

Hint: Rewrite the statement as “All lunches are not free”, and then negate the statement which gives you “Some lunches are free”.

Conditional Statements

Conditional: $A \rightarrow B$

Converse: $B \rightarrow A$

Inverse $\sim A \rightarrow \sim B$

Contrapositive: $\sim B \rightarrow \sim A$

Logical Symbols

Connector	Symbol
And	\wedge
Or	\vee
If-then	\rightarrow
Negation	\sim

Examples from the textbook

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10) We call it the period from noon to sunset

Not a statement

12) An obtuse angle is not an 90^0 angle.

Statement

14) A pencil is a writing implement.

Statement

Negate each statement

20) My dog is a Dalmatian.

Negation: My dog is not a Dalmatian.

22) The jokes are great.

Negation: The jokes are not great.

33) All fish can live under water.

Negation: Some fish can not live under water.

35) Some numbers are not prime numbers.

Negation: All numbers are prime numbers

How to use the three types of connectors

I) Using “or” as a connector

Symbolic representation: p or q ($p \vee q$)

Examples 7

Write the following statement in symbol form: Either you need a 21 on the ACT, or you need a 1000 on the SAT

p = you need a 21 on the ACT

q = you need a 1000 on the SAT

$$p \vee q$$

Example 8

Write the following statement in symbol form: Either I will play basketball or I will go for a swim

p = I will play basketball

q = I go for a swim

$$p \vee q$$

II) Using “and” as a connector

Symbolic representation: p and q ($p \wedge q$)

Example 10

Write the following statement in symbol form: You need to get a 21 on the ACT, and you need to get a 1000 on the SAT

p = you need to get a 21 on the ACT

q = you need to get a 1000 on the SAT

$$p \wedge q$$

Example 11

Write the following statement in symbol form: I like oranges and apples

p = I like oranges

q = I like apples

$p \wedge q$

III) Using the conditional

Symbolic representation: If p , then q ($p \rightarrow q$) where p is the premise or hypothesis, and q is the conclusion.

Example 12

Write the following statement in symbol form: If it rains tomorrow, then I will bring an umbrella to work.

p = If it rains tomorrow (Hypothesis)

q = I will bring an umbrella to work (Conclusion)

$p \rightarrow q$

Example 13

Write the following statement in symbol form: I will sell you my textbook, if you offer me a good price

You can rewrite the statements as “If you offer me a good price, then I will sell my textbook” to help identify the hypothesis and conclusion

p = If you offer me a good price (Hypothesis)

q = I will sell you my textbook (Conclusion)

$p \rightarrow q$

Example 14

Write the inverse, converse, and contrapositive of the condition statement: **If you study the test, then you will pass the test.**

Inverse: If you will pass the test, then you will study for the test

Converse: If you don't study for the test, then you will not pass the test

Contrapositive: If you didn't pass the test, then you didn't study for the test.

Example 15

Write the inverse, converse, and contrapositive of the conditional statement: **If go to Florida for spring break, then you will get a nice tan.**

Inverse: If get a nice tan, then you when to Florida for spring break.

Converse: If didn't go to Florida for spring break, then you didn't get a nice tan.

Contrapositive: If you didn't get a nice tan, then you didn't go to Florida for spring break.

Deductive and Induction Reasoning

Deductive Reasoning is the process of reasoning in which conclusions are based on accepted premises.

Deductive reasoning goes from general to specific

Example 1 (Example of deductive reasoning)

Triangle ABC is isosceles

All isosceles triangles have two equal angles

Therefore, triangle ABC has two equal angles

Syllogism: An argument composed to two statements, **or premise**, which is followed by a **conclusion**.

Example 2 (Example of deductive reasoning)

Molly is a dog
Dogs are very friendly
Therefore, Molly is very friendly

Inductive Reasoning is the process of reasoning in which conclusions are based on experience and experimentation.

Inductive reasoning goes from specific to general.

In deductive reasoning the conclusion is guaranteed.

In inductive reasoning the conclusion is probable, but not necessarily guaranteed

Example 3 (Example of inductive reasoning)

John sneezed around Jill's cat
John sneezed around Jim's cat
Therefore, John sneezes around all cats

Example 4 (Example of inductive reasoning)

Room 295 in Walker Hall is a technology classroom at RU
Room 338 in Currie Hall is a technology classroom at RU
Room 212 in Davis Hall is a technology classroom at RU
Therefore, all classrooms at RU are technology classrooms

Examples 6

Determine if the following arguments use deductive reasoning or inductive reasoning

- 1) All math teachers are strange
Jim Morrison is a math teacher
Therefore, Jim Morrison is strange.

(This is an example of a deductive argument)

- 2) Jimmy likes Mary
Women whom Jimmy likes are pretty
Thus, Mary is pretty

(This is an example of a deductive argument)

- 3) In each of the last five years, the economy has grown by at least two percent.
This year the economy is projected to grow by 2.5 %
Therefore, the economy will always grow by at least 2 %

(This is an example of an inductive argument)

Section 6.3

Truth Tables

Logical Symbols

Connector	Symbol
And	\wedge
Or	\vee
If-then	\rightarrow
Negation	\sim

Truth Tables

A truth table is a chart consisting of all possible combinations of the clauses in the statement.

To full understand truth tables, you must first understand the basic truth table for the basic connectors

Basic Truth Tables

Or “ \vee ”

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

If you let A = John likes apples and B = John likes oranges, then the statement $A \vee B$ or “Either John likes apples or John likes oranges” is always true except when both A and B are false.

And “ \wedge ”

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

If you let A = John likes apples and B = John likes oranges, then the statement $A \wedge B$ or “John likes apples and John likes oranges” is only true when both A and B are true.

If – then “ \rightarrow ”

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Let $A \rightarrow B$ be then conditional statement “If you study for the test, then you will pass the test”. The only time this statement is false is if you would study for the test and not pass the test, which is “ $T \rightarrow F$ ”

Negation

A	$\sim A$
T	F
F	T

More Truth Tables

Use the results from the four above truth tables.

1) $(A \wedge B) \vee A$

A	B	$A \wedge B$	$(A \wedge B) \vee A$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

2) $(A \wedge B) \vee \sim B$

A	B	$\sim B$	$A \wedge B$	$(A \wedge B) \vee \sim B$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

$$3) (A \rightarrow B) \wedge \sim A$$

A	B	$\sim A$	$A \rightarrow B$	$(A \rightarrow B) \wedge \sim A$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$$4) \sim B \rightarrow \sim A$$

A	B	$\sim A$	$\sim B$	$\sim B \rightarrow \sim A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

$$5) (\sim A \rightarrow \sim B) \wedge A$$

A	B	$\sim A$	$\sim B$	$\sim A \rightarrow \sim B$	$(\sim A \rightarrow \sim B) \wedge A$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	T	F

$$6) B \rightarrow A$$

A	B	$B \rightarrow A$
T	T	T
T	F	T
F	T	F
F	F	T

If the last column of an argument result in all true statements, then the argument is a tautology.

Equivalent statements

Two statements are equivalent if they have the same result in last column of their truth tables.

Examples

Show that the following arguments are equivalent: $A \rightarrow B$ and $\sim B \rightarrow \sim A$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$\sim A$	$\sim B$	$\sim B \rightarrow \sim A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Show that the following arguments are equivalent: $\sim A \rightarrow \sim B$ and $B \rightarrow A$

A	B	$B \rightarrow A$
T	T	T
T	F	T
F	T	F
F	F	T

A	B	$\sim A$	$\sim B$	$\sim A \rightarrow \sim B$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

De Morgan's Law

Negation of compound Statements

De Morgan's Law

$$\sim (A \vee B) = \sim A \wedge \sim B$$

$$\sim (A \wedge B) = \sim A \vee \sim B$$

Proof:

Compare the truth tables for $\sim (A \vee B)$ and $\sim A \wedge \sim B$

Truth table for $\sim (A \vee B)$

A	B	$A \vee B$	$\sim (A \vee B)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Truth table for $\sim A \wedge \sim B$

A	B	$\sim A$	$\sim B$	$\sim A \wedge \sim B$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Notice that the last columns of each table are identical. Thus, the arguments are equivalent.

Compare the truth tables for $\sim (A \wedge B)$ and $\sim A \vee \sim B$

Truth table for $\sim (A \wedge B)$

A	B	$A \wedge B$	$\sim (A \wedge B)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Truth table for $\sim A \vee \sim B$

A	B	$\sim A$	$\sim B$	$\sim A \vee \sim B$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Again the truth tables have the same last column. Thus, the statements are equivalent.

Using De Morgan's Law to negate compound statements

Examples

Negate each statement using De Morgan Law

1) $\sim A \vee \sim B$

Negation: $\sim (\sim A \vee \sim B) = \sim (\sim A) \wedge \sim (\sim B) = A \wedge B$

2) $\sim p \vee q$

Negation: $\sim (\sim p \vee q) = \sim (\sim p) \wedge \sim q = p \wedge \sim q$

3) $r \wedge \sim s$

Negation: $\sim (r \wedge \sim s) = \sim r \vee \sim (\sim s) = \sim r \vee s$

4) $r \wedge s$

Negation: $\sim (r \wedge s) = \sim r \vee \sim s = \sim r \vee \sim s$

Validity

An argument is valid if the truth table of the argument in symbolic form is a tautology

1) Determine if the following argument is valid

If it rains on Friday, then I will bring my umbrella to work on Friday

I didn't bring my umbrella to work on Friday

Therefore, it didn't rain on Friday.

Let:

A = It rains on Friday

B = I will bring my umbrella to work.

If it rains on Friday, then I will bring my umbrella to work on Friday ($A \rightarrow B$)

I didn't bring my umbrella to work on Friday $\sim B$

Therefore, it didn't rain on Friday. $\sim A$

Argument in symbol form: $((A \rightarrow B) \wedge \sim B) \rightarrow \sim A$

A	B	$\sim A$	$\sim B$	$(A \rightarrow B)$	$(A \rightarrow B) \wedge \sim B$	$((A \rightarrow B) \wedge \sim B) \rightarrow \sim A$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Since the argument results in all true statements, the argument is a tautology

Thus, the argument is valid

2) Determine if the following argument is valid

A = It is raining

B = the streets are wet

If it is raining, then the streets are wet ($A \rightarrow B$)

It is raining A

Therefore, the streets are wet. B

Argument: $((A \rightarrow B) \wedge A) \rightarrow B$

A	B	$(A \rightarrow B)$	$((A \rightarrow B) \wedge A)$	$((A \rightarrow B) \wedge A) \rightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

This is a tautology. Therefore, the argument is valid.

3)

P: You exercise regularly

Q: You are healthy

If you exercise regularly, then you are healthy ($P \rightarrow Q$)

You are healthy Q

Therefore, you exercise regularly P

Argument: $((P \rightarrow Q) \wedge Q) \rightarrow P$

P	Q	$(P \rightarrow Q)$	$((P \rightarrow Q) \wedge Q)$	$((P \rightarrow Q) \wedge Q) \rightarrow P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

This argument is not a tautology. Thus, the argument is invalid

4)

The senator is not reelected, if she supports new taxes

The senator does not support new taxes

Therefore, the senator is reelected

A = the senator support new taxes

B = the senator is reelected

The senator is not reelected, if she supports new taxes $A \rightarrow \sim B$

The senator does not support new taxes $\sim A$

Therefore, the senator is reelected B

Argument: $((A \rightarrow \sim B) \wedge \sim A) \rightarrow B$

A	B	$\sim A$	$\sim B$	$(A \rightarrow \sim B)$	$(A \rightarrow \sim B) \wedge \sim A$	$((A \rightarrow \sim B) \wedge \sim A) \rightarrow B$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

5)

If you practice hard, you will improve your skills

You didn't improve your skills

Therefore, you did not practice hard.

P = you practice hard

Q = you will improve your skills

If you practice hard, you will improve your skills $(P \rightarrow Q)$

You didn't improve your skills $\sim Q$

Therefore, you did not practice hard. $\sim P$

Argument: $((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$

P	Q	$\sim P$	$\sim Q$	$(P \rightarrow Q)$	$(P \rightarrow Q) \wedge \sim Q$	$((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Valid Argument