

Networks and graphs

Key terms

Vertex (Vertices)

Each point of a graph

Edge

An edge is a segment that connects two vertices.

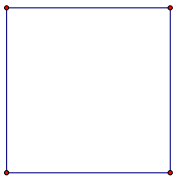
Region

A region is each individual area or separate piece of the plane that is divided up by the network.

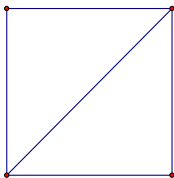
Example 1

Complete a table for the following networks.

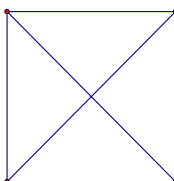
a)



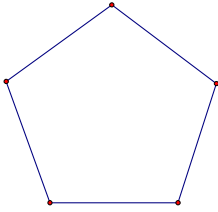
b)



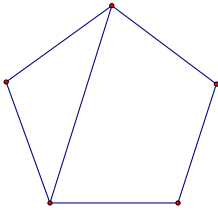
c)



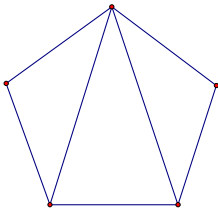
d)



e)



f)



Graph	Edges	Vertices	Regions	$V + R - 2$
A	4	4	2	$4 + 2 - 2 = 4$
B	5	4	3	$4 + 3 - 2 = 5$
C	8	5	5	$5 + 5 - 2 = 8$
D	5	5	2	$5 + 2 - 2 = 5$
E	6	5	3	$5 + 3 - 2 = 6$
F	7	5	4	$5 + 4 - 2 = 7$

A network is said to **traversable** if it can be traced in one sweep without lifting the pencil from the paper and without tracing the same edge more than once.

The **degree of a vertex** is the number of edges that meet at that vertex.

Graph	Number of edges	Degree of each vertex	Sum	Traversable
A	4	2,2,2,2	8	Yes
b	5	3,2,3,2	10	Yes
c	8	3,3,3,3,4	16	No
d	5	2,2,2,2,2	10	Yes
e	6	3,2,2,3,2	12	Yes
f	7	4,2,3,3,2	14	Yes

Rules for the number odd vertices

- 1) If the network has no odd vertices, then the network is traversable and any point is a starting point. The starting point will also turn out to be the ending point.
- 2) If the network has exactly one odd vertex, then the network is not traversable. A network cannot have only one starting point or ending point without the other.
- 3) If the network has two odd vertices, then the network is traversable. One odd vertex must be the starting point and the other odd vertex must be the ending point.
- 4) If the network has more than two odd vertices, then the network is not traversable. A network cannot have more than one starting point and one ending point.

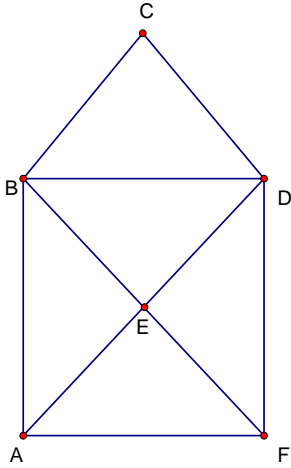
Euler Circuits

Definition

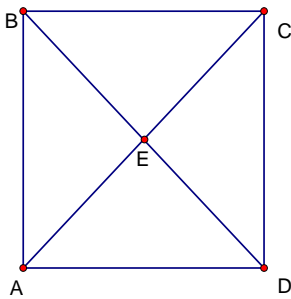
A network is an **Euler circuit** if you can start at one vertex and return to that vertex in one sweep without lifting your pencil and without tracing over the same edge more than once.

Example 2

Network 1



Network 2



Which of the following networks have an Euler circuit?

Network 1 is traversable since the graph has two odd vertices and four even vertices. (See rule above) Vertices A and F are odd and vertices B, C, D, and E are even. However, the network does not have an Euler circuit because the path that is traversable has different starting and ending points.

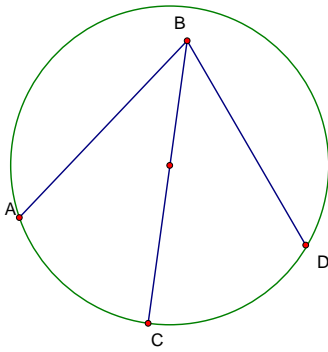
Network 2 is not even traversable because it has four odd vertices which are A, B, C, and D. Thus, the network will not have an Euler circuit.

Hamiltonian cycles

In a Hamilton cycle, you must start from a given vertex and visit each vertex only once, then return to the original vertex.

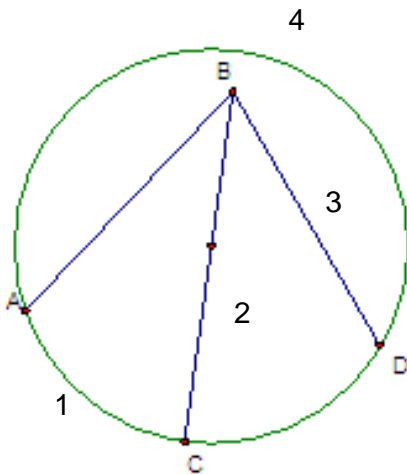
Example

Find the Hamiltonian cycle for the given network

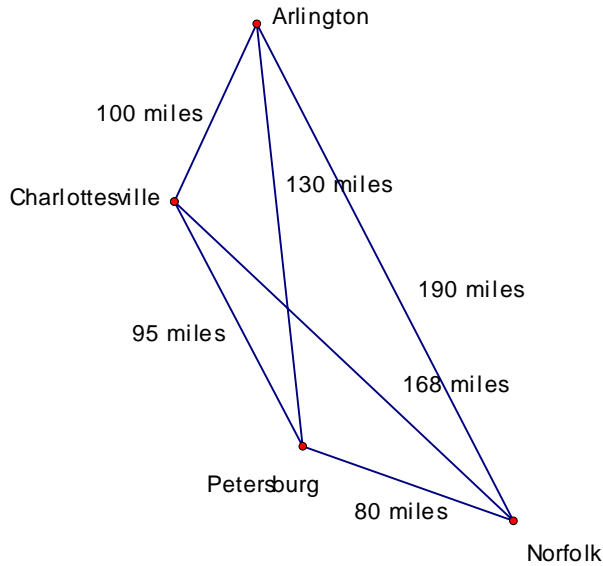


The Hamiltonian cycle would be $A \rightarrow C \rightarrow B \rightarrow D \rightarrow A$

This pattern is shown below



2) A salesman wants to visit four Virginia cities, Arlington, Charlottesville, Petersburg, and Norfolk. Driving distances are shown in figure 1. What is the shortest trip starting and ending in Arlington?



A=Arlington
 N=Norfolk
 P=Petersburg
 C=Charlottesville

Different Paths

$$A \xrightarrow{100} C \xrightarrow{95} P \xrightarrow{80} N \xrightarrow{190} A \quad \text{Total Miles} = 465$$

$$A \xrightarrow{130} P \xrightarrow{80} N \xrightarrow{168} C \xrightarrow{100} A \quad \text{Total Miles} = 478$$

$$A \xrightarrow{190} N \xrightarrow{168} C \xrightarrow{95} P \xrightarrow{130} A \quad \text{Total Miles} = 583$$

The shortest path would be

Arlington → Charlottesville → Petersburg → Norfolk → Arlington

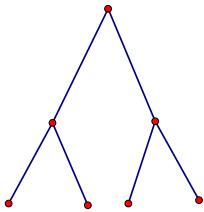
Section 6.2

Trees and Minimum Spanning Trees

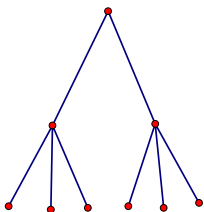
A **tree** is a graph that is connected and has no circuits.

Examples of trees

1)

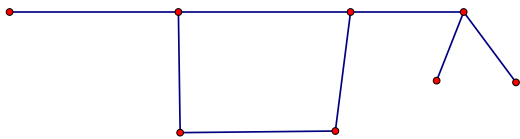


2)



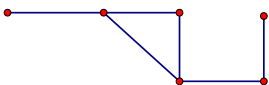
Examples of thing that are not trees

1)



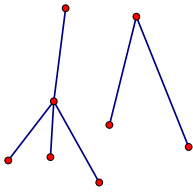
This object contains a circuit.

2)



This object contains a circuit.

3)



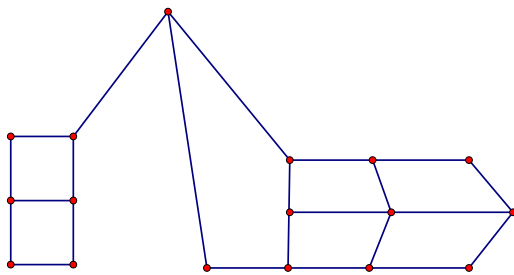
This object is not connected

Spanning Trees

A tree that is created from another graph by removing edges but keeping the path to each vertex is called a **spanning tree**.

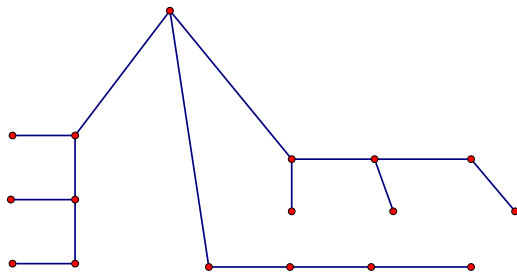
Example 1

Original Graph



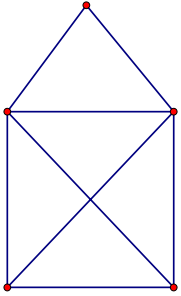
Remove all edges without eliminating any paths to each vertex. This will result in the tree in the diagram 2

Diagram 2

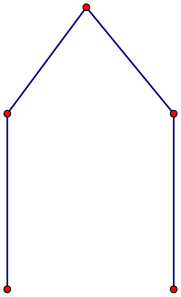


Example 2

Graph



Spanning Tree



Example 3

Graph

