Unit 6
Logic
Math 116

Logic Unit

Statement: A group words or symbols that can be classified as true or false.

Examples of statements

Violets are blue
Five is a natural number
I like Algebra
3 + 7 = 10

Examples of things that are not statements.

Get out of here.
What's up!
Don't worry, be happy

Conditional Statements

Conditional: $A \rightarrow B$

Converse: $B \rightarrow A$

Inverse $\sim A \rightarrow \sim B$

Contrapositive: $\sim B \rightarrow \sim A$

Logical Symbols

Connector	Symbol
And	$\wedge$
Or	$\vee$
If-then	$\rightarrow$
Negation	$\sim$

Negations

Examples:

1) p = I like apples
Negation: ~p (I don't like apples)

Note: The negation of all is some and the negation of some is all.

Examples

2) All RU students love ice cream

Negation: Some RU students do not like ice cream.

3) Some students dislike geometry

Negation: All students like geometry

4) Everyone loves Raymond

Negation: Someone does love Raymond

Write a sentence that is the negation of each statement

a) Her dress is not red
Negation: Her dress is red.

b) Some elephants are pink.
Negation: All elephants are not pink

c) All candy promotes tooth decay.
Negation: Some candy does not promote tooth decay.

d) No lunch is free.
Negation: Some lunches are free.

Examples

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10) We call it the period from noon to sunset
Not a statement

12) An obtuse angle is not an 90^0 angle.

Statement

14) A pencil is a writing implement.

Statement

Negate each statement

20) My dog is a Dalmatian.

Negation: My dog is not a Dalmatian.

22) The jokes are great.

Negation: The jokes are not great.

33) All fish can live under water.

Negation: Some fish can not live under water.

35) Some numbers are not prime numbers.

Negation: All numbers are prime numbers

Write the inverse, converse, and contrapositive of each statement

If you study the test, then you will pass the test

Inverse: If you will pass the test, then you will study for the test

Converse: If you don't study for the test, then you will not pass the test

Contrapositive: If you didn't pass the test, then you didn't study for the test.

When I am a sleep, nothing bothers me

Inverse: If nothing bothers me, then I am sleeping

Converse: If something bothers me, then I am not sleeping.

Contrapositive: If I am not sleeping, then something bothers me.

Section 6.3

Truth Tables

Logical Symbols

Connector	Symbol
And	$\wedge$
Or	$\vee$
If-then	$\rightarrow$
Negation	~

Truth Tables

A truth table is a chart consisting of all possible combinations of the clauses in the statement.

To full understand truth tables, you must first understand the basic truth table for the basic connectors

Basic Truth Tables

Or "$\wedge$"

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

If you let A = John likes apples and B = John likes oranges, then the statement $A \vee B$ or "Either John likes apples or John likes oranges" is always true except when both A and B are false.

And "$\wedge$"

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

If you let A = John likes apples and B = John likes oranges, then the statement $A \wedge B$ or "John likes apples and John likes oranges" is only true when both A and B are true.

If – then "$\rightarrow$"

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Let $A \rightarrow B$ be then conditional statement "If you study for the test, then you will pass the test". The only time this statement is false is if you would study for the test and not pass the test, which is "$T \rightarrow F$"

Negation

A	~A
T	F
F	T

More Truth Tables

Use the results from the four above truth tables.

1) $(A \wedge B) \vee A$

A	B	$A \wedge B$	$(A \wedge B) \vee A$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

2) $(A \wedge B) \vee \sim B$

A	B	~B	$A \wedge B$	$(A \wedge B) \vee \sim B$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

3) $(A \rightarrow B) \wedge \sim A$

A	B	~A	$A \rightarrow B$	$(A \rightarrow B) \vee \sim A$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

4) $\sim B \rightarrow \sim A$

A	B	~A	~B	$\sim B \rightarrow \sim A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

5) $(\sim A \rightarrow \sim B) \wedge A$

A	B	~A	~B	$\sim A \rightarrow \sim B$	$(\sim A \rightarrow \sim B) \wedge A$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	T	F

6) $B \rightarrow A$

A	B	$B \rightarrow A$
T	T	T
T	F	T
F	T	F
F	F	T

If the last column of an argument result in all true statements, then the argument is a tautology.

Equivalent statements

Two statements are equivalent if they have the same result in last column of their truth tables.

Examples

Show that the following arguments are equivalent: $A \rightarrow B$ and $\sim B \rightarrow \sim A$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	~A	~B	$\sim B \rightarrow \sim A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Show that the following arguments are equivalent: $\sim A \rightarrow \sim B$ and $B \rightarrow A$

A	B	$B \rightarrow A$
T	T	T
T	F	T
F	T	F
F	F	T

A	B	~A	~B	$\sim A \rightarrow \sim B$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

De Morgan's Law

Negation of compound Statements

De Morgan's Law

$\sim (A \vee B) = \sim A \wedge \sim B$
$\sim (A \wedge B) = \sim A \vee \sim B$

Proof:

Compare the truth tables for $\sim (A \vee B)$ and $\sim A \wedge \sim B$

Truth table for $\sim (A \vee B)$

A	B	$A \vee B$	$\sim (A \vee B)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Truth table for $\sim A \wedge \sim B$

A	B	~A	~B	$\sim A \wedge \sim B$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Notice that the last columns of each table are identical. Thus, the arguments are equivalent.

Compare the truth tables for $\sim (A \wedge B)$ and $\sim A \vee \sim B$

Truth table for $\sim (A \wedge B)$

A	B	$A \wedge B$	$\sim (A \wedge B)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Truth table for $\sim A \vee \sim B$

A	B	~A	~B	$\sim A \vee \sim B$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Again the truth tables have the same last column. Thus, the statements are equivalent.

Using De Morgan's Law to negate compound statements

Examples

Negate each statement using De Morgan Law

1) $\sim A \vee \sim B$

Negation: $\sim (\sim A \vee \sim B) = \sim (\sim A) \wedge \sim (\sim B) = A \wedge B$

2) $\sim p \vee q$

Negation: $\sim (\sim p \vee q) = \sim (\sim p) \wedge \sim q = p \wedge \sim q$

3) $r \wedge \sim s$

Negation: $\sim (r \wedge \sim s) = \sim r \vee \sim (\sim s) = \sim r \vee s$

4) $r \wedge s$

Negation: $\sim (r \wedge s) = \sim r \vee \sim s = \sim r \vee \sim s$

Validity

An argument is valid if the truth table of the argument in symbolic form is a tautology

1) Determine if the following argument is valid

If it rains on Friday, then I will bring my umbrella to work on Friday
I didn't bring my umbrella to work on Friday
Therefore, it didn't rain on Friday.

Let:

A = It rains on Friday
B = I will bring my umbrella to work.

If it rains on Friday, then I will bring my umbrella to work on Friday ($A \rightarrow B$)
I didn't bring my umbrella to work on Friday $\sim B$
Therefore, it didn't rain on Friday. $\sim A$

Argument in symbol form: $((A \rightarrow B) \wedge \sim B) \rightarrow \sim A$

A	B	~A	~B	$(A \rightarrow B)$	$(A \rightarrow B) \wedge \sim B$	$((A \rightarrow B) \wedge \sim B) \rightarrow \sim A$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Since the argument results in all true statements, the argument is a tautology
Thus, the argument is valid

2) Determine if the following argument is valid

A = It is raining
B = the streets are wet

If it is raining, then the streets are wet $(A \rightarrow B)$
It is raining A
Therefore, the streets are wet. B
Argument: $((A \rightarrow B) \wedge A) \rightarrow B$

A	B	$(A \rightarrow B)$	$((A \rightarrow B) \wedge A)$	$((A \rightarrow B) \wedge A) \rightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

This is a tautology. Therefore, the argument is valid.

3)

P: You exercise regularly
Q: You are healthy

If you exercise regularly, then you are healthy $(P \rightarrow Q)$
You are healthy Q
Therefore, you exercise regularly P

Argument: $((P \rightarrow Q) \wedge Q) \rightarrow P$

P	Q	$(P \rightarrow Q)$	$((P \rightarrow Q) \wedge Q)$	$((P \rightarrow Q) \wedge Q) \rightarrow P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

This argument is not a tautology. Thus, the argument is invalid

4)

The senator is not reelected, if she supports new taxes
The senator does not support new taxes
Therefore, the senator is reelected

A = the senator support new taxes
B = the senator is reelected

The senator is not reelected, if she supports new taxes $A \rightarrow \sim B$
The senator does not support new taxes $\sim A$
Therefore, the senator is reelected B

Argument: $((A \rightarrow \sim B) \wedge \sim A) \rightarrow B$

A	B	~A	~B	$(A \rightarrow \sim B)$	$(A \rightarrow \sim B) \wedge \sim A$	$((A \rightarrow \sim B) \wedge \sim A) \rightarrow B$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

5)

If you practice hard, you will improve your skills
You didn't improve your skills
Therefore, you did not practice hard.

P = you practice hard
Q = you will improve your skills

If you practice hard, you will improve your skills $(P \rightarrow Q)$
You didn't improve your skills $\sim Q$
Therefore, you did not practice hard. $\sim P$

Argument: $((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$

P	Q	~P	~Q	$(P \rightarrow Q)$	$(P \rightarrow Q) \wedge \sim Q$	$((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Valid Argument