Section 6.1

Linear Modeling

The linear model

$y = mx + b$

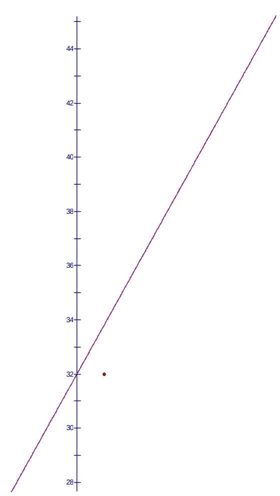
$m = slope$
$b = y - \text{int}$

Example: Converting temperature in Celsius to Fahrenheit

Given the equation $F = \frac{9}{5}C + 32$

1) Sketch a graph of the function

Temperature in Celsius	Temperature in Fahrenheit
0	$F = \frac{9}{5} \cdot 0 + 32 = 0 + 32 = 32$
20	$F = \frac{9}{5}(20) + 32 = 36 + 32 = 68$
80	$F = \frac{9}{5}(80) + 32 = 9(16) + 32 = 144 + 32 = 176$



2) Find the slope: Use $y = mx + b$ which makes the slope $m = \frac{9}{5}$

3) Find the Fahrenheit equivalent of 40° C\

$$F = \frac{9}{5}(40) + 32 = 9(8) + 32 = 72 + 32 = 104^0 F$$

Example 2

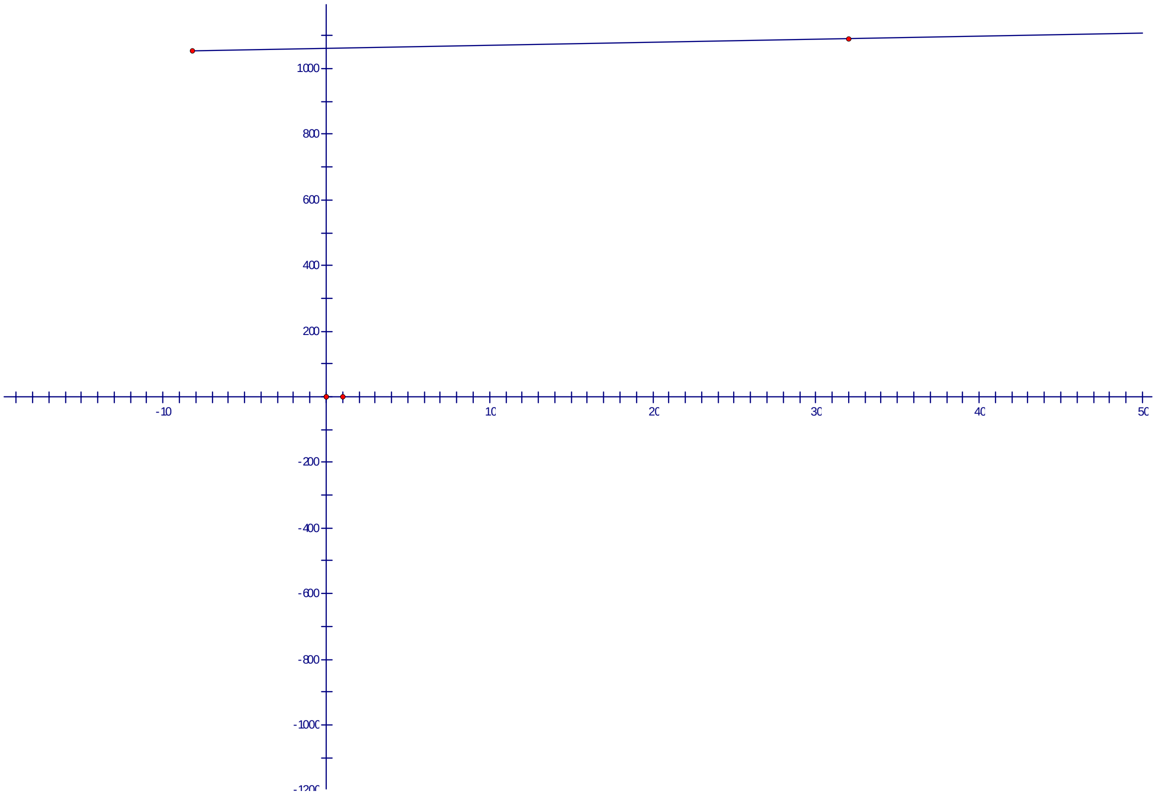
The speed of sound has been calculated to be approximately 1090 ft/s, when the temperature is 32° F. However, as the temperature rises above 32° F, the speed of sound is about 1110 ft/s. Find the linear equation that relates the sped of sound to the Fahrenheit temperature and determine the speed of sound at 100° F

First use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Use the points (32,1090) and (50,1100) to find the slope of the function.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1110 - 1090}{50 - 32} = \frac{20}{18} = \frac{10}{9}$$

find



Find the y-intercept

$$y = mx + b$$
$$1090 = \frac{10}{9}(32) + b$$
$$1090 = \frac{320}{9} + b$$
$$1090 = 35.6 + b$$
$$b = 1090 - 35.6$$
$$b = 1054.4$$

Thus, the linear equation would be $y = mx + b \Rightarrow y = \frac{10}{9}x + 1054.4$

Use this equation to calculated the speed of sound at 100° F

$$y = \frac{10}{9}(100) + 1054.4 = \frac{1000}{9} + 1054.4 = 111.1 + 1054.4 = 1165.4$$

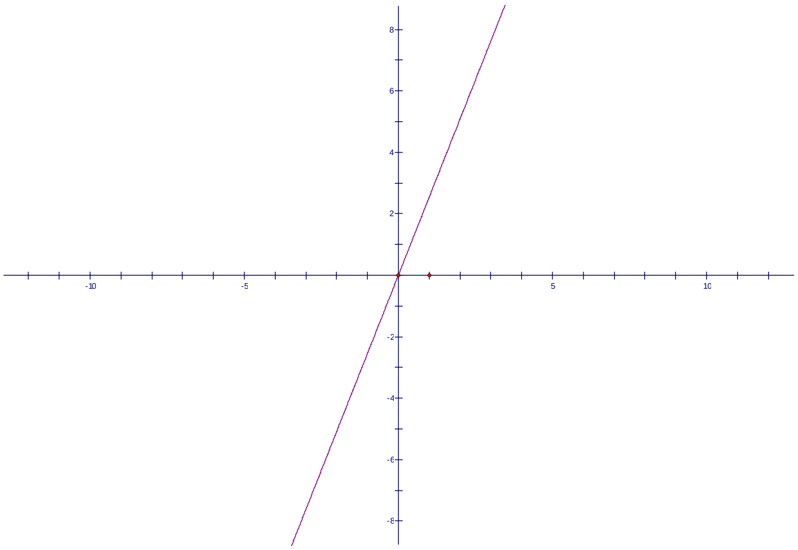
Examples for the book exercises

8) $c = 2.54i$
$m = 2.54$

Make a table of values

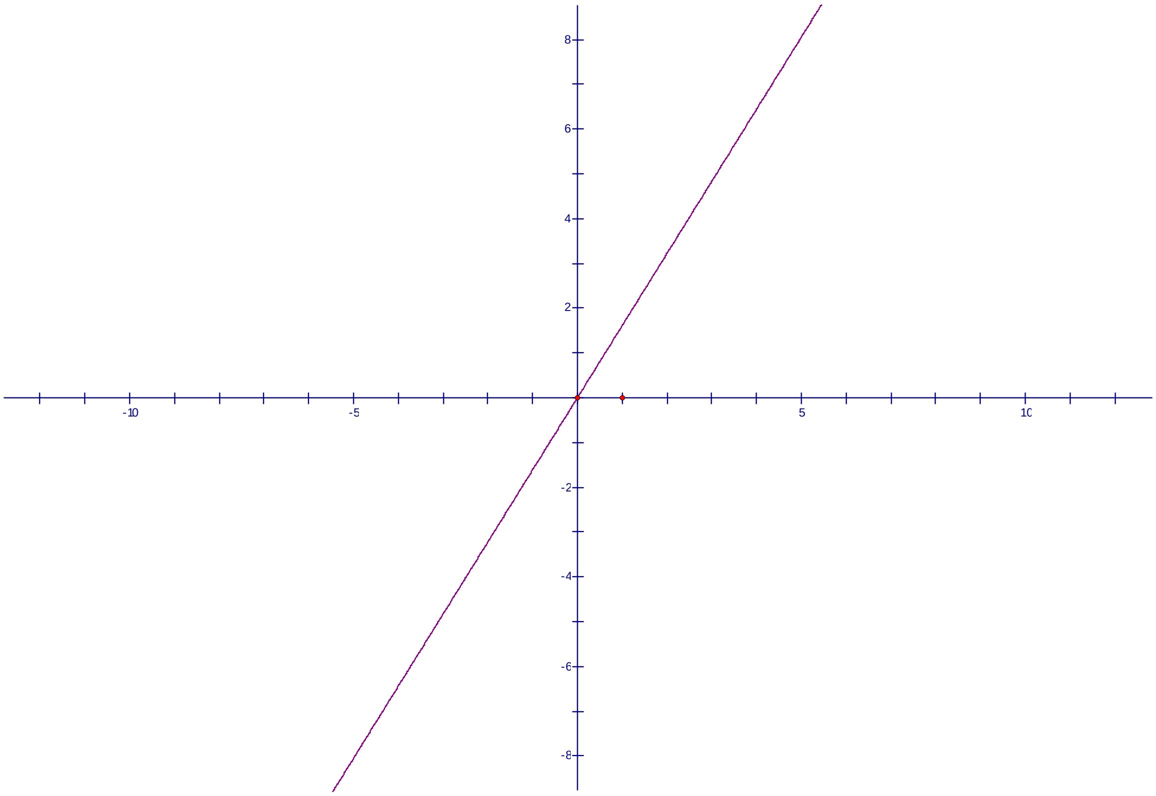
C	I
0	0
2	2.54(2) = 5.08
10	2.54(10) = 25.4

Plot the given points



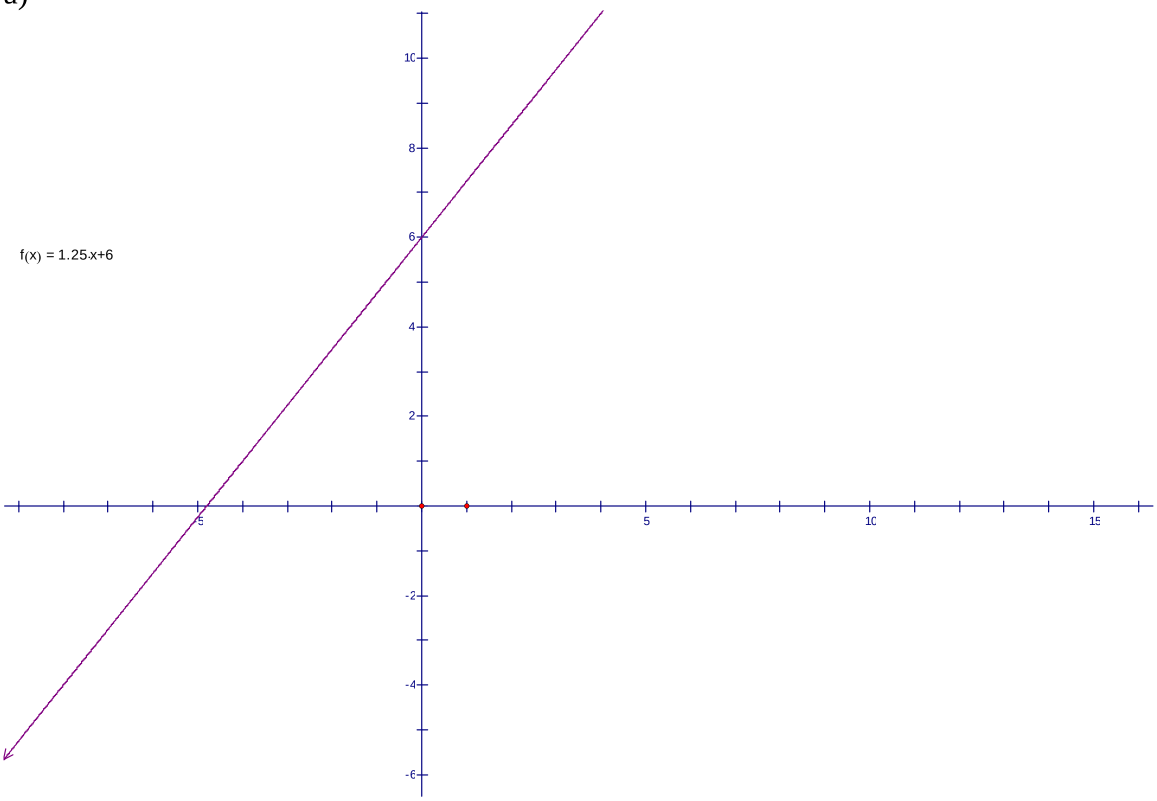
12) $k = \frac{1000}{621} m$

K	M
0	0
10	$\frac{1000}{621}(10) = 16.1$
20	$\frac{1000}{621}(20) = 32.2$



The equation C = 1.25 x + 6 gives the cost C is dollars when x French wine bottles are produced.

a)



b)
What is the slope of the equation
Answer; m=1.25

c) Find the cost of making 100 bottles of wine

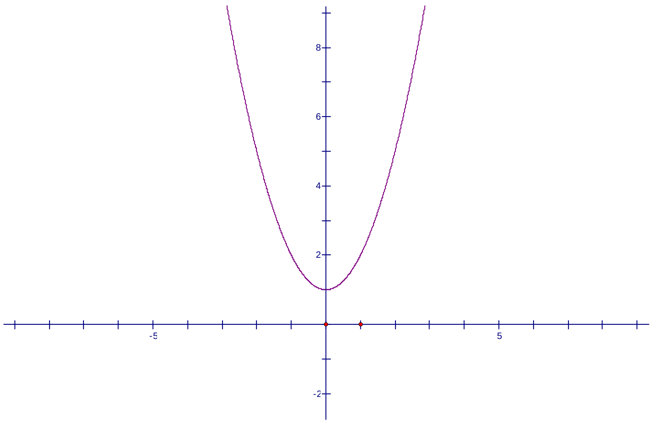
$$y = 1.25(100) + 6 = 125 + 6 = \$131$$

d) How many wine bottles could be made with $1000

$$1000 = 1.25x + 6$$
$$1000 - 6 = 1.25x + 6 - 6$$
$$994 = 1.25x$$
$$x = \frac{994}{1.25}$$
$$x = 795.2$$
$$Answer;\ 795\ bottles$$

Quadratic Models

Graph of Quadratic Models



The parabola

A **quadratic function** is a function where the graph is a parabola and an equation of the form: $y = ax^2 + bx + c$ where $a = 0$

The x coordinate vertex is given by the equation: $x = -\frac{b}{2a}$

Examples

Find the vertex and x-intercepts, then make a sketch of the parabola.

1)
$y = x^2 - 3$

$a = 1, c = -3$

$x = -\frac{0}{2(1)} = -\frac{0}{2} = 0$

$x - \text{int}\,ercepts$

$x^2 - 3 = 0$

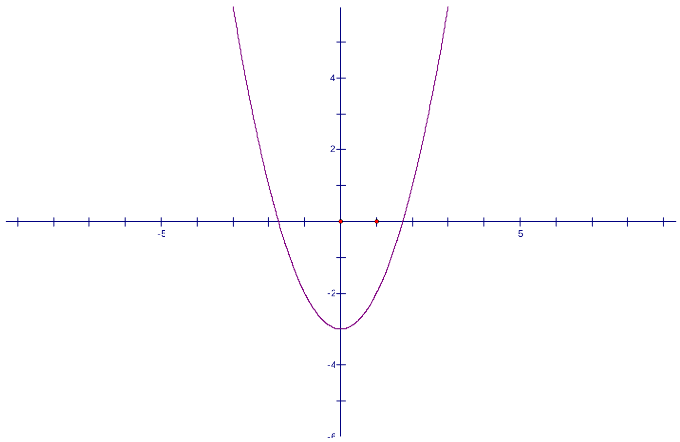
$x^2 = 3$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

$(\sqrt{3}, 0)$ *and* $(-\sqrt{3}, 0)$

Graph



2)

$y = x^2 - 3x$

Vertex

$x = -\frac{-3}{2(1)} = \frac{3}{2}$

$x - \text{int}$

$x^2 - 3x = 0$

$x(x-3) = 0$

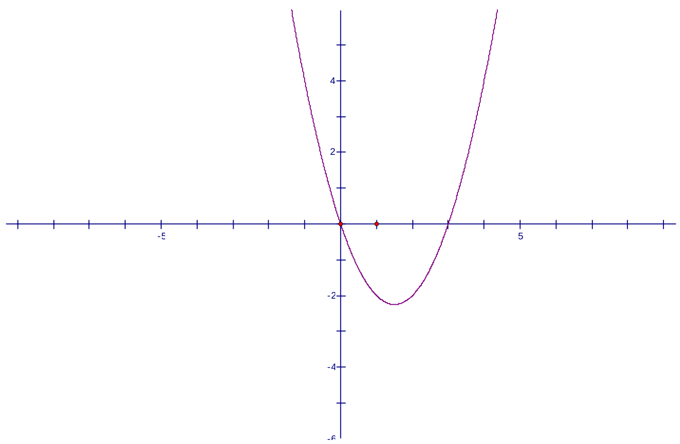
$x = 0 \ \ or \ \ x - 3 = 0$

$x = 0 \qquad x - 3 = 0$

$\qquad\qquad x = 3$

$(0,0) \ \ and \ \ (3,0)$

Graph of the function



Using the quadratic formula to solve an equation

The Quadratic Formula

The solution to the equation $y = ax^2 + bx + c$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1) Solve $x^2 + 5x - 7 = 0$

$$a = 1$$
$$b = 5$$
$$c = -7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 28}}{2} = \frac{-5 \pm \sqrt{53}}{2}$$

2) Solve $x^2 + 7x - 9 = 0$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-9)}}{2(7)} = \frac{-7 \pm \sqrt{49 + 36}}{2(7)} = \frac{-7 \pm \sqrt{85}}{14}$$

Page 301

8) Find the vertex, graph, and x intercepts of each parabola

$$y = -x^2 + 6x - 5.5$$
$$a = -1$$
$$b = 6$$
$$c = -5.5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-5.5)}}{2(-1)} = \frac{-6 \pm \sqrt{36 - 22}}{-2} = \frac{-6 \pm \sqrt{14}}{-2}$$

x-intercepts

$(-6 + \sqrt{14}, 0)$ *and* $(-6 - \sqrt{14}, 0)$

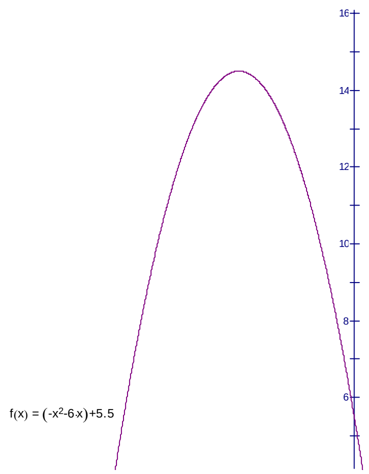
Vertex

$$x = -\frac{6}{2(-1)} = 3$$

$y - coordinate$

$$y = -3^2 + 6(3) - 5.5 = -9 + 18 - 5.5 = 4.5$$

(3,4.5)



12) At a local frog jumping contest. Rivet's jump can be approximated by the equation $y = -\frac{1}{6}x^2 + 2x$ and Croak's jump can be approximate by $y = -\frac{1}{2}x^2 + 4x$, where x = the length of jump in feet and y = the height of the jump in feet.

a) Which frog can jump higher

Rivet's vertex: $x = -\frac{2}{2\left(-\frac{1}{6}\right)} = -\frac{2}{-\frac{1}{3}} = 6$ Height: $y = -\frac{1}{6}(6)^2 + 2(6) = -6 + 12 = 6 \ ft$

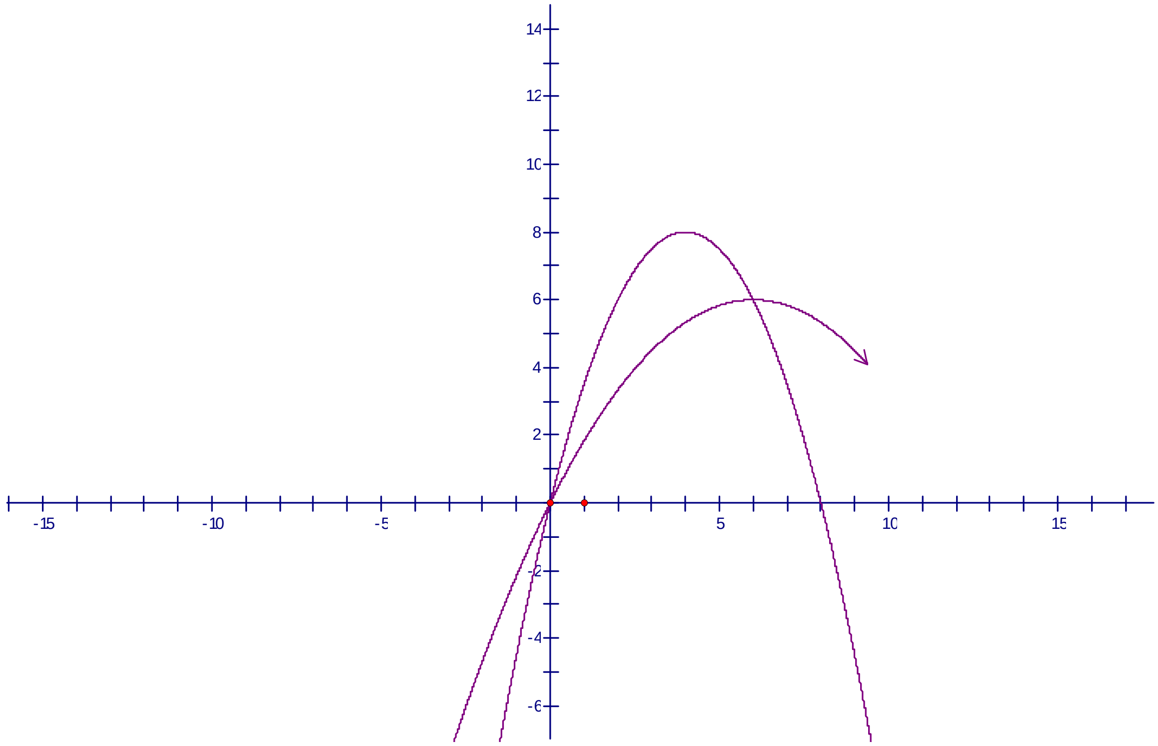
Croak's vertex: $x = -\frac{4}{2\left(-\frac{1}{2}\right)} = \frac{-4}{-1} = 4$ Height: $y = -\frac{1}{2}(4)^2 + 4(4) = -8 + 16 = 8 \ ft$

Croak can jump higher at 8 feet

b) Which frog can jump farther

Rivet's can jump farther at 2(6 ft) = 12 feet

Graph of both frog jumps (Rivet's jump and Croak's jump)



Exponential models

The exponential function

$e \approx 2.718$ "The Euler number"

Examples

$e^2 = 7.39$

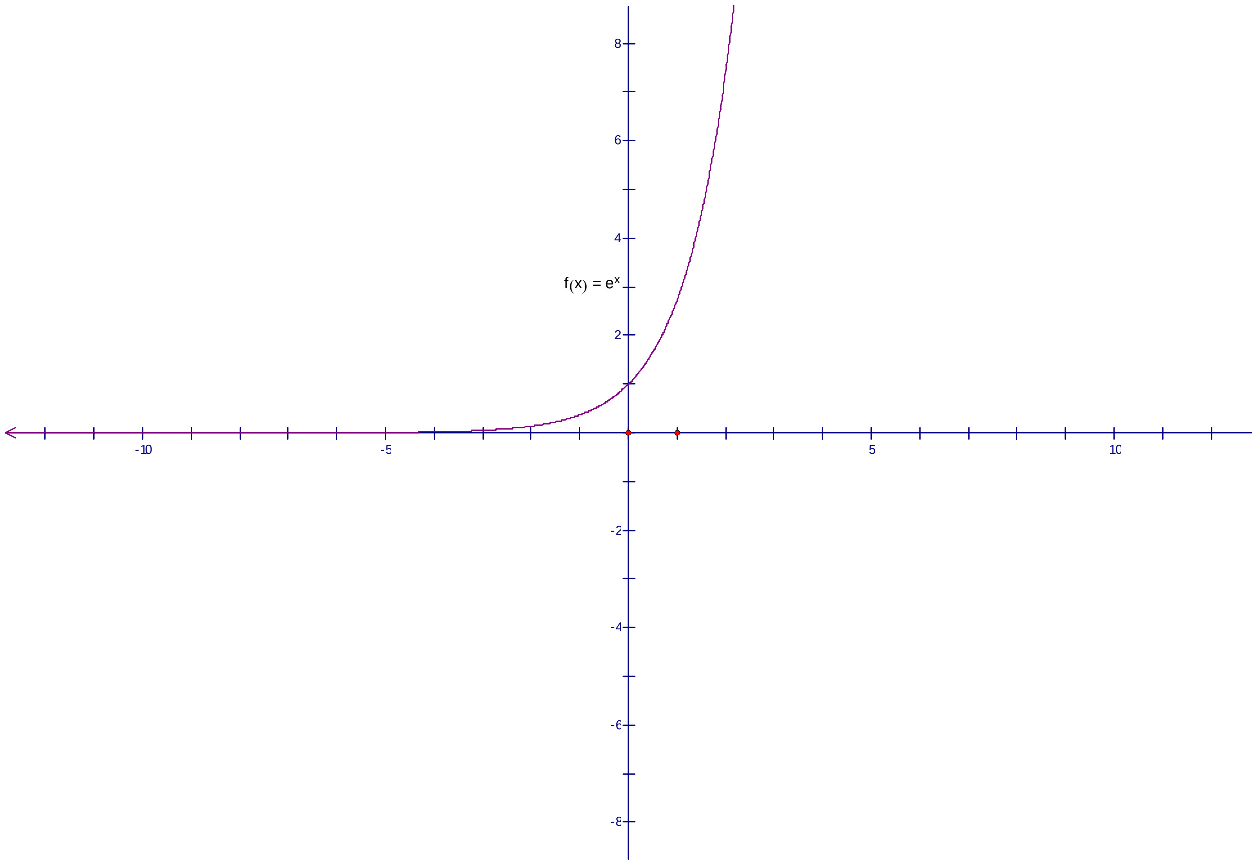
$e^{-3} = \frac{1}{e^3} = .05$

$e^{\frac{1}{3}} = 1.40$

The graph of the exponential function

1) Graph $y = e^x$

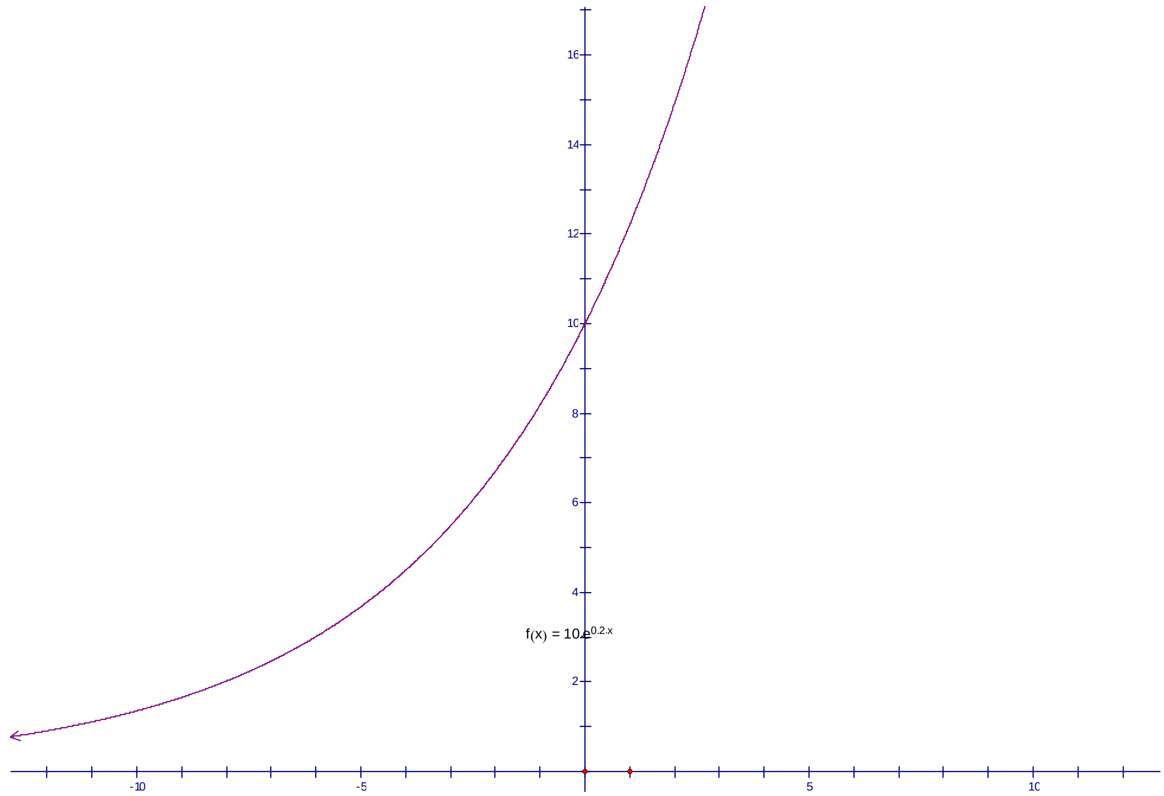
x	y
-2	$y = e^{-2} = .14$
-1	$y = e^{-1} = .37$
0	$y = e^{o} = 1$
1	$y = e^{1} = 2.7$
2	$y = e^{2} = 7.4$



2) Graph $y = 10e^{.2x}$

x	y
-2	$y = 10e^{.2(-2)} = 10e^{-.4} = 6.7$
-1	$y = 10e^{.2(-1)} = 10e^{-.2} = 8.2$
0	$y = 10e^{.2(0)} = 10e^{0} = 10$
1	$y = 10e^{.2(1)} = 10e^{.2} = 12.2$
2	$y = 10e^{.2(2)} = 10e^{.4} = 14.9$

Graph of $y = 10e^{.2x}$



Exponential Models

Exponential Growth

$P = P_0(1+r)^t$
$P = New\ Value$
$P_0 = Original\ Value$
$r = rate$
$t = time$

Examples

1) The population of the United States is 290 million, what would be the population of the U. S. be in 20 years if its population would growth at a steady rate of .7 % for 20 years?

$P = P_0(1+r)^t$
$P_0 = 290{,}000{,}000$
$r = .7\% = .007$
$t = 20$
$P = 290000000(1+.007)^{20} = 290000000(1.007)^{20} = 333416746$

2) The population of Blacksburg, Virginia is 41,000, what would be in 10 years Blacksburg would grow at a rate of 1.1 % per year?

$P = P_0(1+r)^t$
$P_0 = 41000$
$r = 1.1\% = .011$
$t = 10$
$P = 41000(1+.011)^{10} = 41000(1.01)^{10} = 45739.92$

3) In 1995 the United States had greenhouse emissions of about 1400 million tons, where as China had greenhouse emissions of about 850 million tons. If in the next 25 years China greenhouse emission grew by 4 percent and the U. S. greenhouse emission grew by 1.3 percent, what would the emissions in tons for both countries in 2020?

$U.S.\ Emissions\ in\ 2020$

$P = P_0(1+r)^t$

$P_0 = 1400\ million$

$r = 1.3\% = .013$

$t = 25$

$P = 1400(1+.013)^{25} = 1400(1.013)^{25} = 1900\ million\ tons$

$China's\ Emissions\ in\ 2020$

$P = P_0(1+r)^t$

$P_0 = 850\ million$

$r = 4.0\% = .04$

$t = 25$

$P = 850(1+.04)^{25} = 850(1.04)^{25} = 2300\ million\ tons$

4) Using the exponential growth formula, find the amount of money that you would have in a bank account if you deposited $3,000 in the account for 15 years at 1.1 % interest rate.

$P = P_0(1+r)^t$

$P_0 = 3000$

$r = 1.1\% = .011$

$t = 15$

$P = 3000(1+.011)^{15} = 3000(1.011)^{15} = \$3482.91$

Exponential decay

$P = P_0(1-r)^t$

$P = New\ Value$

$P_0 = Original\ Value$

$r = rate$

$t = time$

5) A certain population of black bears in the eastern United States has been decreasing by 3.1 percent per year. If this trend keeps up, what will be the population of bears in 20 years if there is currently 1000 bears.

$P = P_0(1-r)^t$

$P_0 = 1000$

$r = 3.1\% = .031$

$t = 20$

$P = 1000(1-.031)^{20} = 41000(.969)^{20} = 532$

6) A certain isotope decreases at a rate of 5% per years. It there is currently 340 grams of the isotope, how many grams of the isotope will there be in 20 years?

$P = P_0(1-r)^t$

$P_0 = 340$

$r = 5\% = .05$

$t = 25$

$P = 340(1-.05)^{25} = 340(..95)^{25} = 94.3\ grams$

Logarithmic Models

Basic Logarithms

Definition of a logarithm with base b

$\log_b a = x \Leftrightarrow b^x = a$

Examples

1)

Write $4^3 = 64$ as a logarithmic expression

Answer: $\log_4 64 = 3$

2)

Write $5^{-3} = \frac{1}{125}$ as a logarithmic expression

Answer: $\log_5 \left(\frac{1}{125} \right) = -3$

3)

Write $6^4 = 1296$ as a logarithmic expression

Answer: $\log_6 1296 = 4$

4) Write $\log_2 64 = 6$ as a exponent expression

Answer: $2^6 = 64$

Base 10 logarithms

$$\log_{10} a = x \Leftrightarrow 10^x = a$$
$$\log a = x \Leftrightarrow 10^x = a$$

Examples

1)

$$10^4 = 10000 \Leftrightarrow \log 1000 = 4$$

2)

$$10^2 = 100 \Leftrightarrow \log 100 = 2$$

Using logarithms on a scientific calculator

1) Find log(123) using your calculator

log(123) = 2.09

2) Find log(54780)

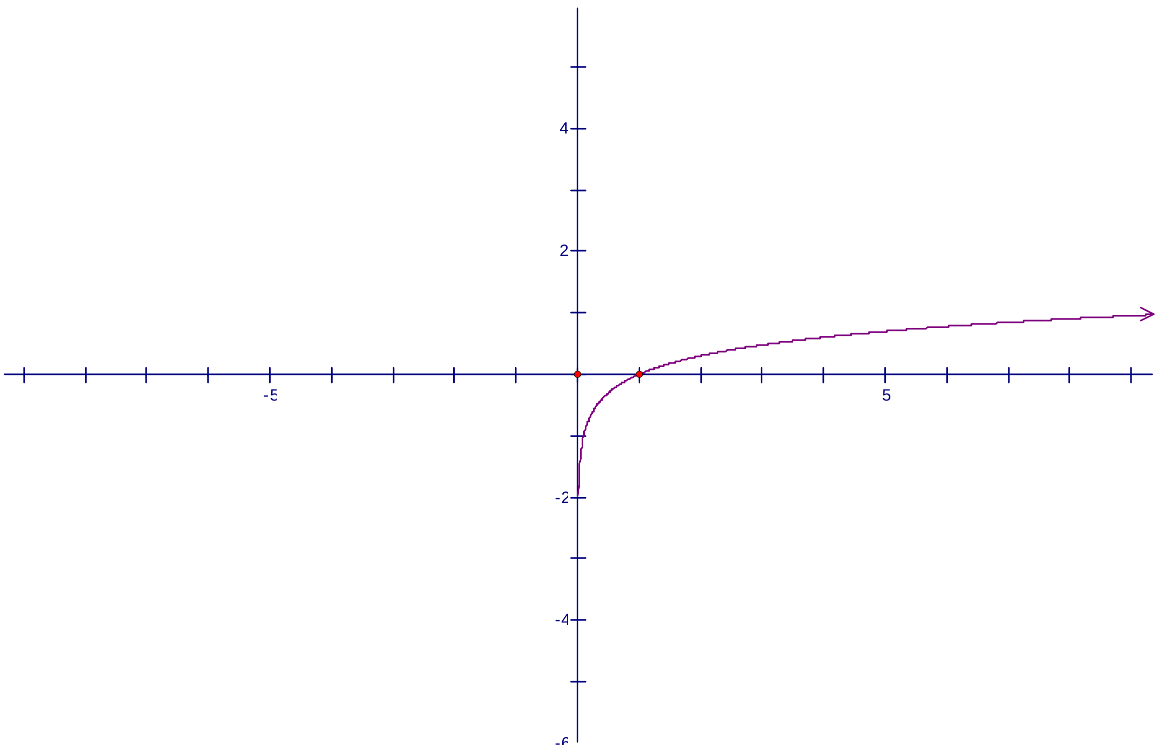
log(54780) = 4.73

Graph of basic logarithms

1) Graph $y = \log x$

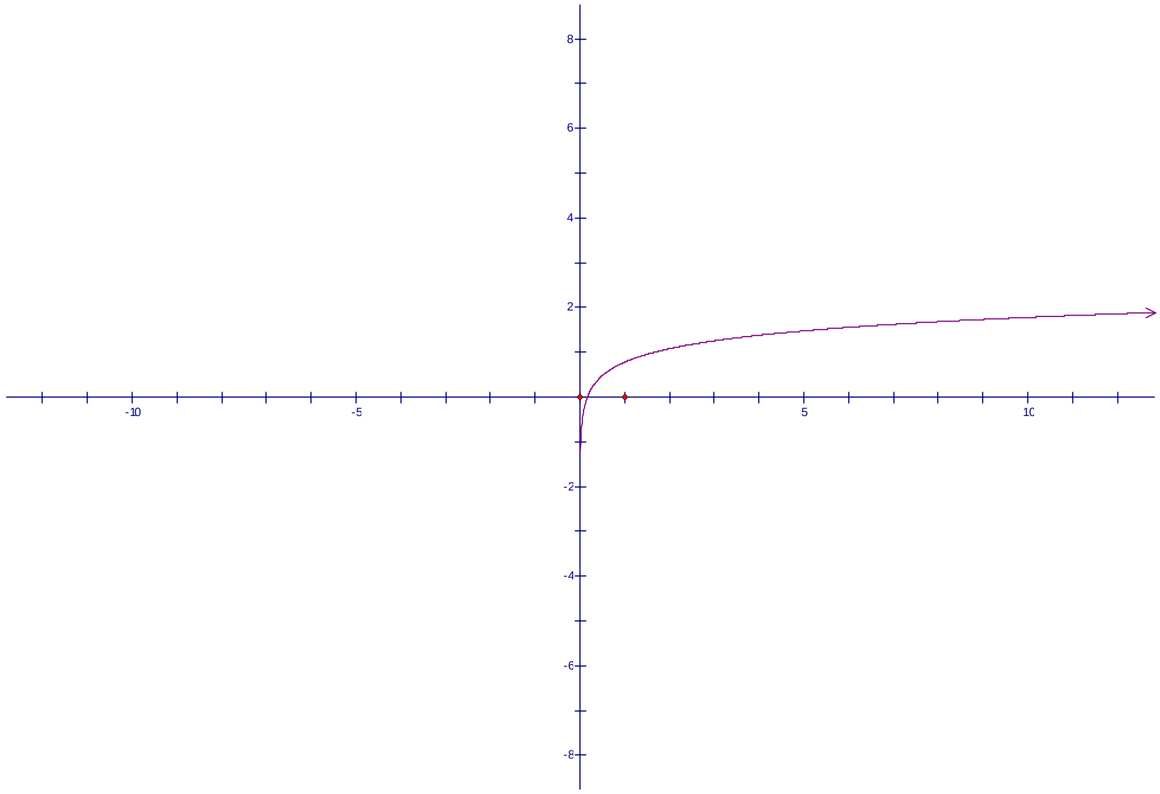
x	y
1	$y = \log(1) = 0$
10	$y = \log(10) = 1$
100	$y = \log(100) = 2$
1000	$y = \log(1000) = 3$

Graph



1) $y = \log 6x$

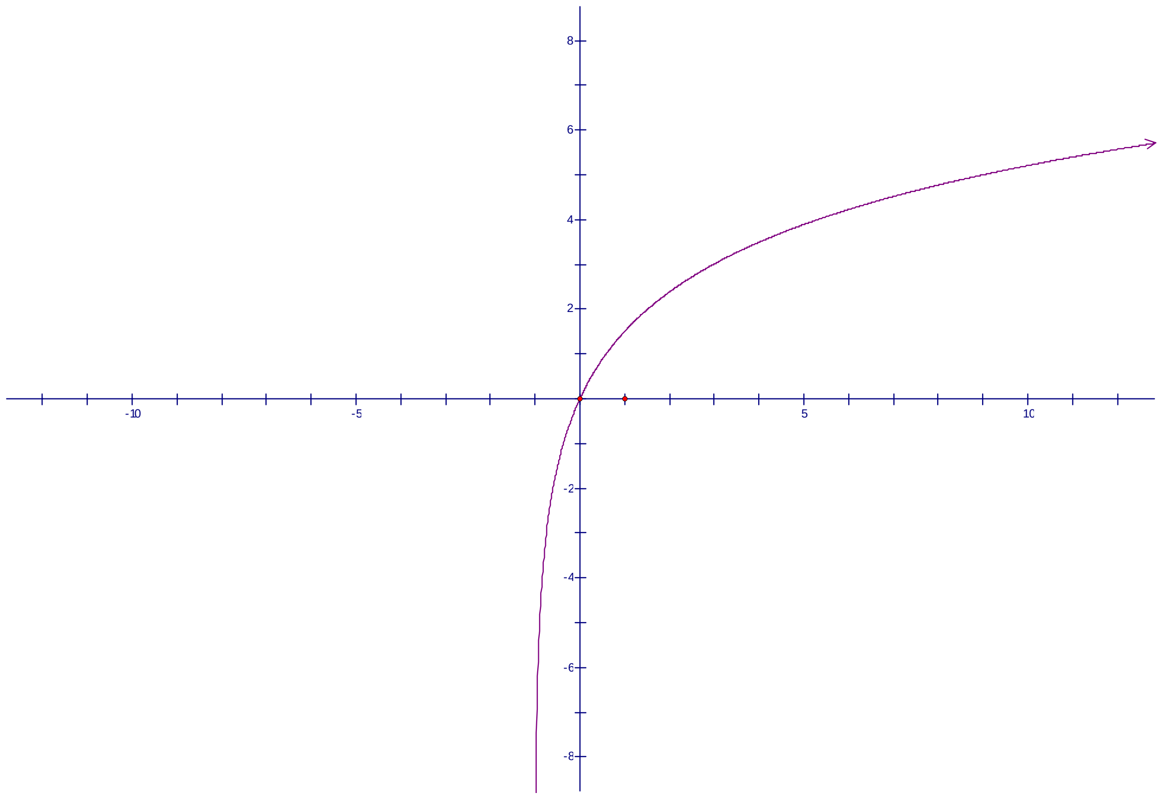
X	Y
2	$y = \log(6(2)) = \log(12) = 1.07$
10	$y = \log(6(10)) = \log(60) = 1.8$
20	$y = \log(6(20)) = \log(120) = 2.1$
40	$y = \log(6(40)) = \log(240) = 2.4$



2)

$y = 5\log(x+1)$

x	y
2	$y = 5\log(2+1) = 5\log(3) = 2.4$
10	$y = 5\log(10+1) = 5\log(11) = 5.2$
20	$y = 5\log(20+1) = 5\log(21) = 6.6$
40	$y = 5\log(40+1) = 5\log(41) = 8.1$



8
6
4
2
-2
-4
-6
-8
-10
-5
5
10

Logarithmic Models

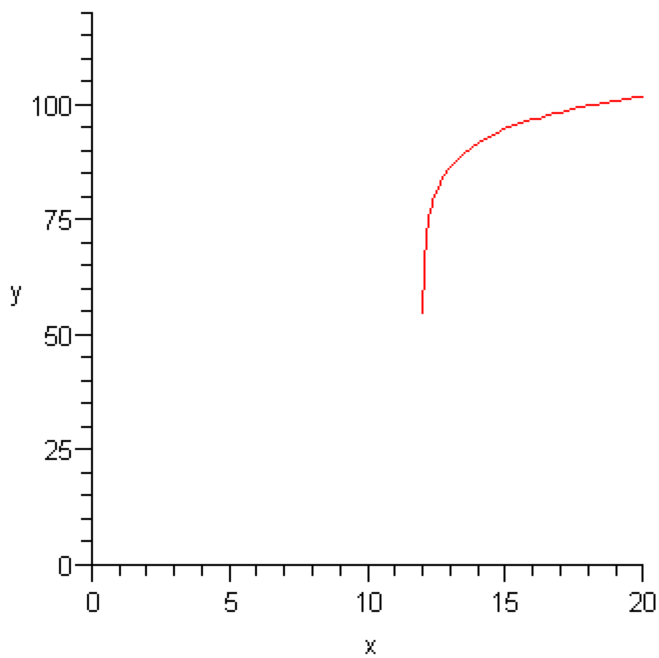
Page 185

16)

A logarithmic model to approximate the percentage P of an adult height a male has reached at an age A form 13 and 18 is $P = 16.7\log(A - 12) + 87$

a) Sketch a graph of this function.

A	P
13	$P = 16.7\log(13 - 12) + 87 = 16.7\log(1) + 87 = 0 + 87 = 87$
14	$P = 16.7\log(14 - 12) + 87 = 16.7\log(2) + 87 = 5 + 87 = 92$
15	$P = 16.7\log(15 - 12) + 87 = 16.7\log(3) + 87 = 8.0 + 87 = 95$
18	$P = 16.7\log(18 - 12) + 87 = 16.7\log(6) + 87 = 13 + 87 = 100$



b) What does the graph tell you about the height of male after age of 18?

Usually males stop growing after age 18

c) Use the model to compute the percentage of the full height of a 15 year old male.

$P = 16.7\log(15-12) + 87 = 16.7\log(3) + 87 = 8.0 + 87 = 95$

95%

Example

The percentage of a girl's full height is given by the equation $y = 58 + 30\log(x-3)$. Use the formula to predict the percentage of her height when the girl is 8 year old.

$$y = 58 + 30\log(x-3)$$
$$y = 58 + 30\log(8-3)$$
$$y = 58 + 30\log(5)$$
$$y = 58 + 21$$
$$y = 79\%$$

Example

Use the following model $n = -694.2 + 231.4\log A$ to find the number of years for the amount of money A to grow to $100,000.

$$n = -694.2 + 231.4\log 100000$$
$$n = -694.2 + 231.4(5)$$
$$n = -694.2 + 1157$$
$$n = 462.8$$