Chapter 4

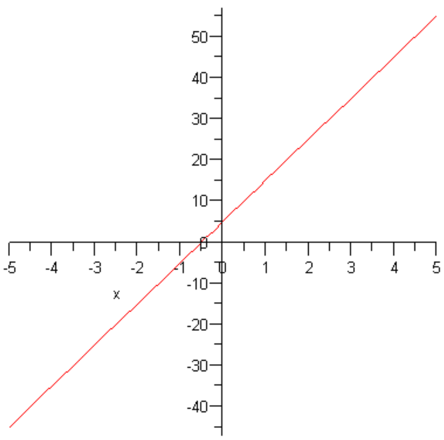
Introduction to Mathematical Modeling

Types of Modeling

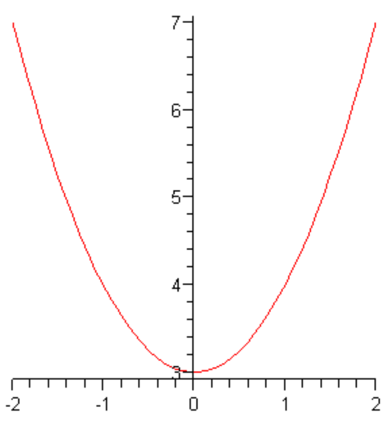
1) Linear Modeling
2) Quadratic Modeling
3) Exponential Modeling
4) Logarithmic Modeling

Each type of modeling in mathematics is determined by the graph of equation for each model. In the next examples, there is a sample graph of each type of modeling

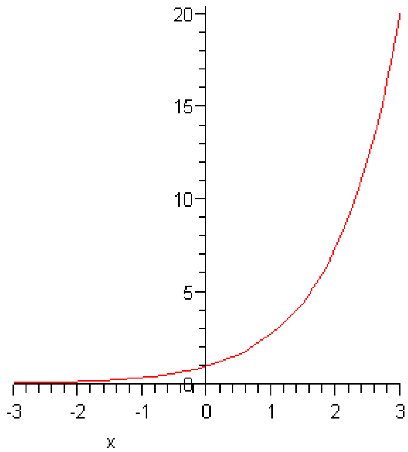
Linear models are described by the following general graph



Quadratic models are described by the following general graph



Exponential models are described by the following general graph



Section 4.1

Linear Models

Before you can study linear models, you must understand so basic concepts in Algebra. One of the main algebra concepts used in linear models is the slope-intercept equation of a line. The slope intercept equation is usually expressed as follows:

Standard linear model

$$y = mx + b$$
$$m = slope$$
$$b = y - Intercept$$

In this equation the variable m represents the slope of the equation and the variable b represents the y-intercept of the line. When studying linear models, you must understand the concept of slope. Slope is usually defined as "rise over run" or "change in y over change in x". In general slope measures the rate in change. Thus, the idea of slope has many applications in mathematics including velocity, temperature change, pay rates, cost rates, and several other rates of change.

Slope

$$Slope = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x}$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Basic Algebra Skills (Slope and y-intercept)

In next examples, we will find the slope of a line given two points on the line.

Example 1

Find the slope between the points (1,3) and (3,2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{3-1} = \frac{-1}{2} = -\frac{1}{2}$$

Example 2

Find the slope between the points (2,3) and (4,6)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-3}{4-2} = \frac{3}{2}$$

Slope and y-intercept also can be found from the equation in slope-intercept, as shown in this next example. Notice that the equation is written in slope-intercept form.

Example 3 Find the slope and y-intercept

$$y = 3x - 2$$

$$m = 3$$
$$b = -2$$

If the equation is not written in slope intercept form, it can be rearranged to slope-intercept form by solving the equation for y. This procedure is shown in the next two examples.

Example 4

Find the slope and y-intercept

$$2x + 3y = 6$$
$$2x - 2x + 3y = -2x + 6$$
$$3y = -2x + 6$$
$$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$$
$$y = -\frac{2}{3}x + 2$$
$$m = -\frac{2}{3}$$
$$b = -2$$

Example 5

Find the slope and y-intercept

$$3x - 5y = 10$$
$$3x - 3x - 5y = -3x + 10$$
$$-5y = -3x + 10$$
$$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{10}{-5}$$
$$y = \frac{3}{5}x - 2$$

$$m = \frac{3}{5}$$
$$b = -2$$

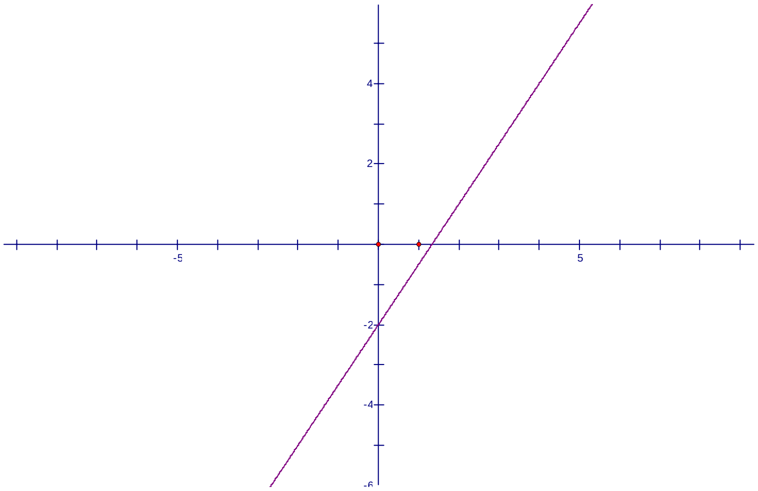
Example 6

Graph the equation $y = \frac{3}{2}x - 2$

First construct a table using 4 arbitrary values of x, and then substitute these x values to the equation $y = \frac{3}{2}x - 2$ to get the corresponding y values.

x	$y = \frac{3}{2}x - 2$
1	$y = \frac{3}{2}(1) - 2 = \frac{3}{2} - 2 = -\frac{1}{2}$
2	$y = \frac{3}{2}(2) - 2 = 3 - 2 = -1$
3	$y = \frac{3}{2}(3) - 2 = \frac{9}{2} - 2 = \frac{5}{2}$
4	$y = \frac{3}{2}(4) - 2 = 6 - 2 = 4$

Next make point using the four points in the above table.



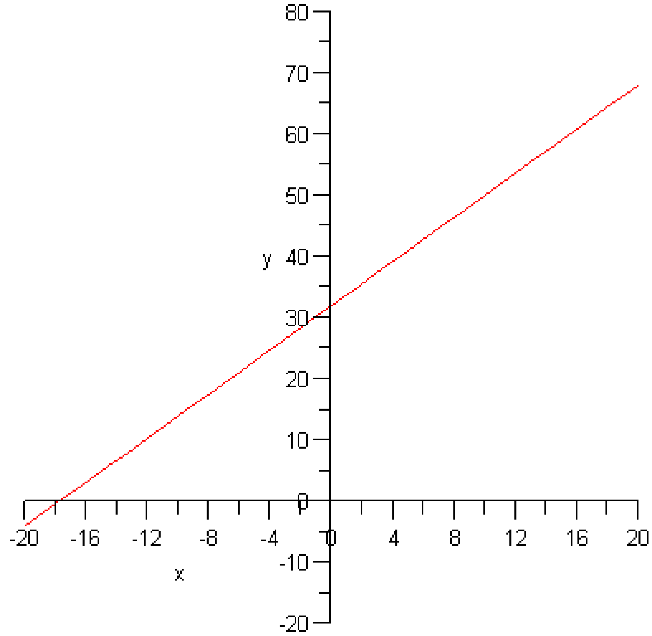
Applications of Linear Equations

Example 6 (Temperature conversion)

$$F = \frac{9}{5}C + 32$$

a) Sketch a graph of $F = \frac{9}{5}C + 32$

C	$F = \frac{9}{5}C + 32$
10	$F = \frac{9}{5}(10) + 32 = 9(2) + 32 = 18 + 32 = 50$
20	$F = \frac{9}{5}(20) + 32 = 9(4) + 32 = 36 + 32 = 68$
30	$F = \frac{9}{5}(30) + 32 = 9(6) + 32 = 54 + 32 = 86$
40	$F = \frac{9}{5}(40) + 32 = 9(8) + 32 = 72 + 32 = 104$



b) Use the model to convert 120 degrees Celsius to degrees Fahrenheit.

$$F = \frac{9}{5}C + 32$$
$$F = \frac{9}{5}(120) + 32$$
$$F = 216 + 32$$
$$F = 248$$

c) Use the model to convert 212 degrees Fahrenheit to Celsius.

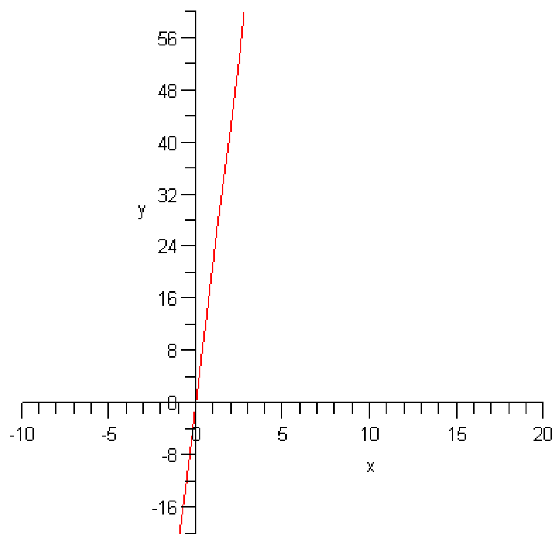
$$F = \frac{9}{5}C + 32$$
$$212 = \frac{9}{5}C + 32$$
$$212 - 32 = \frac{9}{5}C + 32 - 32$$
$$180 = \frac{9}{5}C$$
$$\frac{5}{9}(180) = \frac{5}{9} \cdot \frac{9}{5}C$$
$$C = 100^0 C$$

Example 7 (Business Applications)

The revenue of a company that makes backpacks is given by the formula $R = 21.50x$ where x represents the number of backpacks sold.

a) Graph the linear model $R = 21.50x$

X	$R = 21.50x$
10	$R = 21.50(10) = 215$
20	$R = 21.50(20) = 430$
30	$R = 21.50(30) = 645$
40	$R = 21.50(40) = 860$



b) Use the model to calculate the revenue for selling 50 backpacks

$x = 50$

$R = 21.50x = 21.5(50) = \$1075.0$

c) What is the slope

$m = \$21.50$

d) What is the meaning of the slope?

Cost per unit sold

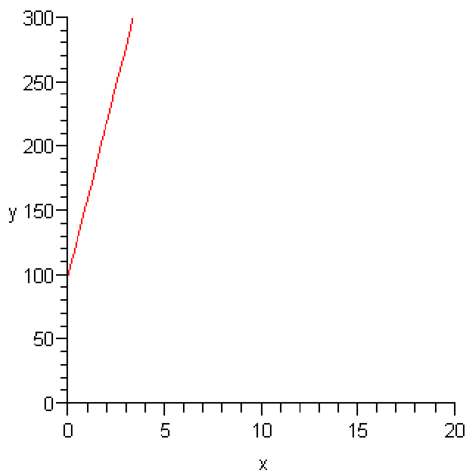
Revenue made per backpack solid

Example 8 (Sales)

A salesperson is paid $100 plus $60 per sale each week. The model $S = 60x + 100$ is used to calculate the salesperson's weekly salary where x is the number of sales per week.

a) Graph $S = 60x + 100$

x	$S = 60x + 100$
2	$S = 60(2) + 100 = 120 + 100 = 220$
4	$S = 60(4) + 100 = 240 + 100 = 340$
6	$S = 60(6) + 100 = 360 + 100 = 460$
8	$S = 60(8) + 100 = 480 + 100 = 580$



b) Use the model to calculate the salespersons weekly salary if he/she makes 8 sales.

$S = 60(8) + 100 = 480 + 100 = \$580.00$

c) What is the slope of the equation

$$m = 60\ \frac{\$}{sale}$$

d) What is the meaning of the slope

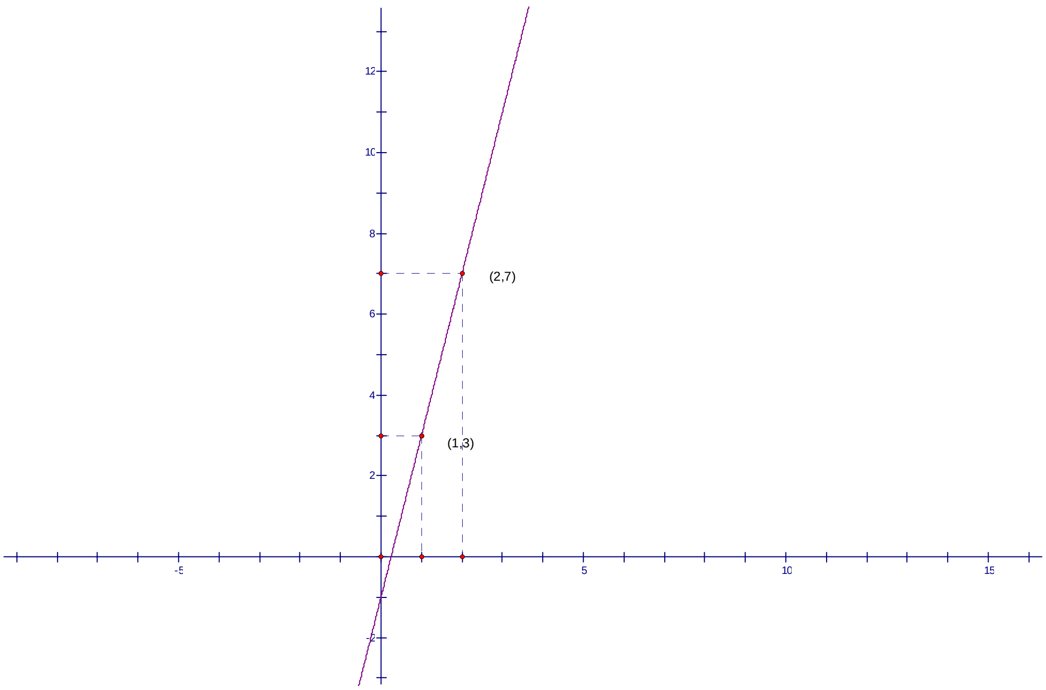
Dollars per each sale

Example 9

Given the following data sketch a graph

Time	Temperature
1 min	3^0C
2 min	7^0C
3 min	11^0C
4 min	14^0C

Sketch a graph of the given data and then compute the slope of the resulting line.



Use the points (1,3) and (2,7) in the above graph to compute the slope

$$m = \frac{7-3}{2-1} = \frac{4}{1} = 4$$

Example 10

An approximate linear model that gives the remaining distance, in miles, a plane must travel from Los Angeles to Paris given by $d = 6000 - 550t$ where d is the remaining distance and t is the hours after the flight begins. Find the remaining distance to Paris after 3 hours and 5 hours.

$$d = 6000 - 550(3)$$
$$d = 6000 - 1650$$
$$d = 4350\,miles$$

$$d = 6000 - 550(5)$$
$$d = 6000 - 2750$$
$$d = 3250\,miles$$

How long should it take for the plane to flight from Los Angeles to Paris?

$$0 = 6000 - 550t$$
$$0 + 550t = 6000 - 550t + 550t$$
$$550t = 6000$$
$$\frac{550t}{550} = \frac{6000}{550}$$
$$t = 10.9 \; hours$$

Problem Set 1

1) Find the slope between the points (1,1) and (3,5)
2) Find the slope between the points (0,0) and (4,5)

Given the equation, find the slope and y-intercept.

3) $y = \frac{3}{4}x - 2$
4) $3x + 4y = 6$
5) $2x - 3y = 6$

Graph the following equations

6) $y = 3x$
7) $y = x + 5$
8) $y = \frac{1}{4}x - 1$
9) $y = -6x$

Linear Models

10) The revenue of a company that makes backpacks is given by the formula $R = 34.50x$ where x represents the number of backpacks sold.

a) Graph the linear model $R = 34.50x$
b) Use the model to calculate the revenue for selling 40 backpacks?
c) What is the slope of the model?
d) What is the meaning of the slope?

11) A salesperson is paid \$100 plus \$30 per sale each week. The model $S = 30x + 100$ is used to calculate the salesperson's weekly salary where x is the number of sales per week.

a) Graph $S = 30x + 100$
b) Use the model to calculate the salespersons weekly salary if he/she makes 8 sales.
c) What is the slope of the equation?
d) What is the meaning of the slope?

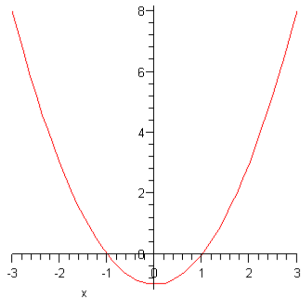
12) A salesperson is paid $200 plus $50 per sale each week. The model $S = 50x + 200$ is used to calculate the salesperson's weekly salary where x is the number of sales per week.

a) Graph $S = 50x + 200$
b) Use the model to calculate the salespersons weekly salary if he/she makes 8 sales.
c) What is the slope of the equation?
d) What is the meaning of the slope?

13) An approximate linear model that gives the remaining distance, in miles, a plane must travel from San Francisco to London given by $d(t) = 5500 - 500t$ where $d(t)$ is the remaining distance and t is the hours after the flight begins. Find the remaining distance to London after 2 hours and 4 hours.

Section 4.2

Quadratic Models
Graph of Quadratic Models



The graph of a quadratic model always results in a **parabola**. The general form of a quadratic function is given in the following definition.

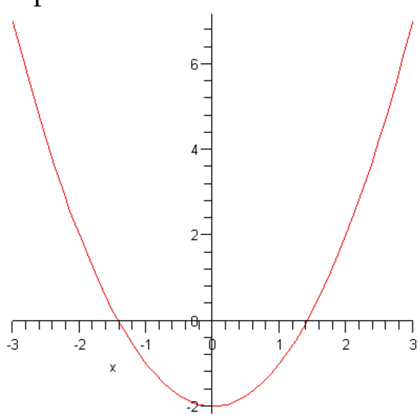
A **quadratic function** is a function where the graph is a parabola and the equation is of the form: $y = ax^2 + bx + c$ where $a = 0$

The x-coordinate of **vertex** is given by the equation: $x = -\dfrac{b}{2a}$

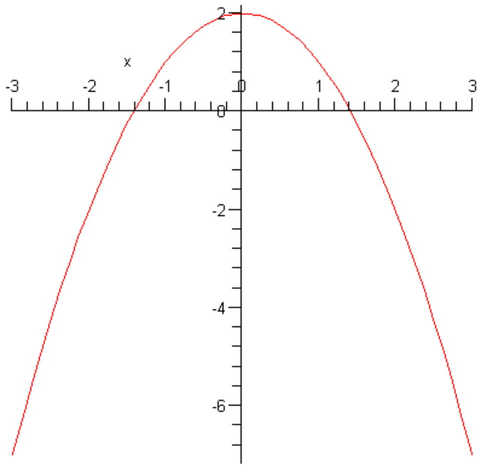
The vertex is the turning point on the graph of a parabola. If the parabola opens upward, then the vertex is the lowest point of the graph. If the parabola opens downward, then the vertex is the highest point on the graph. The direction of the parabola opens can be determined by the sign of the "x^2" term or the a term in the above equation. If $a < 0$, then the parabola open downward. Similarly if $a > 0$, then the parabola opens upward. (See graphs below in figure 1-1)

Figure 1-1

A parabola where $a > 0$ and the vertex is the lowest point on the graph



A parabola where $a < 0$ and the vertex is the highest point on the graph



Here are some examples of finding the vertex and x-intercepts of an exponential equation. The graph of the quadratic equation is also provided in these examples

Example 1

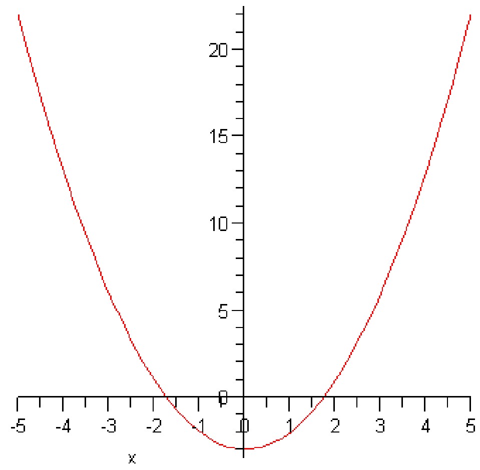
Find the vertex and x-intercepts of the quadratic equation, and then make a sketch of the parabola.

$$y = x^2 - 3$$
$$a = 1, c = -3$$
$$x = -\frac{0}{2(1)} = -\frac{0}{2} = 0$$

x-intercepts:

$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$\sqrt{x^2} = \sqrt{3}$$
$$x = \sqrt{3}$$
$$(\sqrt{3},0) \;\; and \;\; (-\sqrt{3},0)$$

Graph for Example 1



Example 2

Find the vertex and x-intercepts of the quadratic equation, and then make a sketch of the parabola.

$$y = x^2 - 3x$$

Vertex

$$x = -\frac{-3}{2(1)} = \frac{3}{2}$$

$$y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} = -\frac{9}{4}$$

x-intercepts

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

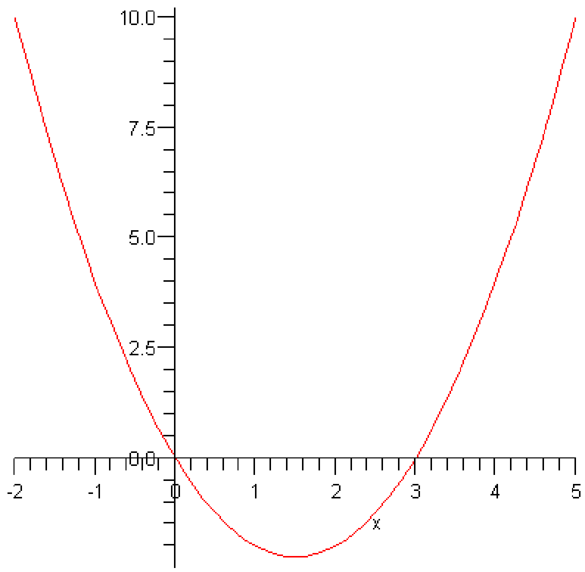
$$x = 0 \ or \ x - 3 = 0$$

$$x = 0 \quad x - 3 = 0$$

$$x = 3$$

$$(0,0) \ \ and \ \ (3,0)$$

Graph of the function



Example 3

Find the vertex and x-intercepts of the quadratic equation, and then make a sketch of the parabola.

$y = 3x^2 - 6x$

Vertex

$$x = -\frac{-(-6)}{2(3)} = \frac{6}{6} = 1$$

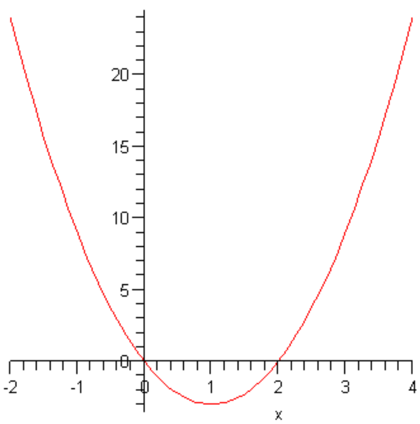
$$y = 3(1)^2 - 6(1) = 3 - 6 = -3$$

$(1,-3)$

x-intercepts

$$3x^2 - 6x = 0$$
$$3x(x-2) = 0$$
$$3x = 0 \; or \; x - 2 = 0$$
$$x = 0 \quad x - 2 = 0$$
$$x = 2$$
$$(0,0) \; and \; (2,0)$$

Graph of $3x^2 - 6x = 0$



More about Quadratic Equations

In some instances, the quadratic equation will not factor properly. In this case, you must use what is called the quadratic formula. In the next few examples, the quadratic formula will be used to find the solutions of a quadratic equation.

The Quadratic Formula

The solution to the equation $y = ax^2 + bx + c$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 4

Solve $x^2 + 5x - 7 = 0$

$a = 1$
$b = 5$
$c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 28}}{2} = \frac{-5 \pm \sqrt{53}}{2}$$

Example 5

Solve $x^2 + 7x - 9 = 0$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-9)}}{2(7)} = \frac{-7 \pm \sqrt{49 + 36}}{2(7)} = \frac{-7 \pm \sqrt{85}}{14}$$

Example 6

At a local frog jumping contest. Rivet's jump can be approximated by the equation $y = -\frac{1}{6}x^2 + 2x$ and Croak's jump can be approximate by $y = -\frac{1}{2}x^2 + 4x$, where x = the length of jump in feet and y = the height of the jump in feet.

a) Which frog can jump higher

Rivet's vertex: $x = -\frac{2}{2\left(-\frac{1}{6}\right)} = -\frac{2}{-\frac{1}{3}} = 6$ Height: $y = -\frac{1}{6}(6)^2 + 2(6) = -6 + 12 = 6\ ft$

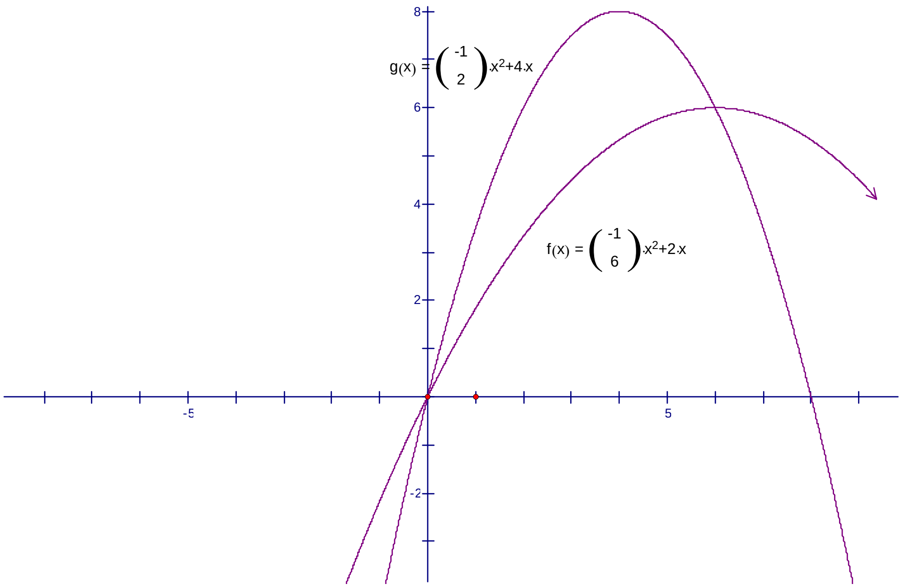
Croak's vertex: $x = -\frac{4}{2\left(-\frac{1}{2}\right)} = \frac{-4}{-1} = 4$ Height: $y = -\frac{1}{2}(4)^2 + 4(4) = -8 + 16 = 8\ ft$

Croak can jump higher at 8 feet

b) Which frog can jump farther

Rivet's can jump farther at 2(6 ft) = 12 feet

Graph of the frogs jumps



Using the parabola to find the maximum or minimum value of a quadratic function

The parabola can be used to find either the maximum value or the minimum value of a quadratic function. (**See figure 1-1**) This can simply be done by find the vertex of the parabola. Remember as stated earlier the vertex will turn out to be either the highest point on the curve or the lowest point on the curve. In the next examples, the vertex of the parabola will be use to find the maximum value.

Example 7

The path of a ball thrown by a boy is given by the equation $y = -.04x^2 + 1.5x$ where x is the horizontal distance the ball travels and y is the height of the ball. Find the maximum height of the ball in yards.

Find the vertex of the ball

$$x = -\frac{1.5}{2(-.04)} = \frac{1.5}{.08} = 18.75$$

$$y = -.04(18.75)^2 + 1.5(18.75) = -14.1 + 28.1 = 14 \ yards$$

Example 8

The path of a cannon ball is given by the equation $y = -.1x^2 + 6.0x$ where x is the horizontal distance the ball travels and y is the height of the cannon ball. Find the maximum height of the cannon ball in feet.

Find the vertex of the cannon ball.

$$x = \frac{-6.0}{2(-.1)} = \frac{-6.0}{-.2} = 30 \Rightarrow y = -.1(30)^2 + 6(30) = -90 + 180 = 90 \ \ feet$$

Problem Set

Find the vertex and x-intercepts of the given parabola, and then make a sketch of the parabola.

1) $y = 2x^2 - 4x$

2) $y = x^2 - 4$

3) $y = x^2 + 2x - 1$

4) $y = x^2 - 4x + 3$

5) $y = x^2 - 16$

6) $y = 3x^2 - 6x$

Quadratic Models

7) The path of a ball thrown by a baseball player is given by the equation $y = -.02x^2 + 1.6x$ where x is the horizontal distance the ball travels and y is the height of the ball. Find the maximum height of the ball in yards.

8) The path of a ball thrown by a boy is given by the equation $y = -.06x^2 + 1.8x$ where x is the horizontal distance the ball travels and y is the height of the ball. Find the maximum height of the ball in yards.

9) The path of a cannon ball is given by the equation $y = -.05x^2 + 6.0x$ where x is the horizontal distance the ball travels and y is the height of the cannon ball. Find the maximum height of the cannon ball in feet.

10) The path of a cannon ball is given by the equation $y = -.1x^2 + 8.0x$ where x is the horizontal distance the ball travels and y is the height of the cannon ball. Find the maximum height of the cannon ball in feet.

Section 4.3

Exponential models

The exponential function
$e \approx 2.718$ "The Euler number"

Example 1

Simplify the following exponential functions

1) $e^2 = 7.39$

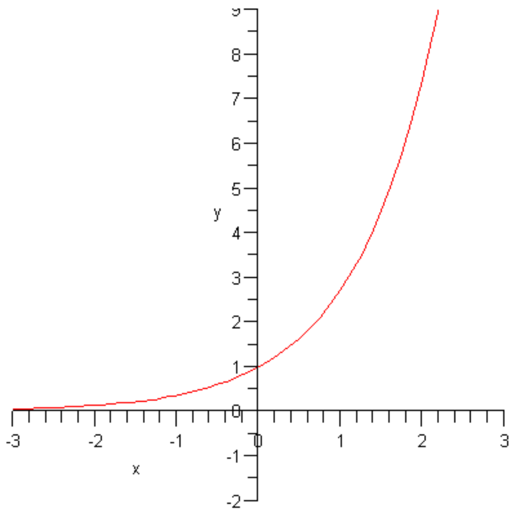
2) $e^{-3} = \frac{1}{e^3} = .05$

3) $e^{\frac{1}{3}} = 1.40$

The graph of the exponential function

Example 2 Graph $y = e^x$

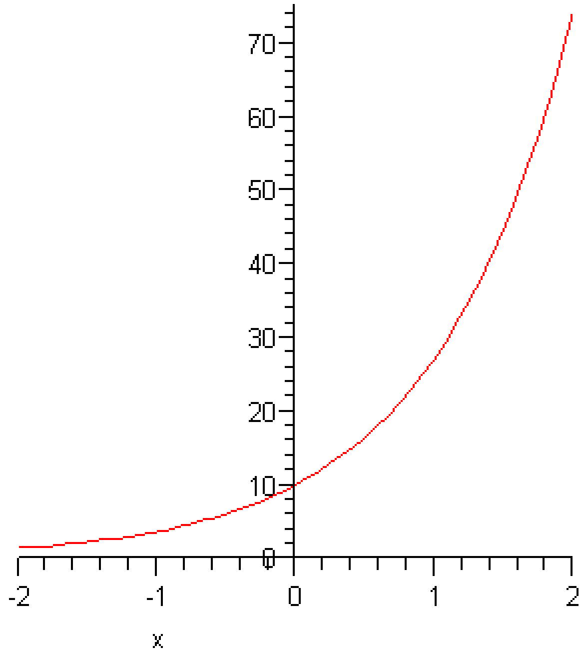
x	$y = e^x$
-2	$y = e^{-2} = .14$
-1	$y = e^{-1} = .37$
0	$y = e^o = 1$
1	$y = e^1 = 2.7$
2	$y = e^2 = 7.4$



Example 3

Graph $y = 10e^{.2x}$

x	y
-2	$y = 10e^{.2(-2)} = 10e^{-.4} = 6.7$
-1	$y = 10e^{.2(-1)} = 10e^{-.2} = 8.2$
0	$y = 10e^{.2(0)} = 10e^0 = 10$
1	$y = 10e^{.2(1)} = 10e^{.2} = 12.2$
2	$y = 10e^{.2(2)} = 10e^{.4} = 14.9$



Exponential Models

Exponential models are used to predict human populations, animal populations, money growth, pollution growth, and other aspects of society that fit exponential models. The variable of an exponential model is found in the exponent of the equation.

Exponential Growth

$P = P_0(1+r)^t$

$P = New\ Value$

$P_0 = Original\ Value$

$r = rate$

$t = time$

Example 4

The population of the United States is 290 million, what would be the population of the U. S. be in 20 years if its population would growth at a steady rate of .7 % for 20 years?

$$P = P_0(1+r)^t$$
$$P_0 = 290,000,000$$
$$r = .7\% = .007$$
$$t = 20$$
$$P = 290000000(1+.007)^{20} = 290000000(1.007)^{20} = 333416746$$

Example 5

The population of Blacksburg, Virginia is 41,000, what would be the population in 10 years if Blacksburg would grow at a rate of 1.1 % per year?

$$P = P_0(1+r)^t$$
$$P_0 = 41000$$
$$r = 1.1\% = .011$$
$$t = 10$$
$$P = 41000(1+.011)^{10} = 41000(1.011)^{10} = 45740$$

Example 6

In 1995 the United States had greenhouse emissions of about 1400 million tons, where as China had greenhouse emissions of about 850 million tons. If in the next 25 years China greenhouse emission grew by 4 percent and the U. S. greenhouse emission grew by 1.3 percent, what would the emissions in tons for both countries in 2020?

$U.S.\ Emissions\ in\ 2020$

$P = P_0(1+r)^t$

$P_0 = 1400\ million$

$r = 1.3\% = .013$

$t = 25$

$P = 1400(1+.013)^{25} = 1400(1.013)^{25} = 1933\ million\ tons$

$China's\ Emissions\ in\ 2020$

$P = P_0(1+r)^t$

$P_0 = 850\ million$

$r = 4.0\% = .04$

$t = 25$

$P = 850(1+.04)^{25} = 850(1.04)^{25} = 2265\ million\ tons$

Example 7

Using the exponential growth formula, find the amount of money that you would have in a bank account if you deposited $3,000 in the account for 15 years at 1.1 % interest rate?

$P = P_0(1+r)^t$

$P_0 = 3000$

$r = 1.1\% = .011$

$t = 15$

$P = 3000(1+.011)^{15} = 3000(1.011)^{15} = \$3482.91$

Exponential decay

Exponential decay models are use to measure radioactive decay, decreasing populations, Half-life, and other elements that fit an exponential model. Again, the one variable in an exponential decay models in found in the exponent.

Exponential Decay Formula

$$P = P_0(1-r)^t$$
$$P = New\ Value$$
$$P_0 = Original\ Value$$
$$r = rate$$
$$t = time$$

Example 8

A certain population of black bears in the eastern United States has been decreasing by 3.1 percent per year. If this trend keeps up, what will be the population of bears in 20 years if there are currently 1000 bears.

$$P = P_0(1-r)^t$$
$$P_0 = 1000$$
$$r = 3.1\% = .031$$
$$t = 20$$
$$P = 1000(1-.031)^{20} = 1000(.969)^{20} = 533$$

Example 9

A certain isotope decreases at a rate of 5% per year. It there is currently 340 grams of the isotope, how many grams of the isotope will there be in 20 years?

$$P = P_0(1-r)^t$$
$$P_0 = 340$$
$$r = 5\% = .05$$
$$t = 25$$
$$P = 340(1-.05)^{20} = 340(.95)^{20} = 122\ grams$$

Problem Set 3

Exponential Functions

Evaluate using a calculator

1) e^2

2) $\frac{1}{2}e$

3) $2e^{\frac{4}{3}}$

Graph the following functions

4) $y = 3^x$

5) $y = e^x - 1$

6) $y = 2e^x$

7) $y = e^{2x}$

Growth Models (Show Work)

8) The current population of Germany is 80,000,000. What would be the population of Germany in 10 years if its population would growth at a steady rate of .9 % for 10 years?

9) The current population of Salem, Virginia is 25,000. What would be the population of Salem in 5 years if Salem would grow at a rate of 1.2 % per year?

10) Using the exponential growth formula, find the amount of money that you would have in a bank account if you deposited $10,000 in the account for 10 years at 1.6 % interest rate?

11) A certain rabbit population is modeled by the equation $P = 2000e^{.03t}$ where t is the time in months. Use the model to predict the population after 20 months.

Decay Models

11) A certain population of Panda Bears in China has been decreasing by 1.0 percent per year. If this trend keeps up, what will be the population of Panda Bears in 10 years if there are currently 2000 bears?

12) A certain isotope decreases at a rate of 4% per year. It there is currently 220 grams of the isotope, how many grams of the isotope will there be in 25 years?

Section 4.5

Basic Logarithms

Definition of a Logarithm

$\log_b a = x \quad \Leftrightarrow \quad b^x = a$

Example 1

i) Write $3^5 = 243$ as a logarithmic expression.

$3^5 = 243 \Rightarrow \log_3 243 = 5$

ii) Write $5^4 = 625$ as a logarithmic expression.

$5^4 = 625 \Rightarrow \log_5 625 = 4$

Example 2

i) Write $\log_4 16 = 2$ as exponential expression.

$\log_4 16 = 2 \Rightarrow 4^2 = 16$

ii) Write $\log_{10} 10{,}000 = 4$ as an exponential expression.

$\log_{10} 10{,}000 = 4 \Rightarrow 10^4 = 10{,}000$

Log base ten

Another way of writing $\log_{10} 1000$ is $\log 1000$.

The way we find the answer to $\log 1000$ is to ask the question of 10 raised to what power gives you 1000? Since we know that $10^4 = 1000$, the answer is 4.

Example 3

i) Find log 100,000

Since $10^5 = 100{,}000$, $\log 100{,}000 = 5$

ii) Find log 100

Since $10^2 = 100$, $\log 100 = 2$

Example 4

Use a scientific calculator to evaluate the following logarithms

i) log 567

Answer: log 567 = 2.754

ii) log 30890

Answer: log 30890 = 4.490

iii) log 456782

Answer: log 456782 = 5.660

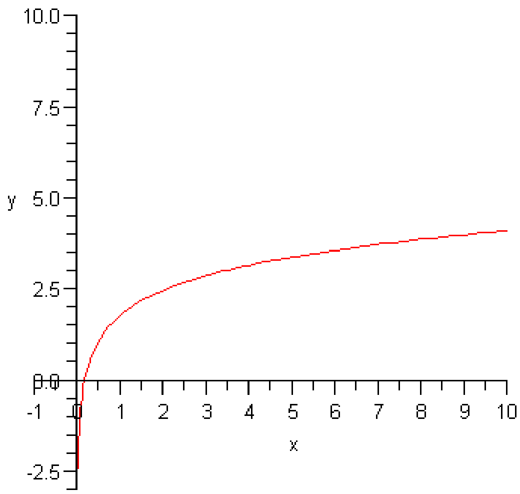
Graph of basic logarithms

Example 5

Graph $y = \log 6x$

x	y
2	$y = \log(6(2)) = \log(12) = 1.07$
10	$y = \log(6(10)) = \log(60) = 1.8$
20	$y = \log(6(20)) = \log(120) = 2.1$
40	$y = \log(6(40)) = \log(240) = 2.4$

Plot the given values from the table gives the following graph

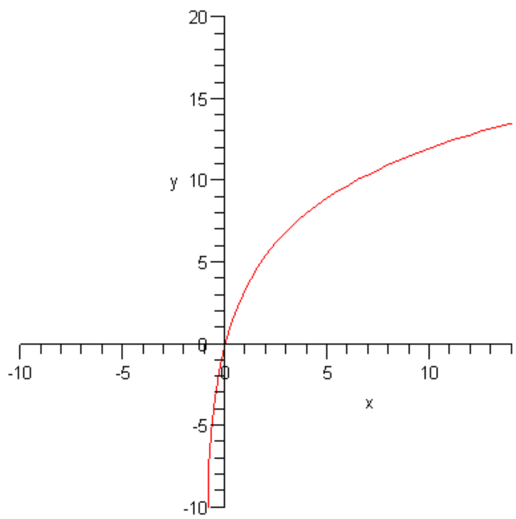


Example 6

Graph $y = 5\log(x+1)$

x	y
2	$y = 5\log(2+1) = 5\log(3) = 2.4$
10	$y = 5\log(10+1) = 5\log(11) = 5.2$
20	$y = 5\log(20+1) = 5\log(21) = 6.6$
40	$y = 5\log(40+1) = 5\log(41) = 8.1$

Plot the given values from the table gives the following graph



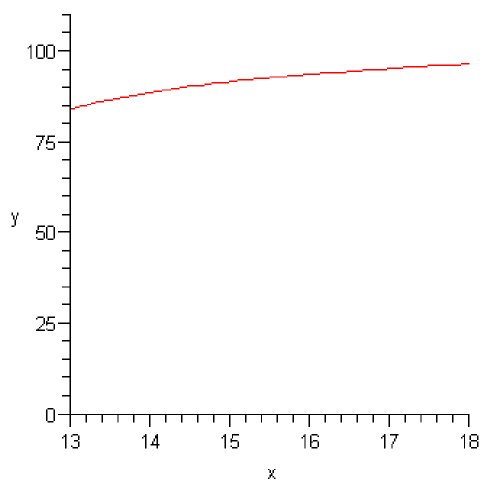
Example 7 (Using logarithmic models to model height)

A logarithmic model to approximate the percentage P of an adult height a male has reached at an age A form 13 and 18 is $P = 16\log(A-12)+84$

1) Sketch a graph of this function.

P	A
13	$P = 16\log(13-12)+84 = 84$
14	$P = 16\log(14-12)+84 = 16\log(2)+84 = 4.8+84 = 88.8$
15	$P = 16\log(15-12)+82 = 16\log(3)+84 = 7.6+84 = 90.6$
18	$P = 16\log(18-12)+84 = 16\log(6)+84 = 12.5+84 = 96.5$

Plot the given values from the table gives the following graph



2) What does the graph tell you about the height of male after age of 18?

Usually males stop growing after age 18

3) Use the model to compute the average height of a 16 year old male.

$P = 16\log(16-12)+84 = 16.\log(4)+84 = 9.6+84 = 93.6$

93.6%

Example 8

Use the following model for $1000 invested in saving account given by $n = -694.2 + 231.4\log(A)$, to find the amount of time (n) for the amount of money A to grow to $100,000.

$$n = -694.2 + 231.4\log 100000$$
$$n = -694.2 + 231.4(5)$$
$$n = -694.2 + 1157$$
$$n = 462.8$$