Chapter 2
Logic Unit
Math 114

Section 2.1

Deductive and Induction Reasoning

Statements

Definition: A **statement** is a group of words or symbols that can be classified as true or false.

Examples of statements

1) Math is a fun subject
2) Cats make good pets
3) The trees in Virginia are beautiful in the fall.

Examples of things that are not statements

1) Get out of here!
2) Stop doing that
3) How are you?

Deductive Reasoning: The application of a general statement to a specific instance.

Deductive reasoning goes from general to specific

Example 1 (Example of Deductive Reasoning)

Triangle ABC is isosceles
All isosceles triangles have two equal angles
Therefore, triangle ABC has two equal angles

This argument is a deductive argument because it goes from general to specific.

Example 2 (Example of Deductive Reasoning)

Solve the following equation:

$$2y + 5 = 17$$
$$2y + 5 - 5 = 17 - 5$$
$$2y = 12$$
$$\frac{2y}{2} = \frac{12}{2}$$
$$y = 6$$

In this example, rules of algebra are used to solve for y. This is an example of deductive reasoning because the answer is the direct result of the application of the rules of algebra.

Syllogism: An argument composed of two statements, **or premise**, which is followed by a **conclusion**.

Example 2

Molly is a dog
<u>Dogs are very friendly</u>
Therefore, Molly is very friendly

Inductive Reasoning: An argument that involves going from a series of specific cases to a general statement.

Inductive reasoning goes from specific to general.

In Deductive Reasoning the conclusion is guaranteed.

In Inductive reasoning the conclusion is probable, but not necessarily guaranteed

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Example 3

Is the following argument a deductive argument or an inductive argument.

John sneezed around Jill's cat
John sneezed around Jim's cat
Therefore, John sneezes around all cats

The argument is inductive because it goes from specific to general

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Example 4

Room 295 in Walker Hall is a technology classroom at RU
Room 338 in Currie Hall is a technology classroom at RU
Room 212 in Davis Hall is a technology classroom at RU
Therefore, all classrooms at RU are technology classrooms

The argument is inductive because it goes from specific to general

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Example 5

Determine if the following arguments use deductive reasoning or inductive reasoning

All math teachers are strange
Jim Morrison is a math teacher
Therefore, Jim Morrison is strange.

(This is an example of a deductive argument)

Example 6

Determine if the following arguments use deductive reasoning or inductive reasoning

Jimmy likes Mary
Women whom Jimmy likes are pretty
Thus, Mary is pretty

(This is an example of a deductive argument)

Example 7

Determine if the following arguments use deductive reasoning or inductive reasoning

> In each of the last five years, the economy has grown by at least two percent.
> This year the economy is projected to grow by 2.5 %
> Therefore, the economy will always grow by at least 2 %
>
> (This is an example of an inductive argument)

Section 2.2

Connectors

Compound Statements

If two or more statements are put together this is referred to as a **compound statement**

Similarly in the subject of English, two sentences put together form a compound sentence.

In order to put statements together, we need to use what are called **connector**s

Types of Connectors

1) Or
2) And
3) If-Then (Conditional or Implication)
4) Negation

Symbols logic and Symbols

Connector	Symbol
Or	$\vee$
And	$\wedge$
If – then (Conditional)	$\rightarrow$
Negation	~

How to use the four types of connectors

I) Using "or" as a connector

Symbolic representation: p or q ($p \vee q$)

Example 1

Write the following statement in symbolic form

Either the sky is blue or the sea is green

p = the sky is blue
q = the sea is green
$p \vee q$

Example 2

Write the following statement in symbolic form

Dogs are loyal or cats are friendly

p = dogs are loyal
q = cats are friendly
$p \vee q$

II) Using “and“ as a connector

Symbolic representation: p and q ($p \wedge q$)

Example 3

Write the following statement in symbolic form

The sky is blue and the sea is green.

p = the sky is blue
q = the sea is green
$p \wedge q$

Example 4

Write the following statement in symbolic form

I like oranges and apples

p = I like oranges
q = I like apples
$p \wedge q$

III) Using the conditional

Symbolic representation: If p, then q ($p \to q$) where p is the premise or hypothesis, and q is the conclusion.

Example 5

Write the following statement in symbolic form
If you study for your geometry test, then you should do well on the test

p = If you study for your geometry test (Hypothesis)
q = you should do well on the test (Conclusion)
$p \to q$

Example 6

Write the following statement in symbolic form

If it rains tomorrow, then I will bring an umbrella to work.

p = If it rains tomorrow (Hypothesis)
q = I will bring an umbrella to work (Conclusion)
$p \to q$

Example 7

Write the following statement in symbolic form

I will sell you my textbook, if you offer me a good price

You can rewrite the statements as "If you offer me a good price, then I will sell my textbook" to help identify the hypothesis and conclusion

p = If you offer me a good price (Hypothesis)
q = I will sell you my textbook (Conclusion)
$p \rightarrow q$

IV) Using the negation

Example 8

Negate the following statement

p = I like apples

Negation: ~p (I don't like apples)

Note: The negation of all is some and the negation of some is all.

Example 8

Negate the following statement

All RU students like ice cream

Negation: Some RU students do not like ice cream.

Example 9

Negate the following statement

Some students dislike geometry

Negation: All students like geometry

Example 10

Negate the following statement

Everyone loves Raymond

Negation: Someone does not love Raymond

Example 11

Which of the following are statements?

a) 3 + 5 = 6 (Statement)
b) Solve the equation 2x + 5 = 3 (This is not a statement, because it can not be classified as true or false)
c) $x^2 + 1 = 0$ has no solution (Statement)
d) $x^2 - 1 = (x+1)(x-1)$ (Statement)
e) Is $\sqrt{2}$ irrational? (Not a Statement)

Example 12

Write a sentence that is the negation of each statement

a) Her dress is not red
Negation: Her dress is red.

b) Some elephants are pink.
Negation: All elephants are not pink

c) All candy promotes tooth decay.
Negation: Some candy does not promote tooth decay.

d) No lunch is free.
Negation: Some lunches are free.

Example 13

Using the symbolic representations:

p: The food is spicy
q: The food is aromatic

Express the following compound statements in symbolic form.

a) The food is aromatic and spicy.
$q \wedge p$
b) If the food isn't spicy, it isn't aromatic
$\sim p \rightarrow \sim q$

c) The food is spicy and it isn't aromatic
$p \wedge \sim q$

d) The food isn't spicy or aromatic.
$\sim (p \vee q)$

Forms of the conditional

The Conditional: $p \rightarrow q$
The Contrapostive: $\sim q \rightarrow \sim p$
The Inverse: $\sim p \rightarrow \sim q$
The Converse: $q \rightarrow p$

Example

Given the conditional below, make complete statements for the contrapositive, converse, and inverse.

Conditional: If you live in Virginia, then you live in the United States

Contrapositive: If you live in the United States, then you live in Virginia

Inverse: If you don't live in Virginia, then you don't live in the United States.

Converse: If you don't live in the United States, then you don't live in Virginia.

De Morgan's Law

Negation of compound Statements

De Morgan's Law

$$\sim(p \vee q) = \sim p \wedge \sim q$$
$$\sim(p \wedge q) = \sim p \vee \sim q$$

Using De Morgan's Law to negate compound statements

Example 14

Negate each statement using De Morgan Law

1) $\sim p \vee \sim q$

Negation: $\sim(\sim p \vee \sim q) = \sim(\sim p) \wedge \sim(\sim q) = p \wedge q$

2) $\sim p \vee q$

Negation: $\sim(\sim p \vee q) = \sim(\sim p) \wedge \sim q = p \wedge \sim q$

3) $r \wedge \sim s$

Negation: $\sim(r \wedge \sim s) = \sim r \vee \sim(\sim s) = \sim r \vee s$

4) $r \wedge s$

Negation: $\sim(r \wedge s) = \sim r \vee \sim s = \sim r \vee \sim s$

5) Either I like apples or I like oranges.

$p \vee q$
$\sim(p \vee q)$
$\sim p \wedge \sim q$

Negation: I don’t like apples and I don’t like oranges

6) The sky is blue and the sea is green

$p \wedge q$
$\sim(p \wedge q)$
$\sim p \vee \sim q$

Negation: Either the sky is not blue or the sea is not green

Section 2.3

Truth Tables

Review of the connectors

Connector	Symbol
Or	$\vee$
And	$\wedge$
If – then (Conditional)	$\rightarrow$
Negation	~

Using the connectors in a truth table

Basic Truth tables

1) $p \vee q$

p	Q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Think of the statement "Either you like apple or you like oranges"

This statement is true unless "you don't like oranges" and "you don't like apples" (See red row in the truth table)

2) $p \wedge q$

p	Q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Using the statement "You like apples and oranges", it turns out that this statement is true only if you both like apples and oranges. (See blue) In the last three cases (rows) the statement is false. (See red)

3) $p \to q$

p	Q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Using the example "If you study for the test, you will pass the test", it turns out that this is all true accept when the hypothesis "If you study for the test" is true, and the conclusion "you will pass the test" is false (See red)

Other examples of Truth Tables

Example 1

Complete a truth table for $(p \wedge q) \to q$

p	Q	$p \wedge q$	$(p \wedge q) \to q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

When a compound statement results with all true statements in the last column it is called a **tautology** (True in all cases)

Example 2

Complete a truth table for $(p \wedge q) \vee p$

p	Q	$p \wedge q$	$(p \wedge q) \vee p$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

Example 3

Complete a truth table for $(p \wedge q) \vee (\sim p)$

p	Q	~p	$p \wedge q$	$(p \wedge q) \vee (\sim p)$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Example 4

Complete a truth table for $\sim (p \vee q)$

p	Q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Example 5

Complete a truth table for $(\sim (p \vee q)) \rightarrow q$

p	Q	$p \vee q$	$\sim (p \vee q)$	$(\sim (p \vee q)) \rightarrow q$
T	T	T	F	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

Example 6

Complete a truth table for $(p \wedge q) \to (p \vee q)$

p	Q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

This is a tautology

Example 7

Complete a truth table for $p \vee (\sim r \wedge q)$

P	Q	r	~r	$(\sim r \wedge q)$	$p \vee (\sim r \wedge q)$
T	T	T	F	F	T
T	T	F	T	F	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

Example 4

Complete a truth table for $(p \wedge q) \to r$

P	Q	r	$(p \wedge q)$	$(p \wedge q) \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Section 2.4 Equivalent Statements and DeMorgan's Laws

Equivalent Statements

Equivalent Statements will have the same result in the last column of their truth tables.

Example 1

Compare the truth tables for $\sim(p \vee q)$ and $\sim p \wedge \sim q$

Truth table for $\sim(p \vee q)$

P	Q	$p \vee q$	$\sim(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Truth table for $\sim p \wedge \sim q$

P	Q	~p	~q	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Notice that the last columns of each table are identical. Thus, the arguments are equivalent.

Example 2

Compare the truth tables for $\sim(p \wedge q)$ and $\sim p \vee \sim q$

Truth table for $\sim(p \wedge q)$

P	Q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Truth table for $\sim p \vee \sim q$

P	Q	~p	~q	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Again the truth tables have the same last column. Thus, the statements are equivalent.

De Morgan's Laws

$\sim(p \vee q) = \sim p \wedge \sim q$

$\sim(p \wedge q) = \sim p \vee \sim q$

Fact: $p \to q \equiv \sim p \vee q$

These are equivalent because the results of their truth table are the same.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

The negation of $p \to q$

$$\sim(p \to q) \equiv \sim(\sim p \vee q) \equiv \sim(\sim p) \wedge \sim q \equiv p \wedge \sim q$$

Section 2.5

Write arguments in symbolic form and valid arguments

Writing an argument in symbolic form

Example 1

I have a college degree (p)
I am lazy (q)

If I have a college degree, then I am not lazy
<u>I don't have a college degree</u>
Therefore, I am lazy

Symbolic form:

If I have a college degree, then I am not lazy $(p \to \sim q)$
<u>I don't have a college degree</u> $(\sim p)$
Therefore, I am lazy q

Hypothesis: $((p \to \sim q) \wedge \sim p)$
Conclusion: q

Argument in symbolic form: $((p \to \sim q) \wedge \sim p) \to q$

To test to see if the argument is valid, we take the argument in symbolic form and construct a truth table. If the last column in the truth table results in all true's, then the argument is valid

P	q	$\sim p$	$\sim q$	$(p \to \sim q)$	$((p \to \sim q) \wedge \sim p)$	$((p \to \sim q) \wedge \sim p) \to q$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

Therefore, this argument is invalid because the last column has a false item.

Example 2

Symbolize the argument, construct a truth table, and determine if the argument is valid.

If I pass the test, then I will graduate.
I graduated
Therefore, I passed the exam

p = pass the exam
g = I will graduate

If I pass the test, then I will graduate. $(p \to g)$
I graduated (g)
Therefore, I passed the exam (p)

Argument: $((p \to g) \land g) \to p$

p	g	$(p \to g)$	$((p \to g) \land g)$	$((p \to g) \land g) \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

This argument is invalid

Example 3

Symbolize the argument, construct a truth table, and determine if the argument is valid.

Jen and Bill will be at the party
Bill was at the party.
Therefore, Jen was at the party
J = Jen will be at the party
B = Bill will be at the party

Jen and Bill will be at the party $J \land B$
Bill was at the party. B
Therefore, Jen was at the party J

Argument in symbolic form: $((J \land B) \land B) \to J$

J	B	$J \wedge B$	$(J \wedge B) \wedge B$	$((J \wedge B) \wedge B) \to J$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

Since the last column is all true, the argument is valid

Example 4

Symbolize the argument, construct a truth table, and determine if the argument is valid.

It will be sunny or cloudy today
<u>It isn't sunny</u>
Therefore, it will be cloudy

S = It will be sunny
C = It will be cloudy

It will be sunny or cloudy today $S \vee C$
<u>It isn't sunny ~S</u>
Therefore, it will be cloudy C

Hypothesis: $(S \vee C) \wedge \sim S$
Conclusion: C

S	C	~S	$S \vee C$	$(S \vee C) \wedge \sim S$	$((S \vee C) \wedge \sim S) \to C$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

This is a valid argument

Example 5

Write in symbolic form

p: The senator supports new taxes.
q: The senator is reelected

The senator is not reelected if she supports new taxes
The senator does not support new taxes
Therefore, the senator is reelected

Symbolic form:

The senator is not reelected if she supports new taxes $p \rightarrow \sim q$
The senator does not support new taxes $\sim p$
Therefore, the senator is reelected q

Hypothesis: $(p \rightarrow \sim q) \wedge \sim p$
Conclusion: q

Argument: $((p \rightarrow \sim q) \wedge \sim p) \rightarrow q$

12) Determine if the argument in problem 6 above is valid

p	q	~q	~p	$p \rightarrow \sim q$	$(p \rightarrow \sim q) \wedge \sim p$	$((p \rightarrow \sim q) \wedge \sim p) \rightarrow q$
T	T	F	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	F

Since the last row results in false, the argument is invalid