

Set and Set Operators

Definition of a set

A **set** is a collection of objects, things or numbers.

The **universal set** is the set of all possible elements of set used in the problem. Denoted by \mathcal{U}

Examples

$\{1,2,3,4,5\}$

$\{Ron, John, Mark, Phil\}$

$\{Virginia, West Virginia, Maryland, Tennessee, Kentucky, North Carolina\}$

Elements are the members of a given set.

\in represents is an element of

\notin represents is not an element of

$3 \in \{1,2,3,4,5\}$

$a \in \{a,b,c,d,e\}$

Roster Notation

$\{a,e,i,o,u\}$

$\{Huron, Ontario, Michigan, Erie, Superior\}$

$\{2,4,6,8,\dots\}$

Builder Set Notation

$\{x \mid x \text{ is a vowel}\}$

$\{x \mid x \text{ is a great lake}\}$

$\{x \mid x \text{ is an even positive number}\}$

A set is **well defined** if the elements of the sets are clearly defined.

If a set is well defined, then there should not be any confusion of what the elements are in the set

Examples of well defined sets

$\{1,3,5,7,9,11,13\}$

$\{m,n,o,p,q,r,s\}$

$\{x \mid x \text{ is a whole number}\}$

Examples of set that are not well defined

$\{x \mid x \text{ is something cool}\}$

$\{x \mid x \text{ is a good football team}\}$

The Empty Set

The **empty set** is a set that contains no elements. The empty set is also referred to as the **null set**.

Symbol representation ϕ or $\{\}$

Important Sets

Natural Numbers or Counting Numbers: $\{1,2,3,4,5,\dots\}$

Whole Numbers: $\{0,1,2,3,4,5,\dots\}$

Integers: $\{\dots -5,-4,-3,-2,-1,0,1,2,3,4,5,\dots\}$

Subsets

A set B is a subset of set C, if every element in B is an element of C. $B \subseteq C$

Proper Subsets

A set B is a proper subset of C, if every element of B is an element of C and there is at least one element of B that is not in C. $B \subset C$

Example 1

$$A = \{1,2,3,4,5\}$$

$$C = \{1,2,3,4,5,6,7\}$$

Is $A \subset C$?

Solution: Since every element in the set A is an element of C, A is a subset of C.

Example 2

Is $\{2,3,4,5,6\}$ a subset of $\{0,1,2,3,4,5\}$?

Solution: no, since the element 6 is not in the set $\{0,1,2,3,4,5\}$

Example 3

List all possible subsets of $\{1,2\}$

Solution: $\phi, \{1\}, \{2\}, \{1,2\}$

Example 4

List all subsets of the set $\{a,b,c\}$

Possible subsets

Solution: $\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}$

Example 5

List all subsets of the set $\{4\}$

Possible sets: $\phi, \{4\}$

Example 6

List all possible subsets of $\{m, a, t, h\}$

$\phi, \{m\}, \{a\}, \{t\}, \{h\}$

$\{m, a\}, \{m, t\}, \{m, h\}, \{a, t\}, \{a, h\}, \{t, h\}$

$\{m, a, t\}, \{m, t, h\}, \{a, t, h\}, \{m, a, h\}$

$\{m, a, t, h\}$

The pattern for subsets

Number of elements	Number of subsets
1	2
2	4
3	8
4	16

Formula to find the number of subsets s of a given set A with n elements: $s = 2^n$

Example 7

How many subsets does a set A with 6 elements have?

Solution: $s = 2^n = 2^6 = 64$

Example 8

Let $A = \{1,2,3\}$, $B = \{1,2\}$, $C = \{1,2,3,4,5\}$

1) Is $A \subset C$?

Yes, every element in A is in C

2) Is $B \subset C$?

Yes

3) Is $3 \in A$?

Yes

4) Is $C \subset A$?

No, 4 and 5 are not in A

5) Is $\emptyset \subset A$?

Yes

Problem Set 1

I) Which of the following sets are well defined?

1) $\{1,2,3,4,5\}$

2) $\{x \mid x \text{ is a US state}\}$

3) $\{x \mid x \text{ is a North America breed of Dog}\}$

4) $\{x \mid x \text{ is a fun game}\}$

II) Write each set in roster form

1) $\{x \mid x \text{ is natural number}\}$

2) $\{x \mid x \text{ is a state that begins with the letter M}\}$

3) $\{x \mid x \text{ is a vowel}\}$

III) Use the following sets to answer each question.

$A = \{a, b, c, d, e, f, g\}$; $B = \{a, b, c, d\}$; $C = \{a, c, e\}$; $D = \{a, e\}$; $E = \{e, f, g, h\}$; $F = \phi$

- 1) Is $C \subset A$?
- 2) Is $c \in A$?
- 3) Is $E \subset A$?
- 4) Is $F \subset A$?
- 5) Is $h \in A$?
- 6) Name three subsets of C
- 7) Is $D \subset C$?
- 8) Is $C \in A$?
- 9) Is $D \subset E$?

IV) Subsets

- 1) List all subsets of $\{a, b\}$
- 2) List all subsets of $\{10, 20, 30\}$
- 3) List all subsets of $\{l, o, v, e\}$
- 4) A set of 8 elements would have how many possible subsets

V) Describe each set as an infinite set or a finite set.

- 1) $\{x \mid x \text{ is one of the 50 state capitals}\}$
- 2) $\{x \mid x \text{ is an even natural number}\}$
- 3) $\{\dots, -5, -3, -1, 1, 3, 5, \dots\}$
- 4) $\{-8, 8, -6, 6, -4, 4, -2, 2, 0\}$

Union of Two Sets

The union of two sets is denoted by $A \cup B$ is $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Intersection of Two Sets

The intersect of two sets is denoted by $A \cap B$ is $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Example 1

$$A = \{1,2,3\}$$

$$C = \{1,3,5,7\}$$

$$1) A \cup C = \{1,2,3,5,7\}$$

$$2) A \cap C = \{1,3\}$$

Example 2: Let $A = \{1,2,3,4,5\}$, $B = \{1,3,5,7\}$, $C = \{1,2,3\}$, $D = \{1,2,3,4,5\}$, and $E = \phi$

1) Is $C \subset A$?

Answer: Yes, every element in C is contained in A

2) Is $B \subset A$?

Answer: No, the element 7 of set B is not contained in A.

3) Is $D \subset A$?

Answer: Yes, every element of D is in A.

4) Is $\phi \subset A$?

Yes, the empty set is a subset of any nonempty every set.

5) Find $A \cap B$

Answer: $A \cap B = \{1,3,5\}$

6) Find $A \cup B$

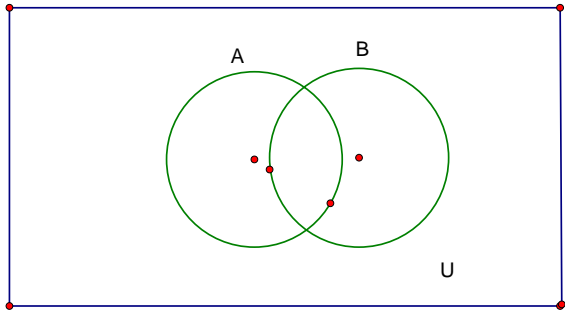
Answer: $A \cup B = \{1,2,3,4,5,7\}$

7) Find $A \cap C$

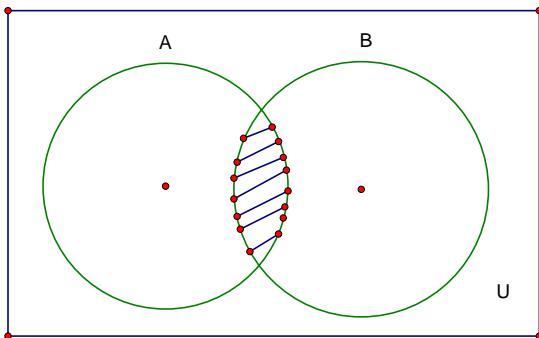
Answer: $A \cap C = \{1,2,3\}$

Venn Diagrams

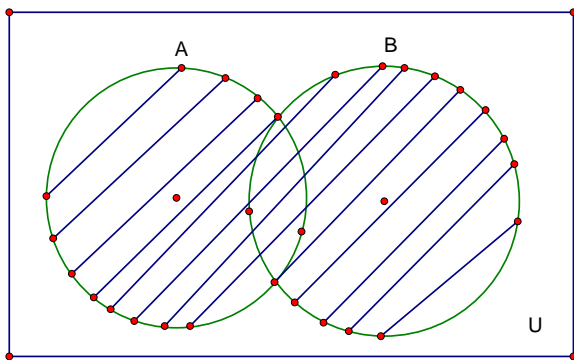
\mathcal{U} = the universal set



$A \cap B$



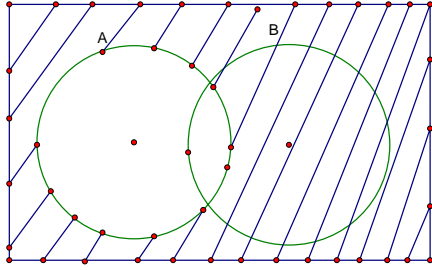
$A \cup B$



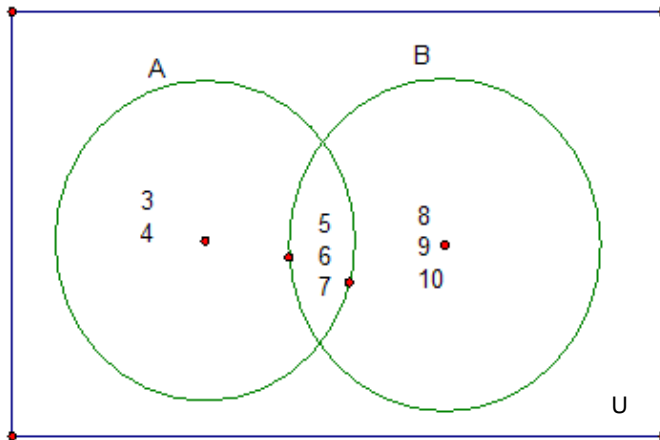
The complement of a set A

The complement of a set A is the set of all elements in the universal set that are not elements of the set A.

$$A' = \{x \mid x \notin A \text{ and } x \in U\}$$



Example 3



- 1) Find $A \cap B$
 $A \cap B = \{5, 6, 7\}$
 - 2) Find $A \cup B$
 $A \cup B = \{3, 4, 5, 6, 7, 8, 9, 10\}$
 - 3) Find A'
 $A' = \{8, 9, 10\}$
-
-

Example 4

Given

$$A = \{1,2,3,4,5,6\}, B = \{4,5,6,7,8\}, U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$$

Find

1) $A \cup B$

$$A \cup B = \{1,2,3,4,5,6,7,8\}$$

2) $A \cap B$

$$A \cap B = \{4,5,6\}$$

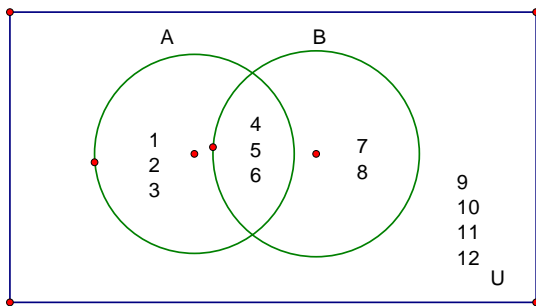
3) A'

$$A' = \{7,8,9,10,11,12\}$$

4) B'

$$B' = \{1,2,3,9,10,11,12\}$$

Make a Venn diagram of A,B, and U



Equivalent Sets

Two sets are equivalent if they have the same number of elements.

Examples of equivalent sets

$\{1,2,3,4\}$

and

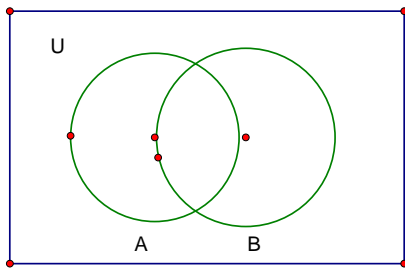
$\{a,b,c,d\}$

$\{john,luke,mark,mathew\}$

and

$\{a,b,c,d\}$

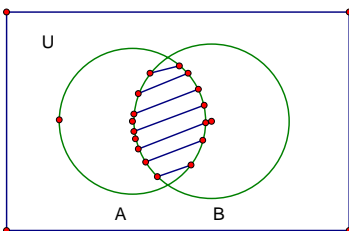
Venn diagrams



Example 5

Shade the region corresponding to the indicated set.

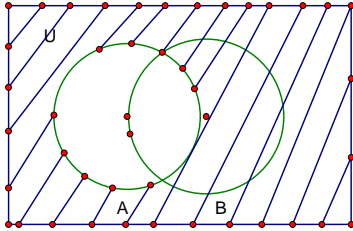
$A \cap B$



Example 6

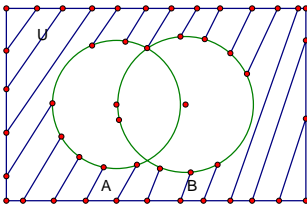
Shade the region corresponding to the indicated set.

$$A'$$

**Example 7**

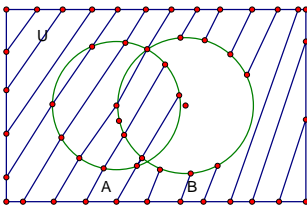
Shade the region corresponding to the indicated set.

$$A' \cap B'$$

**Example 8**

Shade the region corresponding to the indicated set.

$$A \cup B'$$



Cardinality

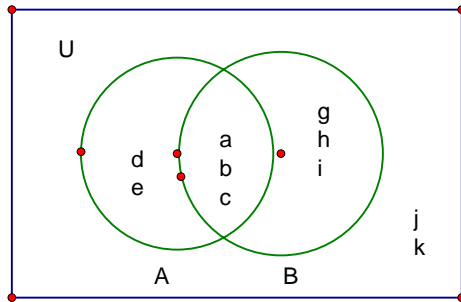
Definition: Cardinality is the number of element in a given set

The number of elements in a set A is denoted by $n(A)$

Example 9

Shade the region corresponding to the indicated set.

$$A = \{a, b, c, d, e\}, B = \{a, b, c, g, h, i\}, U = \{a, b, c, d, e, f, g, h, i, j, k\}$$



1) Find $n(A)$

$$n(A) = 5$$

2) Find $n(B)$

$$n(B) = 6$$

3) Find $n(A \cup B)$

$$n(A \cup B) = 8$$

4) Find $n(A \cap B)$

$$n(A \cap B) = 3$$

Rule for the cardinality for the union of two sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Use this formula to find $n(A \cup B)$ in problem 3.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 5 + 6 - 3 = 11 - 3 = 8$$

This gives the same answer as #3

Problem Set 2

I) Let $A = \{a, b, c, d, e, f\}$, $B = \{b, d, f, h\}$, $C = \{a, b, c\}$, $D = \{e, f, g, h, i\}$

- 1) Find $A \cup B$
- 2) Find $A \cap B$
- 3) Find $A \cup C$
- 4) Find $A \cap D$
- 5) Is $C \subset A$?
- 6) Is $B \subset A$?
- 7) What is $n(A)$?
- 8) What is $n(A \cup B)$?

II) Let $A = \{2, 4, 8, 10, 12\}$, $B = \{3, 5, 7, 9, 11\}$, $C = \{7, 9, 11\}$, $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

- 9) Find $A \cup B$
- 10) Find $A \cap B$
- 11) Find $A \cup D$
- 12) Find $A \cap D$
- 13) Is $C \subset B$?
- 14) Is $B \subset A$?
- 15) What is $n(A)$?
- 16) What is $n(A \cup B)$?
- 17) Draw a Venn Diagram using sets A, B, and U

Subsets

- 1) list all subsets of $\{10, 20, 30\}$
- 2) How many subsets does a set with 20 elements have?