Unit 2

Math 116

Prime numbers, Rational Numbers, and the Pythagorean Theorem

Math 116

Section 2.2

Prime Numbers

A prime number is a natural number that has exactly two divisors.

A natural number that has more that two divisors is called a **composite number.**

Examples of Prime Numbers

5 is a prime number since its only factors are 1 and 5.

7 is prime number since its only factors are 1 and 7

Prime Factorization of Numbers

Examples

Write the prime factorization of the given number or write the factor tree of the number

1) 24

$$24$$
$$12 \cdot 2$$
$$6 \cdot 2 \cdot 2$$
$$3 \cdot 2 \cdot 2 \cdot 2$$
$$2^3 \cdot 3$$

2) 36

$$36$$
$$6 \cdot 6$$
$$2 \cdot 3 \cdot 2 \cdot 3$$
$$2^2 \cdot 3^2$$

3) 90

90

$9 \cdot 10$

$3 \cdot 3 \cdot 5 \cdot 2$

$2 \cdot 3^2 \cdot 5$

Divisibility Rules

Number	**Divisibility Rule**
2	Last digit is divisible by 2
3	Sum of digits is divisible by 3
4	The last two digits are divisible by 4
5	The last digit is 0 or 5
6	The number is divisible by 2 and 3
8	The number formed the last 3 digits is divisible by 8
10	The number is divisible by 2 and 5 Last digit is zero

Divisibility statements

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12)

a) Does 6/39

False, 6 does not divide 39

b) Does 5/30

True, 5 divides 30 six times

c) Does 16/576
True, 16 divides 36

d) Does 3/7,823
False, the sum of the digits is not divisible by 3

Find the prime factorization of the given number

4) 699

699

$233 \cdot 3$

5) 740

740

$74 \cdot 10$

$37 \cdot 2 \cdot 5 \cdot 2$

$2^2 \cdot 5 \cdot 37$

Radicals and the Pythagorean Theorem

Section 2.5

Radicals

Square Roots

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{121} = 11$

Simplifying radicals to lowest form

Rule for multiplication of Radicals

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$
$$or$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Examples

Simplify each radical (Optional)

1)

$\sqrt{8}$

$\sqrt{4 \cdot 2}$

$\sqrt{4} \cdot \sqrt{2}$

$2\sqrt{2}$

2)

27

$\sqrt{9 \cdot 3}$

$\sqrt{9} \cdot \sqrt{3}$

$3\sqrt{3}$

3)

$\sqrt{18}$
$\sqrt{9 \cdot 2}$
$\sqrt{9} \cdot \sqrt{2}$
$3\sqrt{2}$

4)

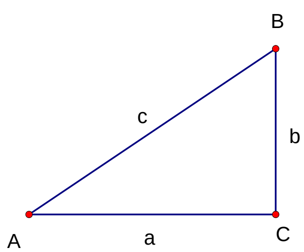
$\sqrt{50}$
$\sqrt{25 \cdot 2}$
$\sqrt{25} \cdot \sqrt{2}$
$5\sqrt{2}$

5)

$\sqrt{48}$
$\sqrt{16 \cdot 3}$
$\sqrt{16} \cdot \sqrt{3}$
$4\sqrt{3}$

Pythagorean Theorem

In a right triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse.



$c^2 = a^2 + b^2$

Examples

1)

$$Given\ a=6, b=8,\ find\ c$$
$$c^2 = 6^2 + 8^2$$
$$c^2 = 36+64$$
$$c^2 = 100$$
$$\sqrt{c^2} = \sqrt{100}$$
$$c = 10$$

2)

$$Given\ a=10, b=8,\ find\ c$$
$$c^2 = 10^2 + 8^2$$
$$c^2 = 100+64$$
$$c^2 = 164$$
$$\sqrt{c^2} = \sqrt{164}$$
$$c = \sqrt{4\cdot 41} = 2\sqrt{41}$$

3)

$$Given\ a=10, c=26,\ find\ b$$
$$26^2 = 10^2 + b^2$$
$$676 = 100 + b^2$$
$$676-100 = 100-100+b^2$$
$$576 = b^2$$
$$\sqrt{b^2} = \sqrt{575}$$
$$b = 24$$

How to determine if a triangle is a right triangle given the lengths of the sides

Converse of the Pythagorean Theorem

Let a, b, and c represent the three sides of a triangle, then:

$$c^2 = a^2 + b^2 \Rightarrow\ \Delta ABC\ is\ a\ right\ triangle$$

The following is also true for acute and obtuse triangles

Let a, b, and c represent the three sides of a triangle, then:

$$c^2 < a^2 + b^2 \Rightarrow \Delta ABC \text{ is an acute triangle}$$
$$c^2 > a^2 + b^2 \Rightarrow \Delta ABC \text{ is an obtuse triangle}$$

Determine the type of triangle given the length of its sides

1)

$$a = 1.5, b = 2, \text{ and } c = 2.5$$
$$c^2 = a^2 + b^2$$
$$2.5^2 = 1.5^2 + 2^2$$
$$6.25 = 2.25 + 4$$
$$6.25 = 6.25 \Rightarrow \Delta ABC \text{ is a right triangle}$$

2)

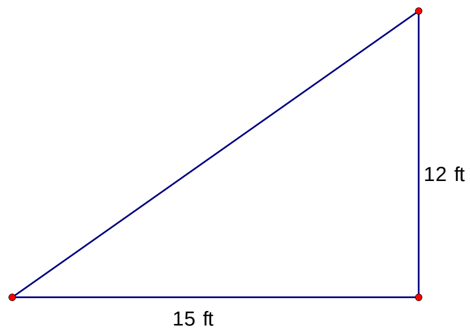
$$a = 20, b = 21, \text{ and } c = 29$$
$$c^2 = a^2 + b^2$$
$$29^2 = 20^2 + 21^2$$
$$841 = 400 + 441$$
$$841 = 841 \Rightarrow \Delta ABC \text{ is a right triangle}$$

3)

$$a = 10, b = 12, \text{ and } c = 16$$
$$c^2 = a^2 + b^2$$
$$16^2 = 10^2 + 12^2$$
$$256 = 100 + 144$$
$$256 > 244 \Rightarrow \Delta ABC \text{ is an obtuse triangle}$$

Applications of the Pythagorean Theorem

1) If a carpenter wants to make sure that a corner of a room is square and measures out 12 ft and 15 ft along the walls. How long should he make the diagonal?



$c^2 = 12^2 + 15^2$

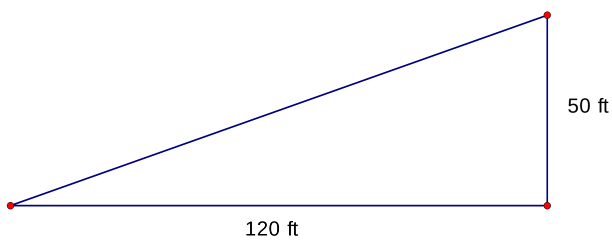
$c^2 = 144 + 225$

$c^2 = 369$

$\sqrt{c^2} = \sqrt{369}$

$c = 19.2\, ft$

2) An empty lot is 120 ft by 50 ft. How many feet would you save walking diagonally across the lot instead of walking length and width?



$c^2 = 120^2 + 50^2$

$c^2 = 14400 + 2500$

$c^2 = 16900$

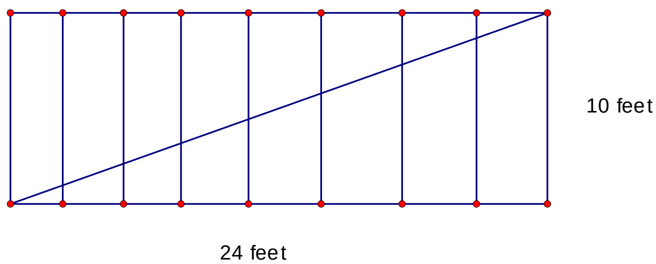
$\sqrt{c^2} = \sqrt{16900}$

$c = 130\, ft$

Compare to walking 120 ft + 50 ft = 170 ft

You would save walking 170 ft – 130 ft = 40 feet

3) A diagonal brace is to be placed in the wall of a room. The height of the wall is 10 feet and the wall is 22 feet long. What is the length of the brace?



$c^2 = 10^2 + 24^2$

$c^2 = 100 + 576$

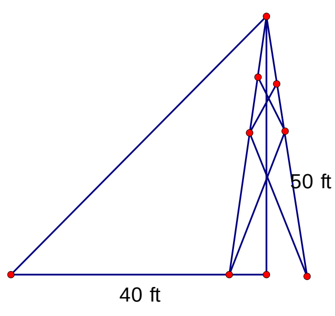
$c^2 = 676$

$\sqrt{c^2} = \sqrt{676}$

$c = 26\ feet$

4)

A television antenna is to be erected and held by guy wires. If the guy wires are 40 ft from the base of the antenna and the antenna is 50 ft high, what is the length of each guy wire?



$c^2 = 40^2 + 50^2$

$c^2 = 1600 + 2500$

$c^2 = 4100$

$\sqrt{c^2} = \sqrt{4100}$

$c \approx 64\ feet$