

## Section 1.1

### Set and Set Operators

#### Definition of a set

A **set** is a collection of objects, things or numbers.

The **universal set** is the set of all possible elements of set used in the problem. Denoted by  $\mathcal{U}$

#### Examples

$\{1,2,3,4,5\}$

$\{Ron, John, Mark, Phil\}$

$\{Virginia, West Virginia, Maryland, Tennessee, Kentucky, North Carolina\}$

Elements are the members of a given set.

$\in$  represents is an element of

$\notin$  represents is not an element of

$3 \in \{1,2,3,4,5\}$

$a \in \{a,b,c,d,e\}$

#### Roster Notation

$\{a, e, i, o, u\}$

$\{Huron, Ontario, Michigan, Erie, Superior\}$

$\{2,4,6,8,\dots\}$

#### Builder Set Notation

$\{x \mid x \text{ is a vowel}\}$

$\{x \mid x \text{ is a great lake}\}$

$\{x \mid x \text{ is an even positive number}\}$

A set is **well defined** if the elements of the sets are clearly defined.

If a set is well defined, then there should not be any confusion of what the elements are in the set

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### Examples of well defined sets

$\{1,3,5,7,9,11,13\}$

$\{m,n,o,p,q,r,s\}$

$\{x \mid x \text{ is a whole number}\}$

### Examples of set that are not well defined

$\{x \mid x \text{ is something cool}\}$

$\{x \mid x \text{ is a good football team}\}$

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### The Empty Set

The **empty set** is a set that contains no elements. The empty set is also referred to as the **null set**.

Symbol representation  $\phi$  or  $\{ \}$

### Important Sets

**Natural Numbers or Counting Numbers:**  $\{1,2,3,4,5,\dots\}$

**Whole Numbers:**  $\{0,1,2,3,4,5,\dots\}$

**Integers:**  $\{\dots -5,-4,-3,-2,-1,0,1,2,3,4,5,\dots\}$

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## Section 1.2

### Subsets

A set B is a subset of set C, if every element in B is an element of C.  $B \subseteq C$

### Proper Subsets

A set B is a proper subset of C, if every element of B is an element of C and there is at least one element of C that is not in B.  $B \subset C$

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**Example 1**

$$A = \{1,2,3,4,5\}$$

$$C = \{1,2,3,4,5,6,7\}$$

Is  $A \subset C$ ?

**Solution:** Since every element in the set A is an element of C, A is a subset of C.

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**Example 2**

Is  $\{2,3,4,5,6\}$  a subset of  $\{0,1,2,3,4,5\}$ ?

**Solution:** no, since the element 6 is not in the set  $\{0,1,2,3,4,5\}$

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**Example 3**

List all possible subsets of  $\{1,2\}$

**Solution:**  $\phi, \{1\}, \{2\}, \{1,2\}$

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**Example 4**

List all subsets of the set  $\{a,b,c\}$

**Possible subsets**

**Solution:**  $\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}$

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**Example 5**

List all subsets of the set  $\{4\}$

**Possible sets:**  $\phi, \{4\}$

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**Example 6**

List all possible subsets of  $\{m, a, t, h\}$

$\phi, \{m\}, \{a\}, \{t\}, \{h\}$

$\{m, a\}, \{m, t\}, \{m, h\}, \{a, t\}, \{a, h\}, \{t, h\}$

$\{m, a, t\}, \{m, t, h\}, \{a, t, h\}, \{m, a, h\}$

$\{m, a, t, h\}$

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**The pattern for subsets**

Number of elements	Number of subsets
1	2
2	4
3	8
4	16

**Formula to find the number of subsets  $s$  of a given set  $A$  with  $n$  elements:  $s = 2^n$**

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**Example 7**

How many subsets does a set  $A$  with 6 elements have?

**Solution:**  $s = 2^n = 2^6 = 64$

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**Example 8**

Let  $A = \{1,2,3\}$ ,  $B = \{1,2\}$ ,  $C = \{1,2,3,4,5\}$

1) Is  $A \subset C$ ?

*Yes, every element in A is in C*

2) Is  $B \subset C$ ?

*Yes*

3) Is  $3 \in A$ ?

*Yes*

4) Is  $C \subset A$ ?

*No, 4 and 5 are not in A*

5) Is  $\phi \subset A$ ?

*Yes*

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**Problem Set 1**

**I) Which of the following sets are well defined?**

1)  $\{1,2,3,4,5\}$

2)  $\{x \mid x \text{ is a U S state}\}$

3)  $\{x \mid x \text{ is a North America breed of Dog}\}$

4)  $\{x \mid x \text{ is a fun game}\}$

**II) Write each set in roster form**

1)  $\{x \mid x \text{ is natural number}\}$

2)  $\{x \mid x \text{ is a state that begins with the letter M}\}$

3)  $\{x \mid x \text{ is a vowel}\}$

**III) Use the following sets to answer each question.**

$A = \{a, b, c, d, e, f, g\}$ ;  $B = \{a, b, c, d\}$ ;  $C = \{a, c, e\}$ ;  $D = \{a, e\}$ ;  $E = \{e, f, g, h\}$ ;  $F = \phi$

- 1) Is  $C \subset A$ ?
- 2) Is  $c \in A$ ?
- 3) Is  $E \subset A$ ?
- 4) Is  $F \subset A$ ?
- 5) Is  $h \in A$ ?
- 6) Name three subsets of  $C$
- 7) Is  $D \subset C$ ?
- 8) Is  $C \in A$ ?
- 9) Is  $D \subset E$ ?

**IV) Subsets**

- 1) List all subsets of  $\{a, b\}$
- 2) List all subsets of  $\{10, 20, 30\}$
- 3) List all subsets of  $\{l, o, v, e\}$
- 4) A set of 8 elements would have how many possible subsets

**V) Describe each set as an infinite set or a finite set.**

- 1)  $\{x \mid x \text{ is one of the 50 state capitals}\}$
- 2)  $\{x \mid x \text{ is an even natural number}\}$
- 3)  $\{\dots, -5, -3, -1, 1, 3, 5, \dots\}$
- 4)  $\{-8, 8, -6, 6, -4, 4, -2, 2, 0\}$

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**Section 1.3**

**Union of Two Sets**

The union of two sets is denoted by  $A \cup B$  is  $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

**Intersection of Two Sets**

The intersect of two sets is denoted by  $A \cap B$  is  $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

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**Example 1**

$$A = \{1,2,3\}$$

$$C = \{1,3,5,7\}$$

$$1) A \cup C = \{1,2,3,5,7\}$$

$$2) A \cap C = \{1,3\}$$

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**Example 2:** Let  $A = \{1,2,3,4,5\}$ ,  $B = \{1,3,5,7\}$ ,  $C = \{1,2,3\}$ ,  $D = \{1,2,3,4,5\}$ , and  $E = \phi$

1) Is  $C \subset A$ ?

**Answer: Yes, every element in C is contained in A**

2) Is  $B \subset A$ ?

**Answer: No, the element 7 of set B is not contained in A.**

3) Is  $D \subset A$ ?

**Answer: Yes, every element of D is in A.**

4) Is  $\phi \subset A$ ?

**Yes, the empty set is a subset of any nonempty every set.**

5) Find  $A \cap B$

**Answer:  $A \cap B = \{1,3,5\}$**

6) Find  $A \cup B$

**Answer:  $A \cup B = \{1,2,3,4,5,7\}$**

7) Find  $A \cap C$

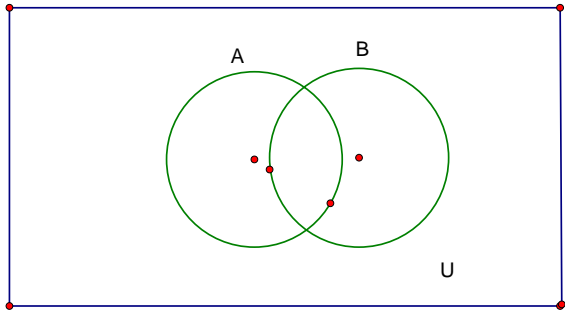
**Answer:  $A \cap C = \{1,2,3\}$**

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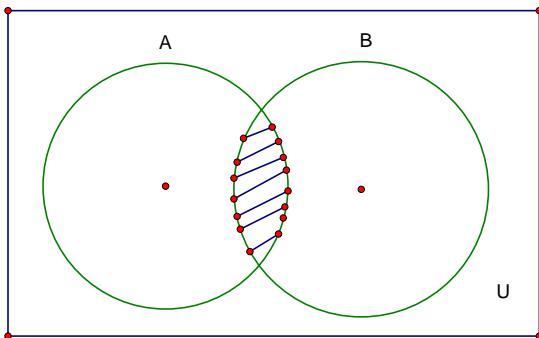
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## Venn Diagrams

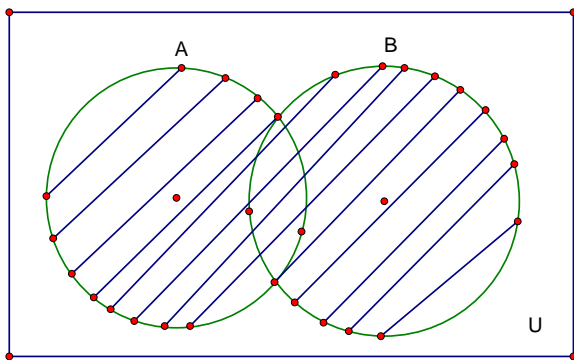
$\mathcal{U}$  = the universal set



$A \cap B$



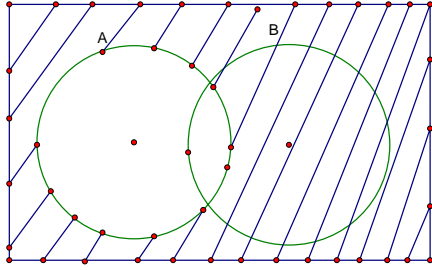
$A \cup B$



## The complement of a set A

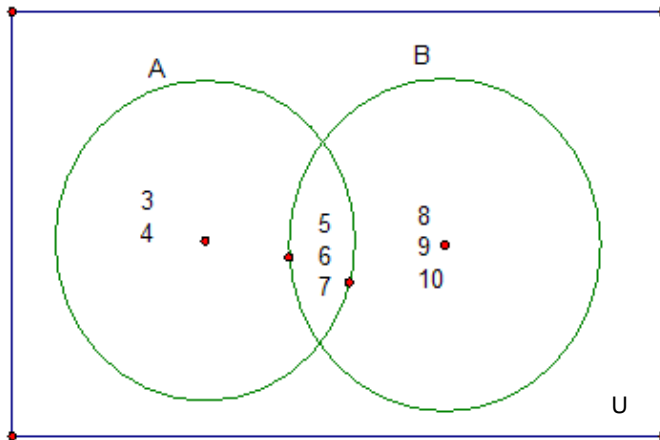
The complement of a set A is the set of all elements in the universal set that are not elements of the set A.

$$A' = \{x \mid x \notin A \text{ and } x \in U\}$$



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### Example 3



- 1) Find  $A \cap B$   
 $A \cap B = \{5, 6, 7\}$
  - 2) Find  $A \cup B$   
 $A \cup B = \{3, 4, 5, 6, 7, 8, 9, 10\}$
  - 3) Find  $A'$   
 $A' = \{8, 9, 10\}$
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### Example 4

*Given*

$$A = \{1,2,3,4,5,6\}, B = \{4,5,6,7,8\}, U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$$

*Find*

1)  $A \cup B$

$$A \cup B = \{1,2,3,4,5,6,7,8\}$$

2)  $A \cap B$

$$A \cap B = \{4,5,6\}$$

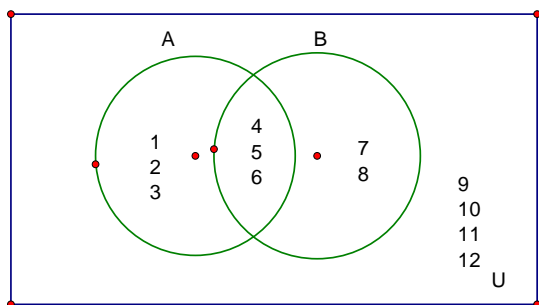
3)  $A'$

$$A' = \{7,8,9,10,11,12\}$$

4)  $B'$

$$B' = \{1,2,3,9,10,11,12\}$$

Make a Venn diagram of A,B, and U



## Equivalent Sets

Two sets are equivalent if they have the same number of elements.

### Examples of equivalent sets

$\{1,2,3,4\}$

and

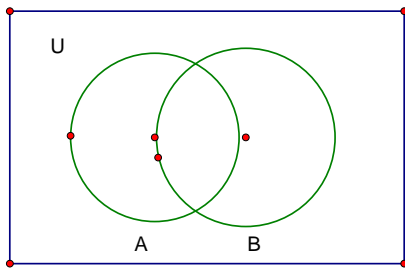
$\{a,b,c,d\}$

$\{john,luke,mark,mathew\}$

and

$\{a,b,c,d\}$

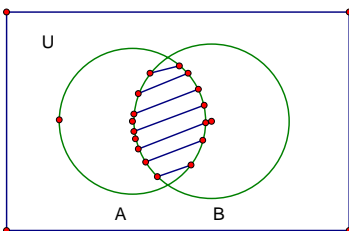
Venn diagrams



### Example 5

Shade the region corresponding to the indicated set.

$A \cap B$

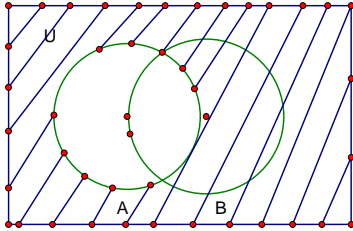


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**Example 6**

Shade the region corresponding to the indicated set.

$$A'$$

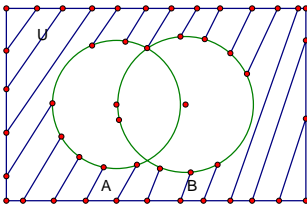


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**Example 7**

Shade the region corresponding to the indicated set.

$$A' \cap B'$$

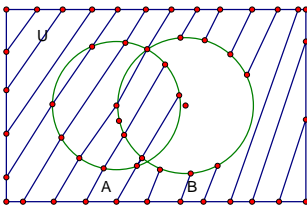


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**Example 8**

Shade the region corresponding to the indicated set.

$$A \cup B'$$



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## Section 1.4

### Cardinality

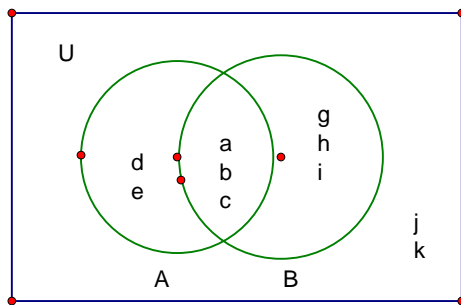
Definition: Cardinality is the number of element in a given set

The number of elements in a set  $A$  is denoted by  $n(A)$

### Example 9

Shade the region corresponding to the indicated set.

$$A = \{a, b, c, d, e\}, B = \{a, b, c, g, h, i\}, U = \{a, b, c, d, e, f, g, h, i, j, k\}$$



1) Find  $n(A)$

$$n(A) = 5$$

2) Find  $n(B)$

$$n(B) = 6$$

3) Find  $n(A \cup B)$

$$n(A \cup B) = 8$$

4) Find  $n(A \cap B)$

$$n(A \cap B) = 3$$

### Rule for the cardinality for the union of two sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Use this formula to find  $n(A \cup B)$  in problem 3.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 5 + 6 - 3 = 11 - 3 = 8$$

This gives the same answer as #3

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### Problem Set 2

I) Let  $A = \{a, b, c, d, e, f\}$ ,  $B = \{b, d, f, h\}$ ,  $C = \{a, b, c\}$ ,  $D = \{e, f, g, h, i\}$

- 1) Find  $A \cup B$
- 2) Find  $A \cap B$
- 3) Find  $A \cup C$
- 4) Find  $A \cap D$
- 5) Is  $C \subset A$ ?
- 6) Is  $B \subset A$ ?
- 7) What is  $n(A)$ ?
- 8) What is  $n(A \cup B)$ ?

II) Let  $A = \{2, 4, 8, 10, 12\}$ ,  $B = \{3, 5, 7, 9, 11\}$ ,  $C = \{7, 9, 11\}$ ,  $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

- 9) Find  $A \cup B$
- 10) Find  $A \cap B$
- 11) Find  $A \cup D$
- 12) Find  $A \cap D$
- 13) Is  $C \subset B$ ?
- 14) Is  $B \subset A$ ?
- 15) What is  $n(A)$ ?
- 16) What is  $n(A \cup B)$ ?
- 17) Draw a Venn Diagram using sets A, B, and U

### Subsets

- 1) list all subsets of  $\{10, 20, 30\}$
- 2) How many subsets does a set with 20 elements have?