Study List Math 151
Solutions

Find the derivative of each function.

1)
$$y = x^4 + 5x^2 + 7x + 6$$
$$y' = 4x^3 + 10x + 7$$

2)
$$y = 5x^4 - 8x^3$$
$$y' = 20x^3 - 24x^2$$

3)
$$f(x) = 8\sin(x)$$
$$f'(x) = 8\cos(x)$$

4)
$$f(x) = \cos(4x^2)$$
$$f'(x) = 8x\cos(4x^2)$$

5)
$$y = \left(x^2 + 3x\right)\left(2x^2 + 6x + 4\right)$$
$$y' = (2x+3)(2x^2 + 6x + 4) + (4x^2 + 6)(x^2 + 3x)$$
$$y' = 4x^3 + 12x^2 + 8x + 6x^2 + 18x + 12 + 4x^4 + 12x^3 + 6x^2 + 18x$$
$$y' = 4x^4 + 16x^3 + 24x^2 + 44x + 12$$

6)
$$y = \frac{x^2 + 3}{3x^2 - 4x}$$
$$y' = \frac{\left(3x^2 - 4x\right)\frac{d}{dx}\left(x^2 - 3\right) - \left(x^2 - 3\right)\frac{d}{dx}\left(3x^2 - 4x\right)}{\left(3x^2 - 4x\right)^2}$$
$$y' = \frac{\left(3x^2 - 4x\right)\left(2x\right) - \left(x^2 - 3\right)\left(6x - 4\right)}{\left(3x^2 - 4x\right)^2}$$
$$y' = \frac{6x^3 - 8x^2 - \left(6x^3 - 4x^2 - 18x + 12\right)}{\left(3x^2 - 4x\right)^2}$$
$$y' = \frac{-4x^2 + 18x - 12}{\left(3x^2 - 4x\right)^2}$$

7)

$$f(x) = e^{x^2} \sin(3x)$$

$$f'(x) = \frac{d}{dx}\left(e^{x^2}\right)\sin(3x) + \frac{d}{dx}\sin(3x)\left(e^{x^2}\right)$$

$$f'(x) = 2xe^{x^2} \sin(3x) + 3e^{x^2} \cos(3x)$$

8)

$$f(x) = 8x^2 e^{x^2-4x}$$

$$f'(x) = \frac{d}{dx}\left(8x^2\right)e^{x^2-8x} + \frac{d}{dx}\left(e^{x^2-8x}\right)\left(8x^2\right)$$

$$f'(x) = 16xe^{x^2-8x} + (2x-8)e^{x^2-8x}\left(8x^2\right)$$

$$f'(x) = 16xe^{x^2-8x} + 16x^3 e^{x^2-8x} - 64x^2 e^{x^2-8x}$$

9)

$$f(x) = \frac{e^{3x} - 2}{e^{-2x}}$$

$$f'(x) = \frac{\left(e^{-2x}\right)\left(3e^{3x}\right) - \left(e^{3x} - 2\right)\left(-2e^{-2x}\right)}{\left(e^{-2x}\right)^2}$$

$$f'(x) = \frac{3e^{x} + 2e^{x} + 4e^{-2x}}{e^{-4x}}$$

$$f'(x) = \frac{5e^{x} + 4e^{-2x}}{e^{-4x}}$$

or

$$f'(x) = e^{4x}\left(5e^{x} + 4e^{-2x}\right)$$

10)

$$y = \left(x^2 + 2x\right)^4$$

$$y' = 4(2x+2)\left(x^2 + 2x\right)^3$$

$$y' = (8x+8)\left(x^2 + 2x\right)^3$$

11)

$$g(x) = \ln(x^2 - 4x + 4)$$

$$g'(x) = \frac{2x - 4}{x^2 - 4x + 4}$$

12)

$$y = \tan(2x^2 + 3x)$$

$$y' = (4x + 3)\sec^2(2x^2 + 3)$$

13)

$$h(x) = 5x\ln(x^2)$$

$$h'(x) = \frac{d}{dx}(5x)\ln(x^2) + \frac{d}{dx}(\ln(x^2))(5x)$$

$$h'(x) = 5\ln(x^2) + \frac{2x}{x^2}(5x)$$

$$h'(x) = 5\ln(x^2) + 10$$

14) $y = \dfrac{\ln(4x) - e^x}{\ln(3x^2)}$

$$y = \frac{\ln(4x) - e^x}{\ln(3x^2)}$$

$$y' = \frac{(\ln(3x^2))\left(\frac{4}{4x} - e^x\right) + (\ln(4x) - e^x)\frac{6x}{3x^2}}{[\ln(3x^2)]^2}$$

$$y' = \frac{(\ln(3x^2))\left(\frac{1}{x} - e^x\right) + (\ln(4x) - e^x)\left(\frac{2}{x}\right)}{[\ln(3x^2)]^2}$$

15)

$$f(x) = x^3 \sin(2x^3)$$
$$f'(x) = \frac{d}{dx}\left(x^3\right)\sin\left(2x^3\right) + \frac{d}{dx}\left(\sin\left(2x^3\right)\right)\left(x^3\right)$$
$$f'(x) = 3x^2 \sin\left(2x^3\right) + 3x^2 \cos\left(2x^3\right)\left(x^3\right)$$
$$f'(x) = 3x^2 \sin\left(2x^3\right) + 3x^5 \cos\left(2x^3\right)$$

16)

$$y = e^{x^2 \sin(2x)}$$
$$y' = \left(\frac{d}{dx}\left(x^2\right)\sin(2x) + \frac{d}{dx}\sin(2x)\left(x^2\right)\right)e^{x^2 \sin 2x}$$
$$y' = \left(2x\sin(2x) + 2\cos(2x)\left(x^2\right)\right)e^{x^2 \sin(2x)}$$
$$y' = \left(2x\sin(2x) + 2x^2 \cos(2x)\right)e^{x^2 \sin 2x}$$

17)

$$f(x) = \cos(\ln(x))$$
$$f'(x) = \frac{d}{dx}\ln(x)(-\sin(\ln(x)))$$
$$f'(x) = -\frac{1}{x}\sin(\ln(x))$$
$$f'(x) = -\frac{\sin(\ln(x))}{x}$$

Find the equation of the tangent line through the given point.

18) $y = x^3 - 2$; $(1,-1)$

$$y = x^3 - 2$$
$$y' = 3x^2$$
$$m = 3(1)^2 = 3(1) = 3$$

$$y - (-1) = 3(x - 1)$$
$$y + 1 = 3x - 3$$
$$y = 3x - 4$$

19) $f(x) = x^2 \sin(x)$; $(\pi, 0)$

$$f(x) = x^2 \sin(x)$$
$$f'(x) = 2x\sin(x) + \cos x\left(x^2\right)$$
$$f'(x) = 2x\sin x + x^2 \cos x$$

$$m = f'(\pi) = 2(\pi)\sin(\pi) + \pi^2 \cos(\pi) = 0 + (-\pi^2) = -\pi^2$$

$$y - 0 = -\pi^2 (x - \pi)$$
$$y = -\pi^2 \left(x - \pi\right)$$

20) $f(x) = e^{x^2 - 1}$; $(1,1)$

$$f(x) = e^{x^2 - 1}$$
$$f(x) = 2xe^{x^2 - 1}$$

$$m = f(1) = 2(1)e^{1^2 - 1} = 2e^{1-1} = 2e^0 = 2$$

$$y - 1 = 2(x - 1)$$
$$y - 1 = 2x - 2$$
$$y = 2x - 1$$

Use implicit differentiation to find the derivative of each function

21) $x^2 + 3xy + y^2 = 8$

$$x^2 + 3xy + y^2 = 8$$
$$\frac{d}{dx}\left(x^2 + 3xy + y^2\right) = \frac{d}{dx}(8)$$
$$2x + 3y + 3xy' + 2yy' = 0$$
$$3xy' + 2yy' = -2x - 3y$$
$$(3x + 2y)y' = -2x - 3y$$
$$y' = \frac{-2x - 3y}{3x + 2y}$$

22)

$$5\sin(x)\cos(y) = 5$$

$$\frac{d}{dx}5\sin x\cos y = \frac{d}{dx}5$$

$$-5\cos x\cos y + 5\sin x\sin y\, y' = 0$$

$$5\sin x\sin y\, y' = 5\cos x\cos y$$

$$y' = \frac{5\cos x\cos y}{5\sin x\sin y}$$

$$y' = \cot x\cot y$$

23)

$$x^3 + 4x^2y^2 = 1 + ye^{x^2}$$

$$\frac{d}{dx}\left(x^3 + 4x^2y^2\right) = \frac{d}{dx}\left(1 + ye^{x^2}\right)$$

$$3x^2 + 8xy^2 + 4x^2\,2yy' = 0 + y'e^{x^2} + 2xe^{x^3}y$$

$$4x^2\,2yy' - y'e^{x^2} = -3x^2 + 2xe^{x^2}y - 8xy^2$$

$$(8x^2y - e^{x^2})y' = -3x^2 + 2xe^{x^2}y - 8xy^2$$

$$y' = \frac{-3x^2 + 2xe^{x^2}y - 8xy^2}{8x^2y - e^{x^2}}$$

Use implicit differentiation to find the equation of a tangent line to the curve at the given point.

24) $x^2 + 2xy + y^2 = 4$; $(1,1)$

$$x^2 + 2xy + y^2 = 4$$
$$\frac{d}{dx}\left(x^2 + 2xy + y^2\right) = \frac{d}{dx}4$$
$$2x + 2y + 2xy' + 2yy' = 0$$
$$2xy' + 2yy' = -2x - 2y$$
$$(2x + 2y)y' = -2x - 2y$$
$$y' = \frac{-2x - 2y}{2x + 2y}$$
$$y' = \frac{-(2x + 2y)}{2x + 2y}$$
$$y' = -1$$

$m = -1$ *when* $x = 1$ *and* $y = 1$

$$y - 1 = -1(x - 1)$$
$$y - 1 = -1x + 1$$
$$y = -x + 2$$