Set and Set Operators

Definition of a set

A **set** is a collection of objects, things or numbers.

The **universal set** is the set of all possible elements of set used in the problem. Denoted by U

Examples

$\{1,2,3,4,5\}$
$\{Ron, John, Mark, Phil\}$
$\{Virginia, West\ Virginia, Maryland, Tennessee, Kentucky, Noth\ Carolina\}$

Elements are the members of a given set.

$\in$ *represents is an element of*
$\notin$ *represents is not an element of*

$3 \in \{1,2,3,4,5\}$
$a \in \{a,b,c,d,e\}$

Roster Notation

$\{a,e,i,o,u\}$
$\{Huron, Ontario, Michigan, Erie, Superior\}$
$\{2,4,6,8,.......\}$

Builder Set Notation

$\{x \mid x\ is\ a\ vowel\}$
$\{x \mid x\ is\ a\ great\ lake\}$
$\{x \mid x\ is\ an\ even\ positive\ number\}$

Subsets

A set B is a subset of set C, if every element in B is an element of C. $B \subset C$

Example

$A = \{1,2,3,4,5\}$
$C = \{1,2,3,4,5,6,7\}$

$Is\ A \subset C$?
Since every element in the set A is an element of C, A is a subset of C.

Union of Two Sets

The union of two sets is denoted by $A \cup B$ is $A \cup B = \{x \mid x \in A\ or\ x \in B\}$

Intersection of Two Sets

The intersect of two sets is denoted by $A \cap B$ is $A \cap B = \{x \mid x \in A\ and\ x \in B\}$

Examples

$A = \{1,2,3\}$
$C = \{1,3,5,7\}$

1) $A \cup C = \{1,2,3,5,7\}$

2) $A \cap C = \{1,3\}$

The Empty Set

The **empty set** is a set that contains no elements. The empty set is also referred to as the **null set**.

Symbol representation ϕ or {}

Example Sets

Example 1

Let A={1,2,3,4,5},B={1,3,5,7},C={1,2,3},D={1,2,3,4,5}, and E=ϕ

1) Is $C \subset A$?
Answer: Yes, every element in C is contained in A

2) Is $B \subset A$?
Answer: No, the element 7 of set B is not contained in A.

3) Is $D \subset A$?
Answer: Yes, every element of D is in A.

4) Is $\phi \subset A$?
Yes, the empty set is a subset of any nonempty every set.

5) Find $A \cap B$
Answer: $A \cap B = \{1,3,5\}$

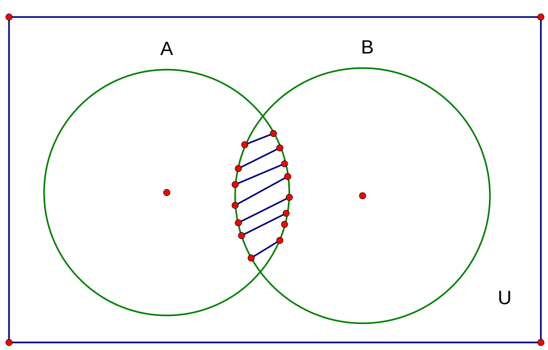
6) Find $A \cup B$
Answer: $A \cup B = \{1,2,3,4,5,7\}$

7) Find $A \cap C$
Answer: $A \cap C = \{1,2,3\}$

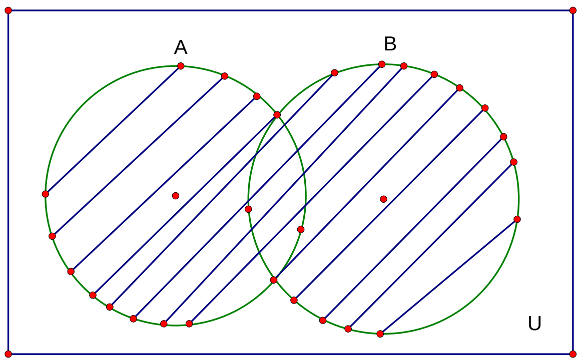
Venn Diagrams

U = the universal set

$A \cap B$



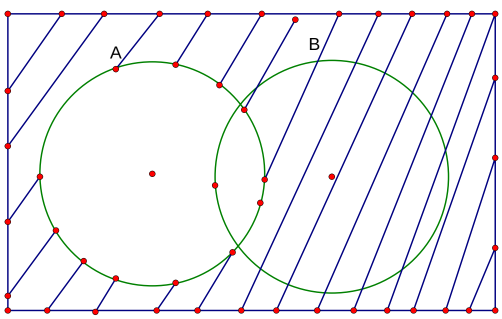
$A \cup B$



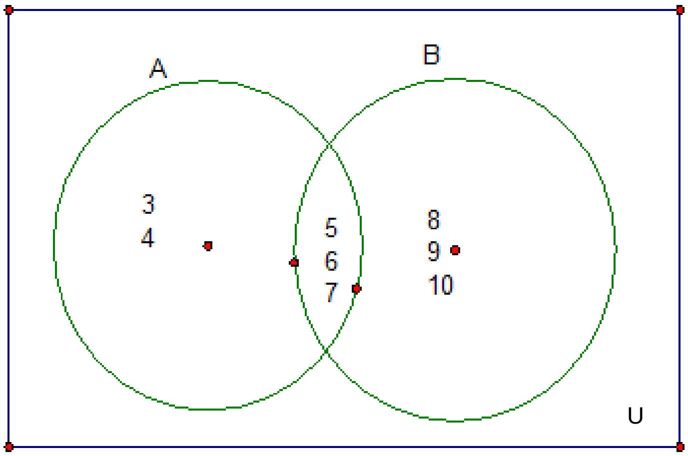
The complement of a set A

The complement of a set A is the set of all elements in the universal that are not elements of the set A.

$A' = \{x \mid x \notin A \text{ and } x \in U\}$



Example 2



1) Find $A \cap B$

$A \cap B = \{5,6,7\}$

2) Find $A \cup B$

$A \cup B = \{3,4,5,6,7,8,9,10\}$

3) Find A'

$A' = \{8,9,10\}$

Example 3

Given

$A = \{1,2,3,4,5,6\}, B = \{4,5,6,7,8\}, U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$

Find

1) $A \cup B$

$A \cup B = \{1,2,3,4,5,6,7,8\}$

2) $A \cap B$

$A \cap B = \{4,5,6\}$

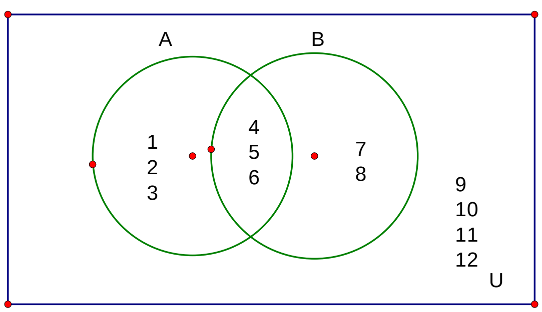
3) A'

$A' = \{7,8,9,10,11,12\}$

4) B'

$B' = \{1,2,3,9,10,11,10\}$

Make a Venn diagram of A,B, and U



Example 4

List all subsets of the set {1,2}

Possible subsets

$\phi, \{1\}, \{2\}, \{1,2\}$

Example 5

List all subsets of the set {a,b,c}

Possible subsets

$\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\{a,b,c\}$

Example 6

List all subsets of the set {4}

Possible sets: $\phi, \{4\}$

The pattern for subsets

Number of elements	Number of subsets
1	2
2	4
3	8
4	16

Formula to find the number of subsets s of a given set A with n elements

$s = 2^n$

Example 6

How many subsets does a set A with 6 elements have?

$s = 2^n$

$s = 2^6$

$s = 64$

Equivalent Sets

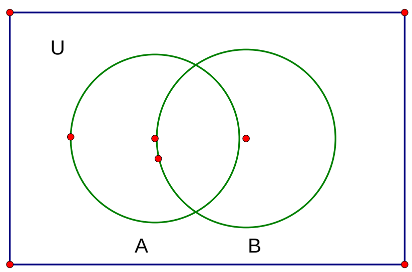
Two sets are equivalent if they have the same number of elements.

Examples of equivalent sets

$\{1,2,3,4\}$
and
$\{a,b,c,d\}$

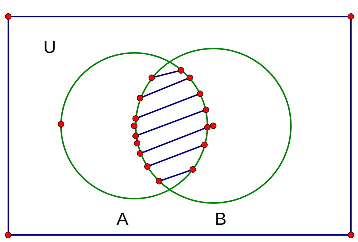
$\{john, luke, mark, mathew\}$
and
$\{a,b,c,d\}$

Venn diagrams

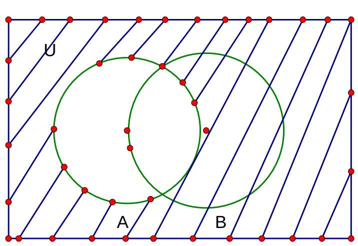


Shade the region corresponding to the indicated set.

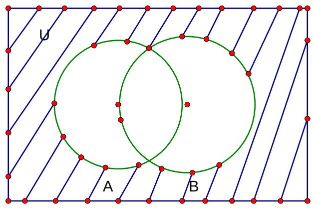
1) $A \cap B$



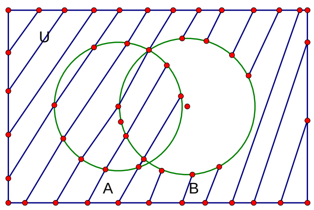
2) A'



3) $A' \cap B'$



4) $A \cup B'$

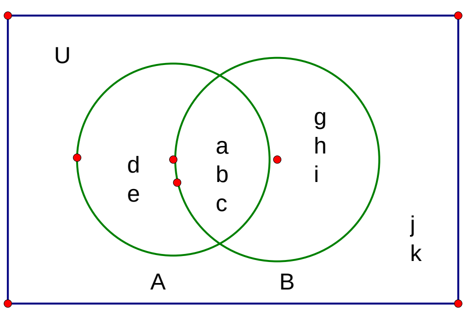


Cardinality

Definition: Cardinality is the number of elements in a given set

The number of elements in a set A is denoted by $n(A)$

$A = \{a,b,c,d,e\}, B = \{a,b,c,g,h,i\}, U = \{a,b,c,d,e,f,g,h,,i,j,k\}$



1) Find $n(A)$

$n(A) = 5$

2) Find $n(B)$

$n(B) = 6$

3) Find $n(A \cup B)$

$$n(A \cup B) = 8$$

4) Find $n(A \cap B)$

$$n(A \cap B) = 3$$

Rule for the cardinality for the union of two sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Use this formula to find $n(A \cup B)$ in problem 3.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 5 + 6 - 3 = 11 - 3 = 8$$

Example

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#33-36

$$U = \{x \mid x \text{ is a state in the United States}\}$$
$$A = \{x \mid x \in U \text{ and } x \text{ begins with } A\}$$
$$I = \{x \mid x \in U \text{ and } x \text{ begins with } I\}$$
$$M = \{x \mid x \in U \text{ and } x \text{ begins with } M\}$$
$$N = \{x \mid x \in U \text{ and } x \text{ begins with } N\}$$
$$O = \{x \mid x \in U \text{ and } x \text{ begins with } O\}$$

$$A = \{Alabama,\ Arkansas,\ Alaska,\ Arizona\}$$
$$I = \{Iowa,\ Indiana,\ Illinois,\ Idaho\}$$
$$M = \{Michigan,\ Minnesota,\ Mississippi,\ Missouri,\ Maryland,\ Maine,\ Montana,\ Massachusetts\}$$
$$N = \left\{\begin{matrix} Nebraska,\ New\ Jersey,\ New\ Mexico,\ New\ York,\ New\ Hamphere,\ North\ Carolina, \\ North\ Dakota,\ Nevada \end{matrix}\right\}$$
$$O = \{Ohio,\ Oklahoma, Oregon\}$$

33) $n(M') = 50 - 8 = 42$

34) $n(A \cup N) = 13$

35) $n(I' \cap O') = 50 - (3 + 4) = 50 - 7 = 43$

36) $n(M \cap I) = 0$

Infinite sets and Cardinality

One-to-one correspondence

Definition: Two sets are in one-to-one correspondence if each element in the first is paired with exactly one element in the second set, and each element of the second set is paired with exactly one element form the first set

Examples

1) The sets $\{1,2,3,4\}$ *and* $\{a,b,c,d\}$ are in one-to-one correspondence as shown in this diagram.

$$\begin{array}{cccc} \{1, & 2, & 3, & 4\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{a, & b, & c, & d\} \end{array}$$

2) The sets $\{Viriginia,\ Maryland, North\ Carolina\}$ and $\{Richmond, Annapolis, Raliegh\}$ are in one-to-one correspondence as shown in this diagram.

$$\begin{array}{ccc} \{Viriginia, & Maryland, & North\ Carolina\} \\ \updownarrow & \updownarrow & \updownarrow \\ \{Richmond, & Annapolis, & Raliegh\} \end{array}$$

Infinite sets

The natural numbers

$N = \{1,2,3,4,5,6,..........\}$

The whole numbers

$W = \{0,1,2,3,4,5,..........\}$

The integers

$J = \{.....,-4,-3,-2,-1,0,1,2,3,4,...........\}$

The rational numbers

$$Q = \left\{ x \mid x = \frac{a}{b}, where\ a\ and\ b \in I \right\}$$

Other infinite sets

The irrational numbers

The real numbers

The real numbers are the rational number and irrational combine as one set.

History of Infinity

Aristotle

Aristotle distinguished between the potential infinite and the actual infinite. Aristotle actually claimed that the natural numbers where potentially infinite because they have no greatest element, but he would not allow them to be actually infinite.

Galileo

Galileo noticed the fact that you could take the natural numbers and remove half of them and remaining set is still as large as the original set.

Galileo's Paradox

If you take the natural numbers and remove the odd elements, the resulting set, the even numbers, is equinumerous to the natural numbers.

In the 1870's **Georg Cantor** discovered that it is possible to determine if two infinite sets are the same size or equinumerous by seeking to find a one-to-one match up between the elements of each set.

Cantor's definition of set

A set is **infinite** if we can remove some of its elements without reducing its size.

Countable sets

A set is countable if you establish a one-to-one correspondence form the given set to the natural numbers.

Examples

1) Are the even natural numbers countable?

$$\begin{array}{l}\{2,4,6,8,10,12,............2n\} \\ \updownarrow\updownarrow\updownarrow\updownarrow\ \updownarrow\ \updownarrow \qquad\qquad \updownarrow \\ \{1,2,3,4,5,\ 6,................n\}\end{array}$$

The even natural can be put in a one-to-one correspondence with the natural numbers by using the mapping $n \leftrightarrow 2n$

2) Are the integers countable?

$$J = \{.....,-4,-3,-2,-1,0,1,2,3,4,...........\}$$

The mapping would go as follows:

$$\begin{array}{l} 0 \leftrightarrow 1 \\ 1 \leftrightarrow 2 \\ -1 \leftrightarrow 3 \\ 2 \leftrightarrow 4 \\ -2 \leftrightarrow 5 \\ 3 \leftrightarrow 6 \\ -3 \leftrightarrow 7 \\ etc. \end{array}$$

Use this mapping

$$n \leftrightarrow \frac{n}{2} \ if \ \ n \ is \ even$$

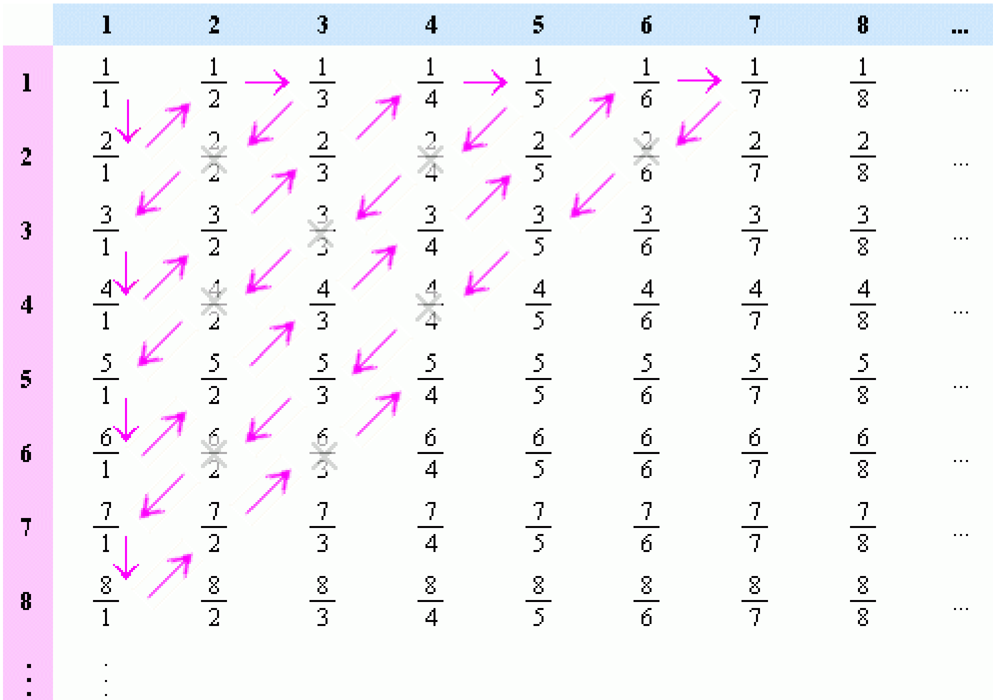
$$n \leftrightarrow \frac{1-n}{2} \ if \ \ n \ is \ odd$$

Therefore, there exist a one-to-one correspondence between the integers and the natural numbers.

Thus, the integers are countable.

3) Are the rational numbers countable?

Look at the following diagram



http://www.homeschoolmath.net/other_topics/rational-numbers-countable.php

This allow the following ordering of numbers

$1 \to 1$

$2 \to 2$

$\frac{1}{2} \to 3$

$\frac{1}{3} \to 4$

$3 \to 5$

......

This shows that each element of the rational number can be paired with one element of the natural numbers. Thus, it is possible to establish a one-to-one correspondence with the natural numbers. This provides an interesting result which is that the rational numbers turn out to countable.