Section 6.6
Logarithms and Logarithmic Models

Definition of a logarithm

$\log_b a = x \Leftrightarrow b^x = a$

Example 1

Write each logarithm expression as an exponent expression.

a) $\log_5 125 = 3$

Solution: $5^3 = 125$

b) $\log_6 36 = 2$

Solution: $6^2 = 36$

c) $\log_6\left(\frac{1}{216}\right) = -3$

Solution: $6^{-3} = \frac{1}{216}$

Example 2

Write each exponential expression as an logarithmic expression.

a) $2^5 = 32$

Solution: $\log_2 32 = 5$

b) $4^4 = 256$

Solution: $\log_4 256 = 4$

Example 3

Evaluate each logarithm

a) Find $\log_4 64$

$\log_4 64 = x$

$4^x = 64$

$4^x = 4^3$

$x = 3$

b) Find $\log_3\left(\frac{1}{81}\right)$

$\log_3\left(\frac{1}{81}\right) = x$

$3^x = \frac{1}{81}$

$3^x = \frac{1}{3^4}$

$3^x = 3^{-4}$

c) Find $\log_{10} 10000$ *or* $\log 10000$

$\log_{10} 10000 = x$

$10^x = 10000$

$10^x = 10^4$

$x = 4$

Example 4

Solve the equation for x

$$\log_3 x = 5$$
$$3^5 = x$$
$$x = 243$$

The natural logarithm

$$\ln a = x \Leftrightarrow e^x = a$$
$$Note: \ \ln a = \log_e a$$

Example 5

Write the logarithm expression as an exponent expression

$$\ln 7.39 = 2$$

Solution: $e^2 = 7.39$

pH Models and Logarithm scales

Chemistry uses logarithm to determine the pH of liquid. The pH of a liquid measures the acidity or alkalinity of a liquid. A liquid with a pH of 1 is a very strong acid and a liquid with a pH of 14 is a very strong base. Specifically, the acidity of a substance is a function of its hydrogen-ion concentration. The pH of substance can be determined by the taking the log of its hydrogen concentration. H^+

$$pH = -\log[H^+]$$

Example 6

Find the pH of a sample of orange juice that has a hydrogen-ion concentration of $3.0 \ x \ 10^{-4} \ mole \ per \ liter$

$$pH = -\log[H^+] = -\log[3.0 \ x \ 10^{-4}] =$$

World Oil Supply

Example 7

The time it will take the world's oil supply to be depleted can be modeled by the following formula where r is the estimated oil reserves in billions of barrels.

$T(r) = 14.29\ln(0.0041r + 1)$

a) Use the model to find out how much time it will take to use 500 billions barrels.

$$T(r) = 14.29\ln(0.0041r + 1) = 14.29\ln(0.0041(500) + 1) = 14.29(\ln(2.05) + 1) = 14.29(1.718) \approx 25 \ \textit{years}$$

b) How many barrels of oil are necessary to last 40 years?

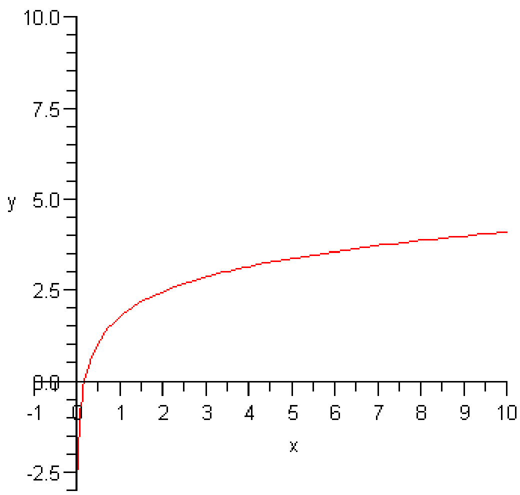
$$T(r) = 14.29\ln(0.0041r + 1)$$
$$40 = 14.29\ln(0.0041r + 1)$$
$$\frac{40}{14.29} = \frac{14.29\ln(0.041r + 1)}{14.29}$$
$$2.799 = \ln(0.0041r + 1)$$
$$\Rightarrow e^{2.799} = 0.0041r + 1$$
$$\Rightarrow 0.0041r = e^{2.799} - 1$$
$$\Rightarrow r = \frac{e^{2.799} - 1}{0.0041}$$
$$\Rightarrow r \approx 3763$$

Example 8

Graph $y = \log 6x$

X	y
2	$y = \log(6(2)) = \log(12) = 1.07$
10	$y = \log(6(10)) = \log(60) = 1.8$
20	$y = \log(6(20)) = \log(120) = 2.1$
40	$y = \log(6(40)) = \log(240) = 2.4$

Plot the given values from the table gives the following graph

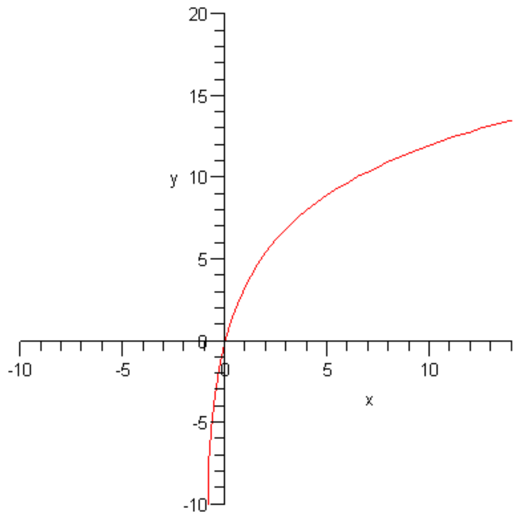


Example 9

Graph $y = 5\log(x+1)$

x	Y
2	$y = 5\log(2+1) = 5\log(3) = 2.4$
10	$y = 5\log(10+1) = 5\log(11) = 5.2$
20	$y = 5\log(20+1) = 5\log(21) = 6.6$
40	$y = 5\log(40+1) = 5\log(41) = 8.1$

Plot the given values from the table gives the following graph


20
15
y 10
5
0
-10
-5
5
10
x
-5
-10