

Number Systems

Section 4.3

Binary numbers

Babylonians system was based on 60

The Mayan system was based on 20

Our number system is based on 10

Computer use a number system based on 2 (Binary)

System	Base	Digits	Place Values
Binary	2	0,1	1,2,4,8,16,32
Quintary	5	0,1,2,3,4	1,5,25,125,625
Octal	8	0,1,2,3,4,5,6,7	1,8,64,512,4096

Converting a based number other than base 10 to base 10

Write each of the following on a decimal numeral

1) Optional

253_7

$$2 \cdot 7^2 + 5 \cdot 7^1 + 3 \cdot 7^0$$

$$2 \cdot 49 + 5 \cdot 7 + 3 \cdot 1$$

$$98 + 35 + 3$$

$$136$$

2) Optional

1068_9

$$1 \cdot 9^3 + 0 \cdot 9^2 + 6 \cdot 9^1 + 8 \cdot 9^0$$

$$729 + 0 + 54 + 8$$

$$791$$

3) Convert to base 10

1111_2

$$1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$8 + 4 + 2 + 1$$

15

4) Convert to base 10

101010_2

$$1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 \quad]$$

$$32 + 0 + 16 + 0 + 2$$

50

Binary Numbers (Base Two)

5) Convert 243 to a base 2 number (**Binary Number**)

First Check all power of 2 that divide 243

$$2^0 = 2$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256 \text{ which is too big, so use 128 or } 2^7$$

Write 243 as a difference (Note : $243 - 128 = 115$)

$$\text{Thus, } 243 = 128 + 115$$

$$\text{Then, } 243 = 2^7 + 115$$

Find the greatest power of two that divides 115 which is $2^6 = 64$

$$\text{So, } 243 = 2^7 + 64 + 51 \quad (\text{Note : } 115 = 64 + 51)$$

Keep repeating the process until the remainder is 1 or 0

$$\Rightarrow 243 = 2^7 + 2^6 + 32 + 19$$

$$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 19$$

$$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 16 + 3$$

$$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 2^4 + 3$$

$$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 2^4 + 2 + 1$$

$$\Rightarrow 243 = 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$\Rightarrow \text{The binary number is } 11110011_2$$

6) Convert 165 to a binary number

First Check all power of 2 that divide 165

$$2^0 = 2$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

Then, rewrite $165 = 128 + 37$

$$\Rightarrow 165 = 2^7 + 37$$

$$\Rightarrow 165 = 2^7 + 32 + 5$$

$$\Rightarrow 165 = 2^7 + 2^5 + 4 + 1$$

$$\Rightarrow 165 = 2^7 + 2^5 + 2^2 + 2^0$$

$$\Rightarrow 165 = 1 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$\Rightarrow 10100101_2$$

7) Convert 111_2 to a base ten number

$$111_2 \Rightarrow 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 4 + 2 + 1 = 7$$

8) Convert 11011_2 to a base 10 number

$$11011_2$$

$$\Rightarrow 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 16 + 8 + 0 + 2 + 1 = 27$$

9) Convert 245 into a base 5 number (**Optional**)

List powers of 5

$$5^1 = 5$$

$$5^2 = 25$$

$$5^3 = 125$$

$$5^4 = 625$$

$$245 = 125 + 120$$

$$\Rightarrow 245 = 5^3 + 120$$

$$\Rightarrow 245 = 5^3 + 4 \cdot 25 + 20$$

$$\Rightarrow 245 = 5^3 + 4 \cdot 5^2 + 4 \cdot 5$$

$$\Rightarrow 245 = 1 \cdot 5^3 + 4 \cdot 5^2 + 4 \cdot 5 + 0 \cdot 5^0$$

This converts to 1440_5

8) Convert 49 into a base 6 number (**Optional**)

List powers of 6

$$6^0 = 1$$

$$6^1 = 6$$

$$6^2 = 36$$

$$6^3 = 216$$

$$49 = 36 + 13$$

$$\Rightarrow 49 = 6^2 + 13$$

$$\Rightarrow 49 = 6^2 + 2 \cdot 6 + 1$$

$$\Rightarrow 49 = 6^2 + 2 \cdot 6^1 + 6^0$$

$$\Rightarrow 49 = 1 \cdot 6^2 + 2 \cdot 6^1 + 1 \cdot 6^0$$

This converts to 121_6