

Section 3.5

Write arguments in symbolic form and valid arguments

Writing an argument in symbolic form

I have a college degree (p)

I am lazy (q)

If I have a college degree, then I am not lazy

I don't have a college degree

Therefore, I am lazy

Symbolic form:

If I have a college degree, then I am not lazy ($p \rightarrow \sim q$)

I don't have a college degree ($\sim p$)

Therefore, I am lazy q

Hypothesis: $((p \rightarrow \sim q) \wedge \sim p)$

Conclusion: q

Argument in symbolic form: $((p \rightarrow \sim q) \wedge \sim p) \rightarrow q$

To test to see if the argument is valid, we take the argument in symbolic form and construct a truth table. If the last column in the truth table results in all true's, then the argument is valid

p	q	$\sim p$	$\sim q$	$(p \rightarrow \sim q)$	$((p \rightarrow \sim q) \wedge \sim p)$	$((p \rightarrow \sim q) \wedge \sim p) \rightarrow q$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

Therefore, this argument is invalid because the last column has a false item.

Example 1

Symbolize the argument, construct a truth table, and determine if the argument is valid.

If I pass the exam, then I will graduate.

I graduated

Therefore, I passed the exam

p = pass the exam

g = I will graduate

If I pass the exam, then I will graduate. ($p \rightarrow g$)

I graduated (g)

Therefore, I passed the exam (p)

Argument: $((p \rightarrow g) \wedge g) \rightarrow p$

P	g	$(p \rightarrow g)$	$((p \rightarrow g) \wedge g)$	$((p \rightarrow g) \wedge g) \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

This argument is invalid

Example 2

Symbolize the argument, construct a truth table, and determine if the argument is valid.

Jen and Bill will be at the party

Bill was at the party.

Therefore, Jen was at the party

J = Jen will be at the party

B = Bill will be at the party

Jen and Bill will be at the party $J \wedge B$

Bill was at the party. B

Therefore, Jen was at the party J

Argument in symbolic form: $((J \wedge B) \wedge B) \rightarrow J$

J	B	$J \wedge B$	$(J \wedge B) \wedge B$	$((J \wedge B) \wedge B) \rightarrow J$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

Since the last column is all true, the argument is valid

Example 3

Symbolize the argument, construct a truth table, and determine if the argument is valid.

It will be sunny or cloudy today

It isn't sunny

Therefore, it will be cloudy

S = It will be sunny

C = It will be cloudy

It will be sunny or cloudy today $S \vee C$

It isn't sunny $\sim S$

Therefore, it will be cloudy C

Hypothesis: $(S \vee C) \wedge \sim S$

Conclusion: C

S	C	$\sim S$	$S \vee C$	$(S \vee C) \wedge \sim S$	$((S \vee C) \wedge \sim S) \rightarrow C$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

This is a valid argument

Example 4

Write in symbolic form

p: The senator supports new taxes.

q: The senator is reelected

The senator is not reelected if she supports new taxes

The senator does not support new taxes

Therefore, the senator is reelected

Symbolic form:

The senator is not reelected if she supports new taxes $p \rightarrow \sim q$

The senator does not support new taxes $\sim p$

Therefore, the senator is reelected q

Hypothesis: $(p \rightarrow \sim q) \wedge \sim p$

Conclusion: q

Argument: $((p \rightarrow \sim q) \wedge \sim p) \rightarrow q$

Determine if the argument in problem 6 above is valid

p	q	$\sim q$	$\sim p$	$p \rightarrow \sim q$	$(p \rightarrow \sim q) \wedge \sim p$	$((p \rightarrow \sim q) \wedge \sim p) \rightarrow q$
T	T	F	F	F	F	T
T	F	T	F	T	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	F

Since the last row results in false, the argument is invalid

Example 5

If you complete your computer training, you can be eligible to work in the computer lab. You did not finish your computer training. Therefore, you can not work in the computer lab.

p : You finished your computer training

q : You can work in the computer lab

Argument in symbolic: $((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$((p \rightarrow q) \wedge \sim p)$	$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Invalid Arguments