Section 2.4

Applications of Sets

Definition:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Example 1

Given $n(A) = 340, n(B) = 240, \text{ } and \text{ } n(A \cap B) = 80$, find $n(A \cup B)$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 340 + 240 - 80 = 580 - 80 = 500$$

Example 2

Given $n(A) = 30, n(B) = 28, \text{ } and \text{ } n(A \cup B) = 50$, find $n(A \cap B)$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$50 = 30 + 28 - n(A \cap B)$$
$$50 = 58 - n(A \cap B)$$
$$-8 = -n(A \cap B)$$
$$n(A \cap B) = 8$$

Example 3

Given $n(A) = 88, n(B) = 65, \text{ } and \text{ } n(A \cup B) = 120$, find $n(A \cap B)$

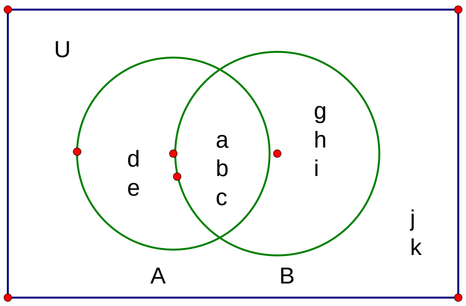
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$120 = 88 + 65 - n(A \cap B)$$
$$120 = 153 - n(A \cap B)$$
$$-33 = -n(A \cap B)$$
$$n(A \cap B) = 33$$

Cardinality

Definition: Cardinality is the number of elements in a given set

The number of elements in a set A is denoted by $n(A)$

$A = \{a,b,c,d,e\}, B = \{a,b,c,g,h,i\}, U = \{a,b,c,d,e,f,g,h,,i,j,k\}$



1) Find $n(A)$

$$n(A) = 5$$

2) Find $n(B)$

$$n(B) = 6$$

3) Find $n(A \cup B)$

$$n(A \cup B) = 8$$

4) Find $n(A \cap B)$

$$n(A \cap B) = 3$$

Rules for the cardinality for the union of two sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Use this formula to find $n(A \cup B)$ in problem 3.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 5 + 6 - 3 = 11 - 3 = 8$$

Example 5

Let
$U = \{x \mid x \ is \ a \ state \ in \ the \ United \ States\}$
$A = \{x \mid x \in U \ and \ x \ begins \ with \ A\}$
$I = \{x \mid x \in U \ and \ x \ begins \ with \ I\}$
$M = \{x \mid x \in U \ and \ x \ begins \ with \ M\}$
$N = \{x \mid x \in U \ and \ x \ begins \ with \ N\}$
$O = \{x \mid x \in U \ and \ x \ begins \ with \ O\}$

$A = \{Alabama, \ Arkansas, \ Alaska, \ Arizona\}$
$I = \{Iowa, \ Indiana, \ Illinois, \ Idaho\}$
$M = \{Michigan, \ Minnesota, \ Mississippi, \ Missouri, \ Maryland, \ Maine, \ Mon\tan a, \ Massachusetts\}$
$N = \left\{ \begin{array}{l} Nebraska, \ New Jersey, \ New \ Mexico, \ New York, \ New \ Hamphere, \ North \ Carolina, \\ North \ Dakota, \ Nevada \end{array} \right\}$
$O = \{Ohio, \ Okla\hom a, Oregon\}$

33) $Find \ n(M') = 50 - 8 = 42$
34) $Find \ n(A \cup N) = 13$
35) $Find \ n(I' \cap O') = 50 - (3+4) = 50 - 7 = 43$
36) $Find \ n(M \cap I) = 0$

Example 4

Let
$U = \{English, \ Math, \ History, Drama, \ Physics, \ Spanish, \ Philosohpy, \ Chemistry, \ Latin, \ French\}$
$A = \{English, \ History, \ Chemistry, \ Spanish\}$
$B = \{History, \ Math, \ Chemistry, \ French\}$
$C = \{Physics, \ English, \ French, \ Math\}$

1) Find $A \cup B$

$A \cup B = \{English, \ History, \ Chemistry, \ Spanish, \ Math, \ French\}$

2) Find $A \cap B$

$A \bigcap B = \{English, \ Chemistry\}$

3) Find $n(A \cup B)$

$n(A \cup B) = 6$

4) Find $n(A \cap B)$

$n(A \cap B) = 2$

5) Find $n(A) + n(B)$

$n(A) + n(B) = 4 + 4 - 2 = 6$

6) Find $n(A) + n(B) + n(C)$

$n(A) + n(B) + n(C) = 4 + 4 + 4 - 2 - 2 = 8$

Section 2.5

Infinite Sets

Infinite sets and Cardinality

Equivalent Sets

Two sets are equivalent if they have the same number of elements.

Examples of equivalent sets

$$\{1,2,3,4\}$$
and
$$\{a,b,c,d\}$$

$$\{john, luke, mark, mathew\}$$
and
$$\{a,b,c,d\}$$

Cardinality

Definition: Cardinality is the number of elements in a given set

One-to-one correspondence

Definition: Two sets are in one-to-one correspondence if each element in the first is paired with exactly one element in the second set, and each element of the second set is paired with exactly one element form the first set

Examples

1) The sets $\{1,2,3,4\}$ *and* $\{a,b,c,d\}$ are in one-to-one correspondence as shown in this diagram.

$$\begin{array}{cccc} \{1, & 2, & 3, & 4\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{a, & b, & c, & d\} \end{array}$$

2) The sets $\{Viriginia,\ Maryland, North\ Carolina\}$ and $\{Richmond, Annapolis, Raliegh\}$ are in one-to-one correspondence as shown in this diagram.

$$\begin{array}{ccc} \{Viriginia, & Maryland, & North\ Carolina\} \\ \updownarrow & \updownarrow & \updownarrow \\ \{Richmond, & Annapolis, & Raliegh\} \end{array}$$

Cantor's definition of set

A set is **infinite** if we can remove some of its elements without reducing its size.

Countable sets

A set is countable if you establish a one-to-one correspondence form the given set to the natural numbers.

Examples

1) Are the even natural numbers countable?

$$\begin{array}{l} \{2,4,6,8,10,12,.............2n\} \\ \updownarrow\updownarrow\updownarrow\updownarrow\ \updownarrow\ \updownarrow \qquad\qquad\quad \updownarrow \\ \{1,2,3,4,5,\ 6,.................n\} \end{array}$$

The even natural can be put in a one-to-one correspondence with the natural numbers by using the mapping $n \leftrightarrow 2n$

2) Are the integers countable?

$$J = \{.....,-4,-3,-2,-1,0,1,2,3,4,...........\}$$

The mapping would go as follows:

$$0 \leftrightarrow 1$$
$$1 \leftrightarrow 2$$
$$-1 \leftrightarrow 3$$
$$2 \leftrightarrow 4$$
$$-2 \leftrightarrow 5$$
$$3 \leftrightarrow 6$$
$$-3 \leftrightarrow 7$$
etc.

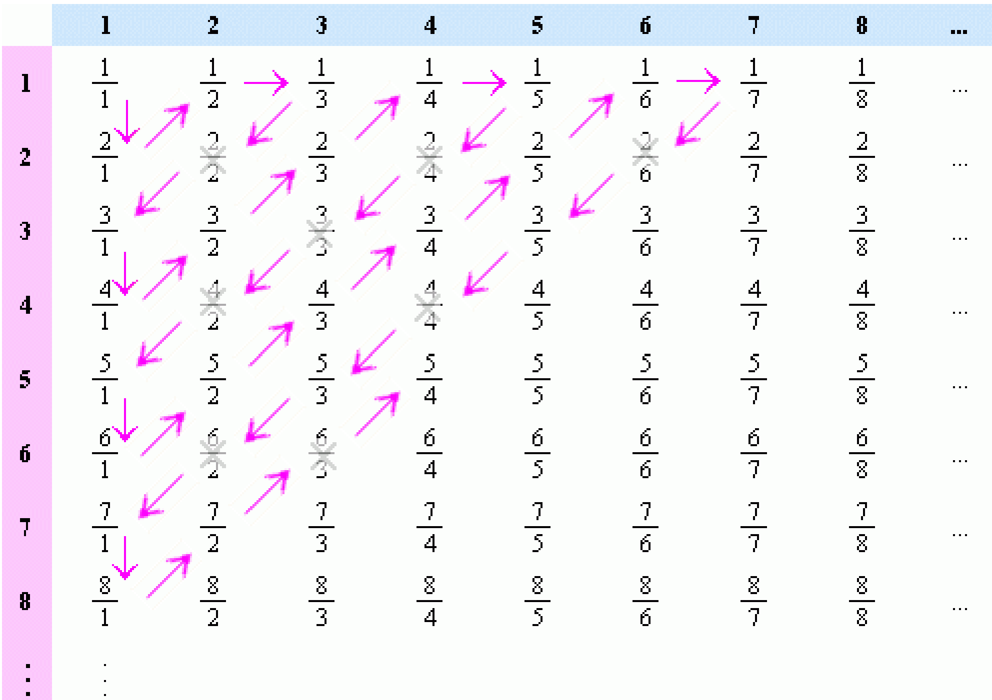
Use this mapping

$$n \leftrightarrow \frac{n}{2} \; if \;\; n \; is \; even$$
$$n \leftrightarrow \frac{1-n}{2} \; if \;\; n \; is \; odd$$

Therefore, there exist a one-to-one correspondence between the integers and the natural numbers. Thus, the integers are countable.

3) Are the rational numbers countable?

Look at the following diagram



http://www.homeschoolmath.net/other_topics/rational-numbers-countable.php

This allow the following ordering of numbers

$1 \to 1$

$2 \to 2$

$\frac{1}{2} \to 3$

$\frac{1}{3} \to 4$

$3 \to 5$

......

This shows that each element of the rational number can be paired with one element of the natural numbers. Thus, it is possible to establish a one-to-one correspondence with the natural numbers. This provides an interesting result which is that the rational numbers turn out to countable.