

Section 2.2

Subsets and Improper Subsets

Key Terms

The **empty set** is a set that contains no elements. The empty set is also referred to as the **null set**.

Subsets

A set B is a subset of set C, if every element in B is an element of C. $B \subset C$

Proper Subsets

A set B is a proper subset of C, if every element of B is an element of C and there is at least one element of B that is not in C. $B \subset C$

Example 1

$$A = \{1,2,3,4,5\}$$

$$C = \{1,2,3,4,5,6,7\}$$

Is $A \subset C$?

Solution: Since every element in the set A is an element of C, A is a subset of C.

Example 2

Is $\{4,5,6\}$ a subset of $\{0,1,2,3,4,5\}$?

Solution: no, since the element 6 is not in the set $\{0,1,2,3,4,5\}$

Example 3

List all possible subsets of $\{a, m\}$

Solution: $\phi, \{a\}, \{m\}, \{a, m\}$

Example 4

List all subsets of the set {2,3,4}

Possible subsets

Solution: $\phi, \{2\}, \{3\}, \{4\}, \{2,3\}, \{3,4\}, \{2,4\}, \{2,3,4\}$

Example 5

List all subsets of the set {6}

Possible sets: $\phi, \{6\}$

The pattern for subsets

| Number of elements | Number of subsets |
|--------------------|-------------------|
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

Formula to find the number of subsets s of a given set A with n elements

$$s = 2^n$$

Example 6

How many subsets does a set A with 10 elements have?

$$s = 2^n$$

$$s = 2^{10}$$

$$s = 1024$$

The **universal set** is the set of all possible elements of set used in the problem. Denoted by U

The complement of a set A

The complement of a set A is the set of all elements in the universal that are not elements of the set A .

$$A' = \{x \mid x \notin A \text{ and } x \in U\}$$

Example 7

Find the compliment of each set. The that the universal set is $U = \{0,1,2,3,4,5,6,7,8,9,10\}$

1) $A = \{2,3,4,5\}$

$$A' = \{0,1,6,7,8,9,10\}$$

2) The odd natural numbers less than 10: $\{1,3,5,7,9\}$

$$\text{Compliment} = \{0,2,4,6,8\}$$

3) $\{1,4,7,8,9,10\}$

$$\text{Compliment} = \{0,2,3,5,6\}$$
