Math 151

Section 1.7

Continuity and One Sided Limits

Definition: A function f is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a)$$

Notice that the definition implicitly requires three things if f is continuous at a number a:

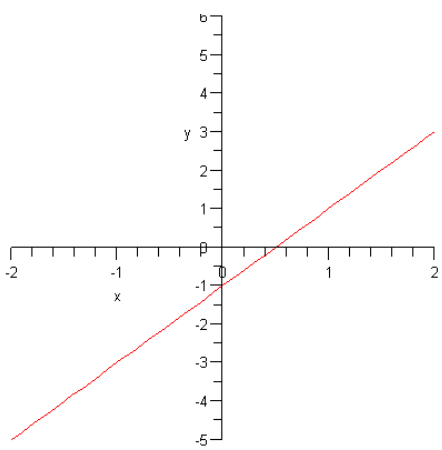
1) $f(a)$ *is defined*

2) $\lim_{x \to a} f(x)$ *exist*

3) $\lim_{x \to a} f(x) = f(a)$

Example 1

Discuss the continuity of $f(x)$ at $x = 1$



1) $f(1) = 1$, *so is defined at* $x = 1$

2) $\lim_{x \to 1} f(x) = 1$, so the limit exist at $x = 1$

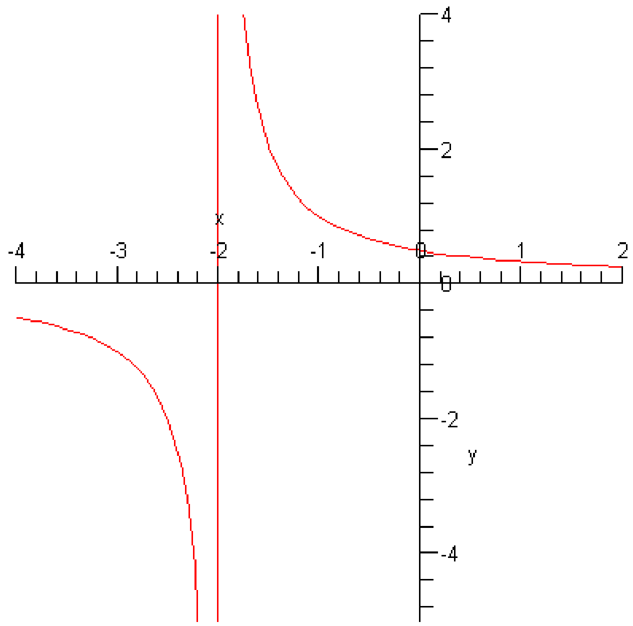
3) $\lim_{x \to 1} f(x) = f(1) = 1$

Therefore, the function is continuous at $x = 1$.

Example 2

Use the graph to determine the limit, and discuss the continuity of the function.

$a)\ \lim_{x\to -2^{+}} f(x) \quad b)\ \lim_{x\to -2^{-}} f(x) \quad c)\ \lim_{x\to -2} f(x)$



$a)\ \lim_{x\to -2^{+}} f(x)$ *undefined*

$b)\ \lim_{x\to -2^{-}} f(x)$ *undefined*

$c)\ \lim_{x\to -2} f(x)$ *undefined*

Is the function continuous?

1) $f(-2)$ *is undefined*

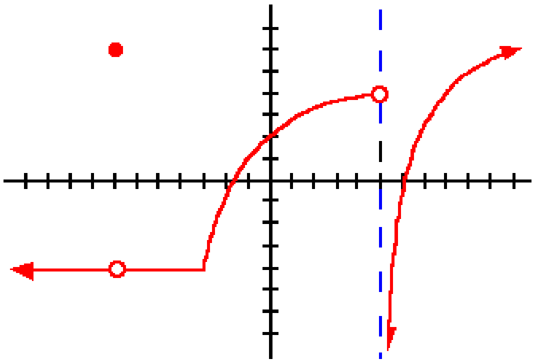
2) $\lim_{x\to -2} f(x)$ *does not exist*

3) $\lim_{x\to -2} f(x) \neq f(1)$

Therefore, the function is not continuous at $x = -2$.

Example 3

Discuss the continuity of $f(x)$ at $x=-6$ and $x=5$



Look $x=-6$

1) $f(-6)=6$ *is defined*

2) $\lim_{x\to -6} f(x)$ *exist*

3) $\lim_{x\to --6} f(x) \neq f(-6)$

Therefore, the function is not continuous at $x=-6$.

Look $x=5$

1) $f(5)$ *is undefined*

2) $\lim_{x\to 5} f(x)$ *exist*

3) $\lim_{x\to 5} f(x) \neq f(5)$

Therefore, the function is not continuous at $x=5$.

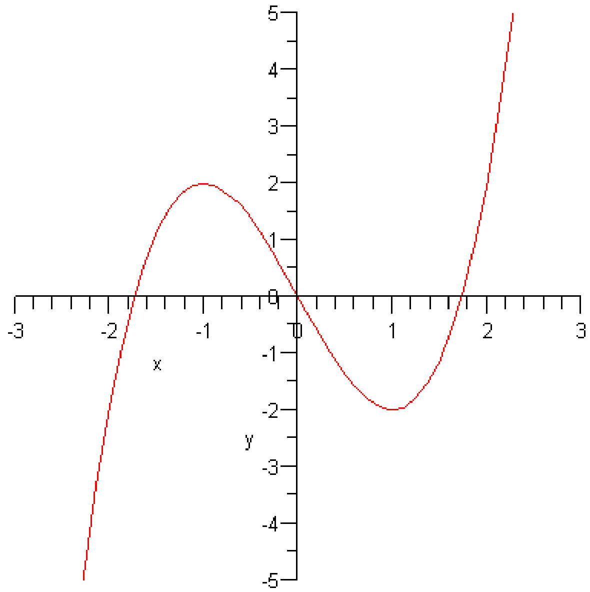
The graph of a continuous function is piecewise smooth.

Continuous Intervals

A function is continuous on an interval if the function is continuous at every point of the interval.

Example 4

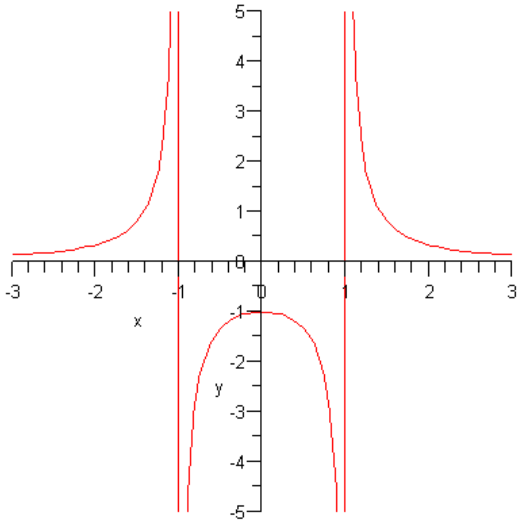
Discuss the continuity of the given function.



The graph is piece wise smooth, since it has breaks, holes, or asymptotes. Therefore, the function is continuous on $(-\infty, \infty)$

Example 5

Discuss the continuity of the function.



The function is discontinuous at $x = -1$ and $x = -1$
The function is continuous on $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Theorem: Polynomial functions are continuous everywhere.

Theorem: Rational functions are continuous on their domain

Example 6

Find the values of x where the function is discontinuous.

$f(x) = x^3 + 2x^2$

The function has no points of discontinuity.

Function is continuous on $(-\infty, \infty)$

Example 7

Give the intervals where the function is continuous.

$$f(x) = \frac{1}{x^2 - 4}$$

$$f(x) = \frac{1}{x^2 - 4} = \frac{1}{(x-2)(x+2)} \Rightarrow f \textit{ is undefined at } x = -2 \textit{ and } x = 2$$

The function is discontinuous at $x = -2$ and $x = 2$

Continuous on $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Example 8

Give the intervals where the function is continuous.

$$f(x) = \frac{x-3}{x^2 + x - 12}$$

$$f(x) = \frac{x-3}{x^2 + x - 12} = \frac{x-3}{(x-3)(x+4)} \Rightarrow f \textit{ is undefined at } x = 3 \textit{ and } x = -4$$

The function is discontinuous at $x = -4$ and $x = 3$

Continuous on $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

Example 9

Give the intervals where the function is continuous.

$$f(x) = x - \cos x$$

The function has no points of discontinuity

Continuous on $(-\infty, \infty)$

Example 10: Give the intervals where the function is continuous.

$f(x) = x + e^{x+3}$

The function has no points of discontinuity

Solution: Continuous on $(-\infty, \infty)$

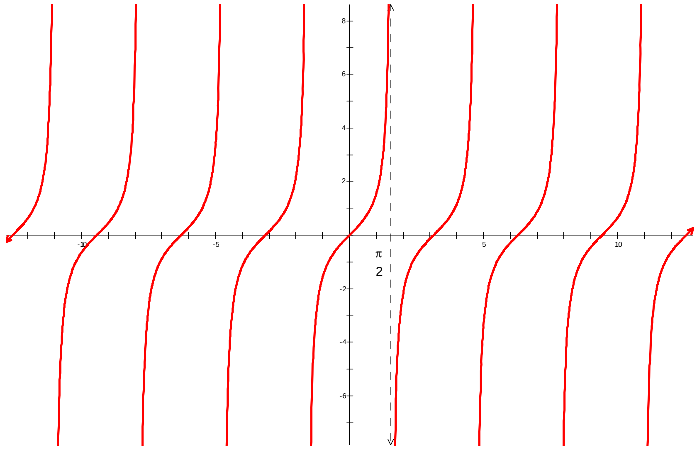
Example 11

Find the limit, if it exists. If the limit doesn't exist then explain why?

a) $\lim_{x \to \frac{\pi}{2}} \tan x$

$\lim_{x \to \frac{\pi}{2}} \tan x \ \ Does\ not\ exist$

At the value $\frac{\pi}{2}$, the graph of tan x has an asymptote.



b) $\lim_{x \to 3^-} \ln(x-3)$

$\lim_{x \to 3^-} \ln(x-3)$ does not exist. The function $x-3$ is negative as x approaches 3 from the left. The domain of the natural log function is restricted to positive real numbers. Therefore, the limit of the function does not exist.