Section 1.5 (Limits)

Definition of a Limit

$\lim_{x \to a} f(x) = L$

The limit of $f(x)$, as x approaches a, equals L

We can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a (on either side of a) but not equal to a.

Evaluating limits using the graph of a function

Approximating Limits

$\lim_{x \to 1} x^2 + 1$

x	.9	.99	.999	1.1	1.01	1.001
$f(x)$	$f(.9)$ $= .9^2 + 1$ $= 1.81$	$f(.9)$ $= .99^2 + 1$ $= 1.98$	$f(.9)$ $= .999^2 + 1$ $= 1.998$	$f(.9)$ $= 1.1^2 + 1$ $= 2.21$	$f(1.01)$ $= 1.01^2 + 1$ $= 2.02$	$f(1.001)$ $= 1.001^2 + 1$ $= 2.002$

The limit as x approaches from the right or "Right hand limit"

$\lim_{x \to 1^+} x^2 + 1 = 2$

The limit as x approaches from the left or "Left hand limit"

$\lim_{x \to 1^-} x^2 + 1 = 2$

The regular limit

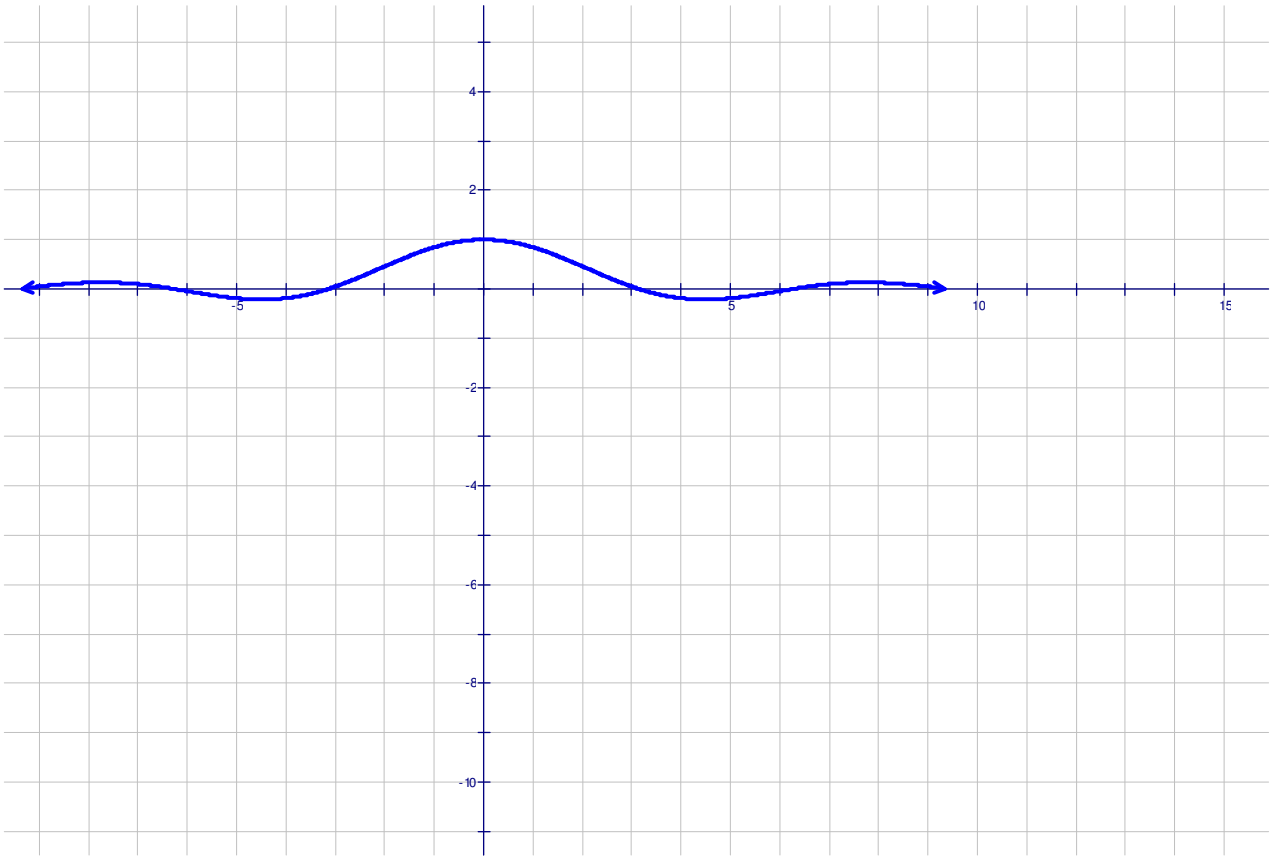
$\lim_{x \to 1} x^2 + 1 = 2$

Example 1

$$\lim_{x\to 0}\frac{\sin x}{x}$$

x	$-.1$	$-.01$	$-.001$	1.1	1.01	1.001
$f(x)$	$f(-.1)$ $=\frac{\sin(-.1)}{-.1}$ $=.988$	$f(-.01)$ $=\frac{\sin(-.01)}{-.01}$ $=.999$	$f(-.001)$ $=\frac{\sin(-.001)}{-.001}$ $=.9999$	$f(0.1)$ $=\frac{\sin(0.1)}{0.1}$ $=.988$	$f(0.01)$ $=\frac{\sin(0.01)}{0.01}$ $=.999$	$f(0.001)$ $=\frac{\sin(0.001)}{0.001}$ $=.9999$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

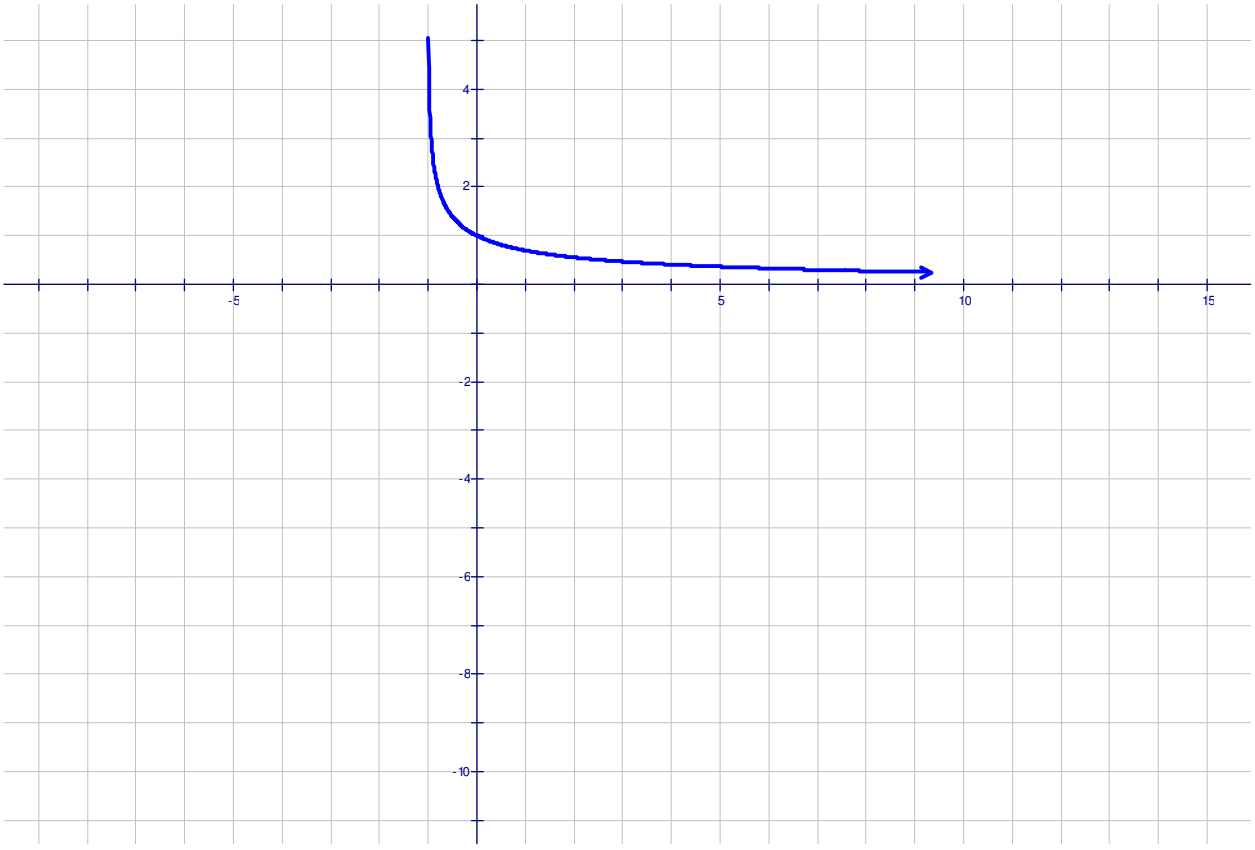


Example 2

$$\lim_{x\to 0}\frac{\ln(x+1)}{x}$$

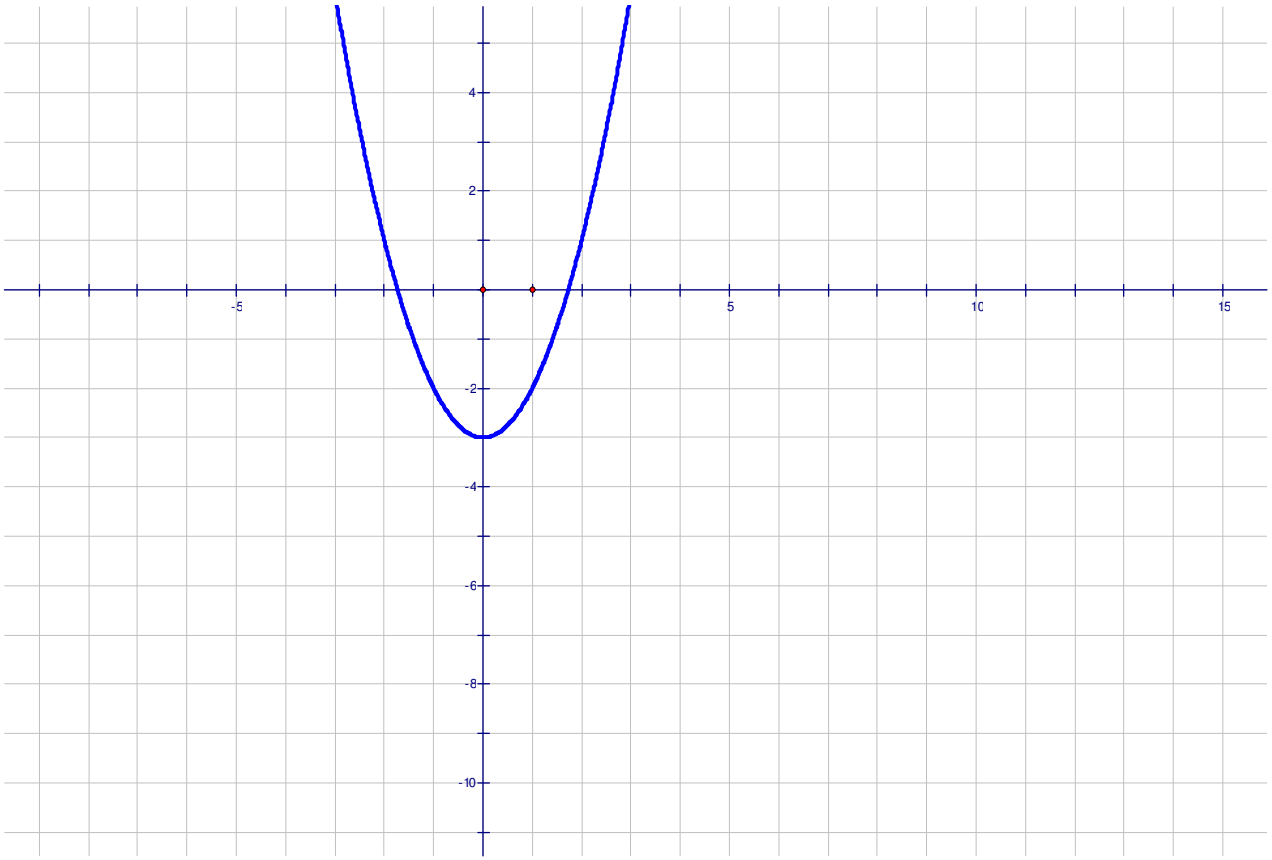
x	.1	.01	.001	$-.1$	$-.01$	$-.001$
$f(x)$	$f(.1)=\frac{\ln(.1+1)}{.1}=.953$	$f(.01)=\frac{\ln(.01+1)}{.01}=.995$	$f(.001)=\frac{\ln(.001+1)}{.001}=.9995$	$f(-.1)=\frac{\ln(-.1+1)}{-.1}=1.05$	$f(-.01)=\frac{\ln(-.01+1)}{-.01}=1.005$	$f(-.001)=\frac{\ln(-.001+1)}{-.001}=1.0005$

$$\lim_{x\to 0}\frac{\ln(x+1)}{x}=1$$



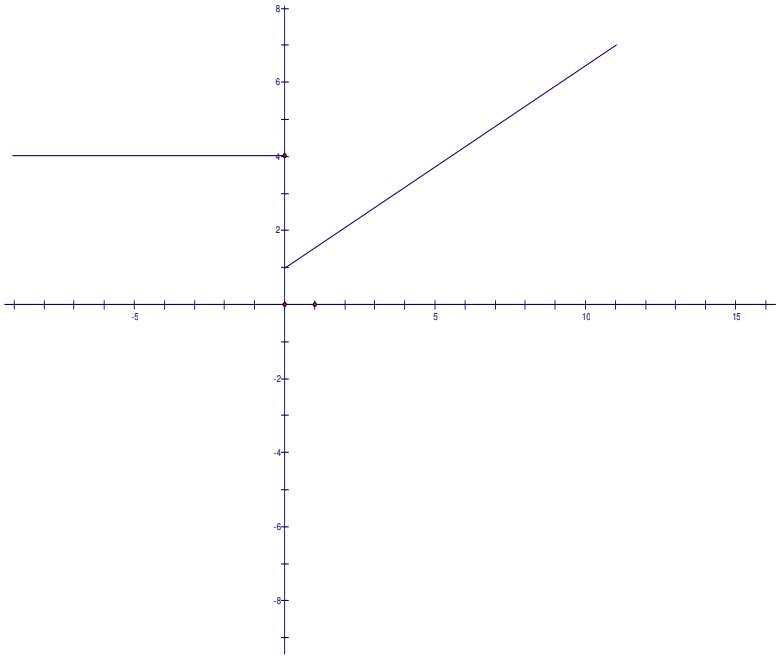
Example 3

$$\lim_{x \to 1} x^2 - 3$$



$$\lim_{x \to 1} x^2 - 3 = 2$$

Example 4



a) Find $\lim_{x\to 2} f(x)$

Solution: $\lim_{x\to 2} f(x) = 2$

b) Find $\lim_{x\to -3} f(x)$

Solution: $\lim_{x\to -3} f(x) = 4$

c) Find $\lim_{x\to 0^+} f(x)$

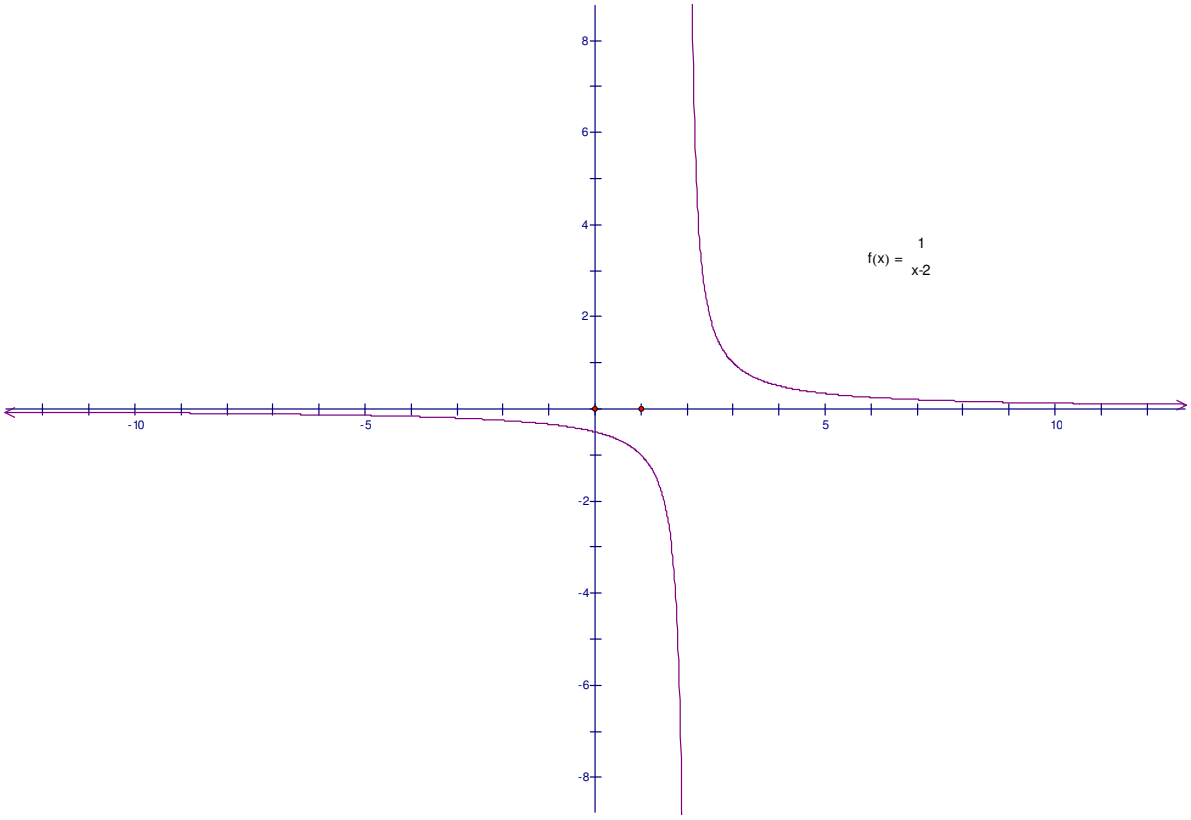
Solution $\lim_{x\to 0^+} f(x) = 1$ **Left hand limit (The y value x approaches from the left)**

d) Find $\lim_{x\to 0^-} f(x)$

Solution: $\lim_{x\to 0^-} f(x) = 4$ **Right hand limit (The y value x approaches from the right)**

e) Find $\lim_{x\to 0} f(x)$: *Does Not Exist* **The limit doesn't exist since the right hand and the left hand limits are different.**

Example 5



a) Find $\lim_{x \to 2} f(x)$:

Solution: No Limit (Limit does not exist)

b) Find $\lim_{x \to 0} f(x)$:

Solution: $\lim_{x \to 0} f(x) = \frac{1}{2}$

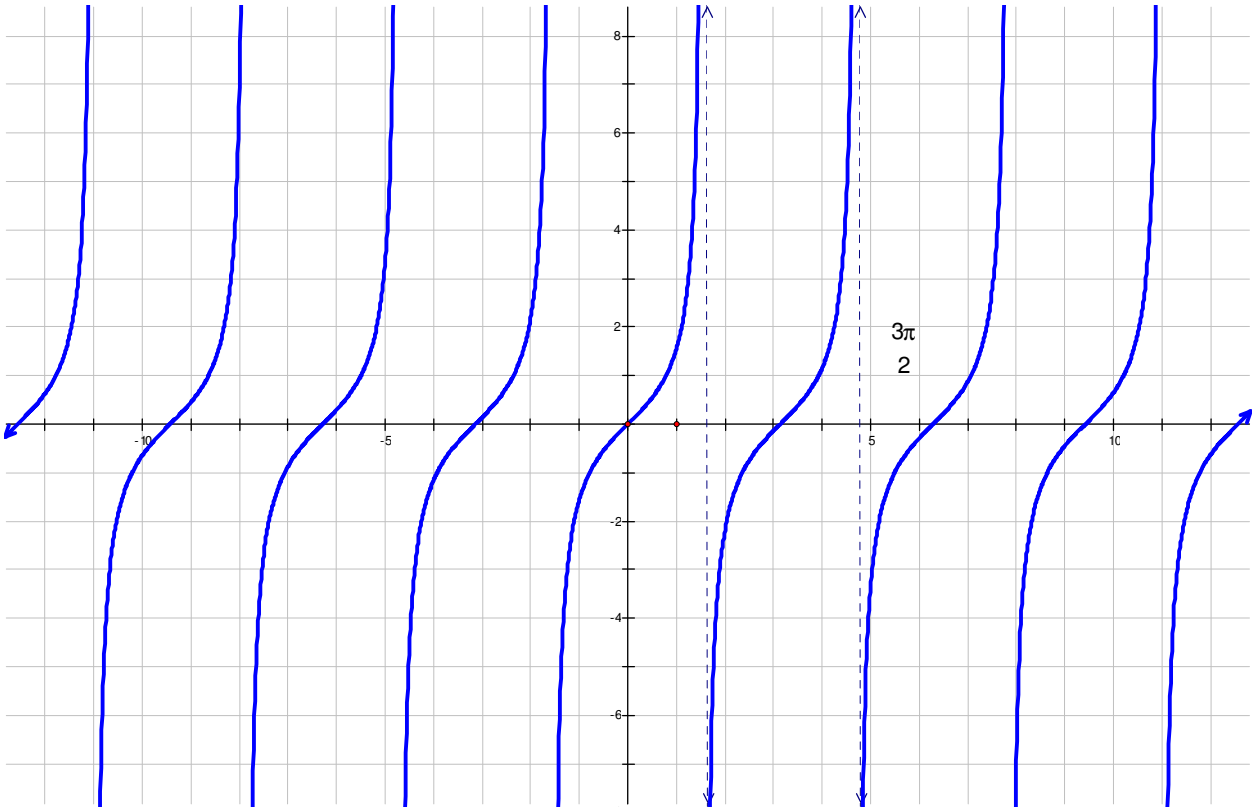
c) Find $\lim_{x \to 3} f(x)$

Solution: $\lim_{x \to 3} f(x) = 1$

Example 6

Find the limit using the graph of the function

$$\lim_{x \to \frac{3\pi}{2}} \tan x$$



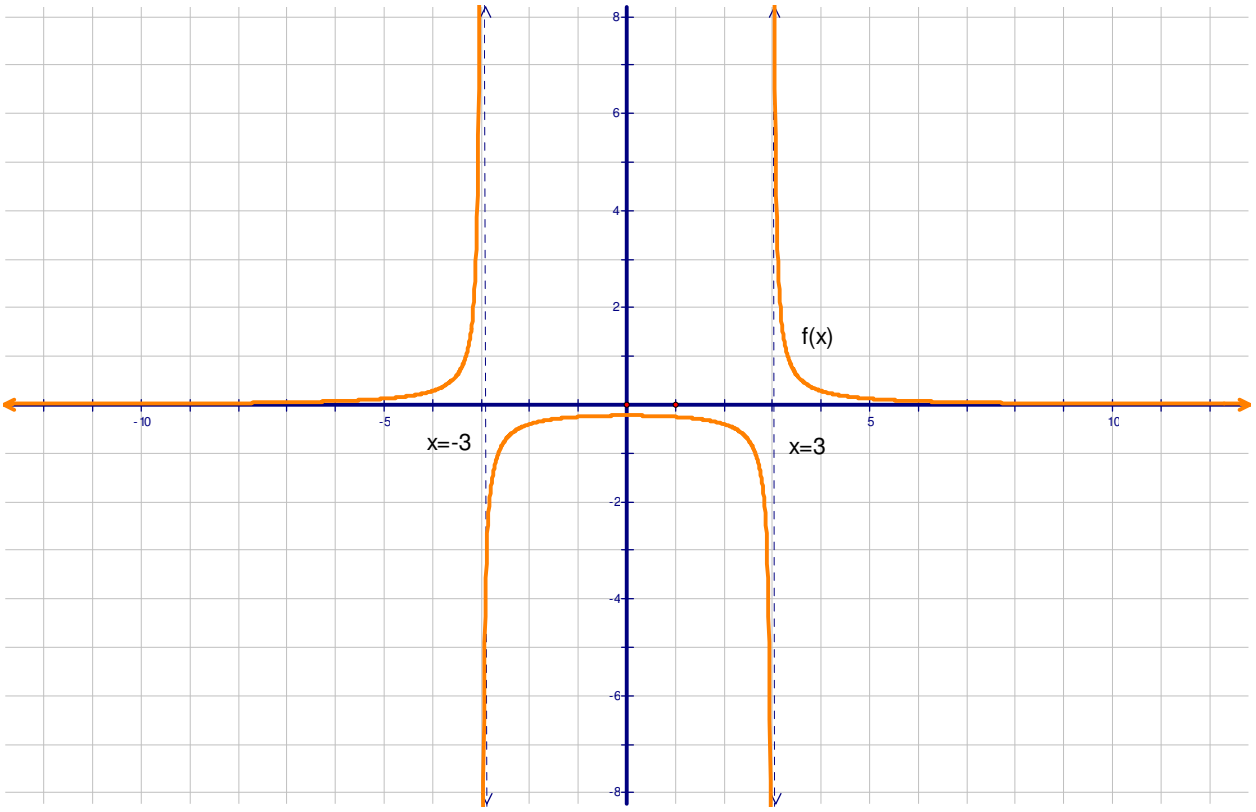
$$\lim_{x \to \frac{3\pi}{2}} \tan x$$ (Does not exist)

Example 7

Find each limit.

a) $\lim_{x \to -3} f(x) =$ _____

b) $\lim_{x \to 3} f(x) =$ _____



Solution:

$\lim_{x \to -3} f(x)$ *Does not exist*

$\lim_{x \to 3} f(x)$ *Does not exist*

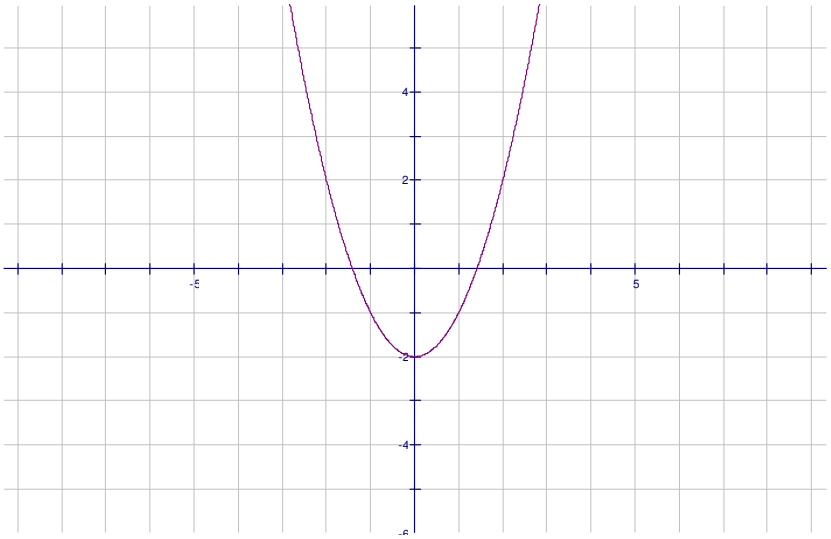
Example 7

Using the graph below evaluate each limit.

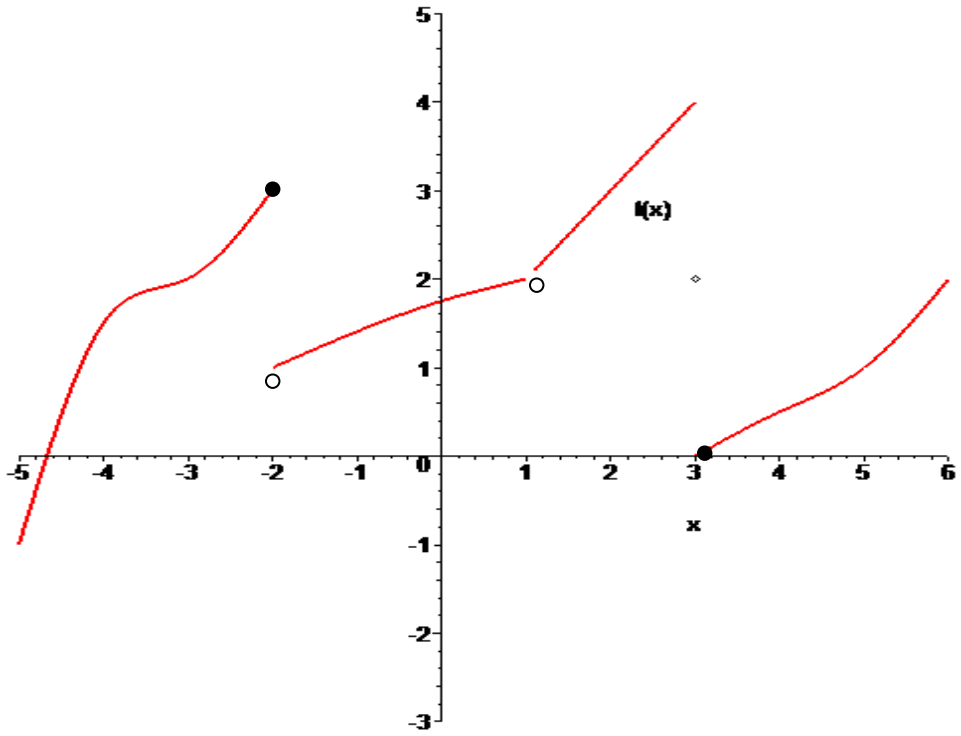
a) Find $\lim_{x\to 2} f(x)$: **Solution** $\lim_{x\to 2} f(x) = 2$

b) Find $\lim_{x\to -0} f(x)$: **Solution** $\lim_{x\to 0} f(x) = -2$

c) Find $\lim_{x\to -2} f(x)$: **Solution** $\lim_{x\to -2} f(x) = 2$



Example 8



a) Find $\lim_{x\to -2^-} f(x)$: **Solution** $\lim_{x\to -2^-} f(x) = 3$

b) Find $\lim_{x\to -2^+} f(x)$: **Solution** $\lim_{x\to -2^-} f(x) = 1$

c) Find $\lim_{x\to -2} f(x)$: **Solution** $\lim_{x\to -2} f(x)$ *DNE*

d) Find $\lim_{x\to 1^-} f(x)$: **Solution** $\lim_{x\to -1^-} f(x) = 2$

e) Find $\lim_{x\to 1^+} f(x)$: **Solution** $\lim_{x\to -1^+} f(x) = 2$

f) Find $\lim_{x\to 3^-} f(x)$: **Solution** $\lim_{x\to 3^+} f(x) = 4$

g) Find $\lim_{x\to 3^+} f(x)$: **Solution** $\lim_{x\to 3^+} f(x) = 0$

h) Find $\lim_{x\to 3} f(x)$: **Solution** $\lim_{x\to 3^+} f(x)$ *DNE*

Example 9

Using the graph below evaluate each limit.

a) Find $\lim_{x\to 5} f(x)$: **Solution** $\lim_{x\to 5} f(x) = \textit{Does Not Exist}$

b) Find $\lim_{x\to -7} f(x)$: **Solution** $\lim_{x\to -7} f(x) = -4$

c) Find $\lim_{x\to 0} f(x)$: **Solution** $\lim_{x\to 0} f(x) = 2$