Section 1.5

Exponential Functions

General Form of an exponential function $f(x) = ca^n$

Where $a^n = \underbrace{a \cdot a \cdot a \cdot a \cdot a \cdot a ....a}_{n-times}$

Example $6^4 = 6 \cdot 6 \cdot 6 \cdot 6 = 1296$

Laws of Exponents

1) $a^x a^y = a^{x+y}$

2) $\frac{a^x}{a^y} = a^{x-y}$

3) $(a^x)^y = a^{xy}$

4) $(ab)^x a^x a^y$

5) $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

6) $a^0 = 1$

Example 1

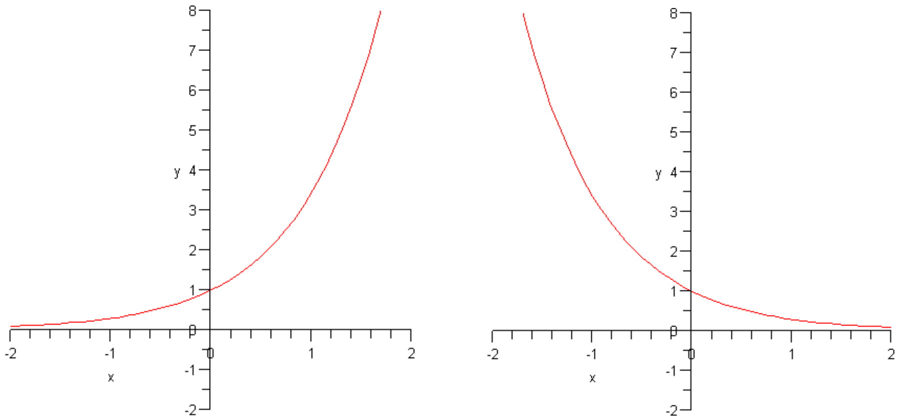
a) Simplify $(2x^4)(5x^8)$
Solution: $(2x^4)(5x^8) = 10x^{4+8} = 10x^{12}$

b) Simplify $(5x^3)^2$
Solution: $(5x^3)^2 = 5^2(x^3)^2 = 25x^6$

c) Simplify $16^{\frac{3}{2}}$
Solution: $16^{\frac{3}{2}} = (\sqrt{16})^3 = 4^3 = 64$

Graphs of exponential functions

Exponent curves are sometimes referred to having a “hockey stick shape”
Here are some examples of exponential functions.

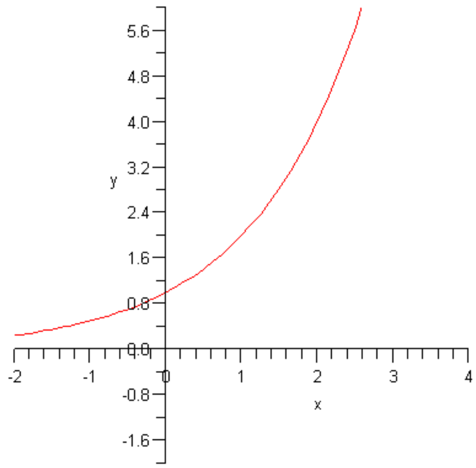


Example 2

Graph the following function $f(x) = 2^x$

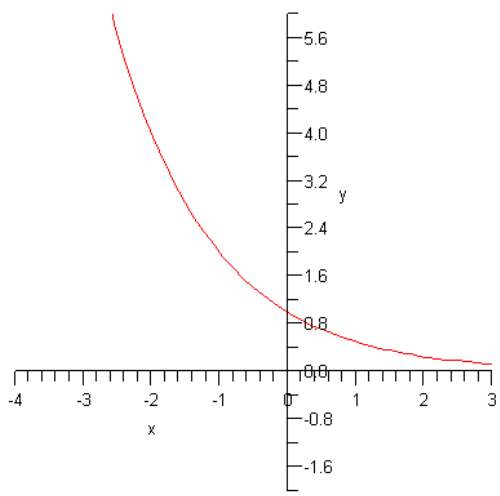
To the graph the function we will use the strategy of choosing five x values and making a table of values.

x	$f(x)$
-2	$f(-2) = (2)^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$f(-1) = (2)^{-1} = \frac{1}{2^1} = \frac{1}{2}$
0	$f(0) = (2)^0 = 1$
1	$f(1) = (2)^1 = 2$
2	$f(2) = 2^2 = 4$



Example: Graph the following function $f(x) = 2^{-x}$

x	$f(x)$
-2	$f(-2) = 2^{-(-2)} = 2^2 = 4$
-1	$f(-1) = (2)^{-(-1)} = 2^1 = 2$
0	$f(0) = (2)^0 = 1$
1	$f(1) = (2)^{-1} = \frac{1}{2^1} = \frac{1}{2}$
2	$f(2) = (2)^{-2} = \frac{1}{2^2} = \frac{1}{4}$



The Exponential Function

The Euler Number: $e \approx 2.718$

Example 3

a) Find e^2
Solution: $e^2 = 7.39$

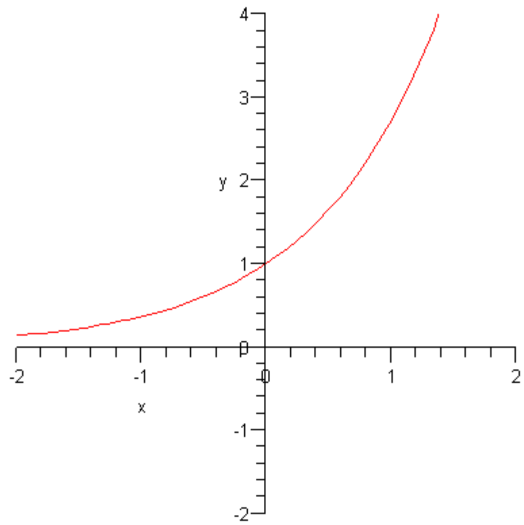
b) Find e^{-3}
Solution: $e^{-3} = .050$

Graphs of the exponential function

Example 4

Graph $f(x) = e^x$

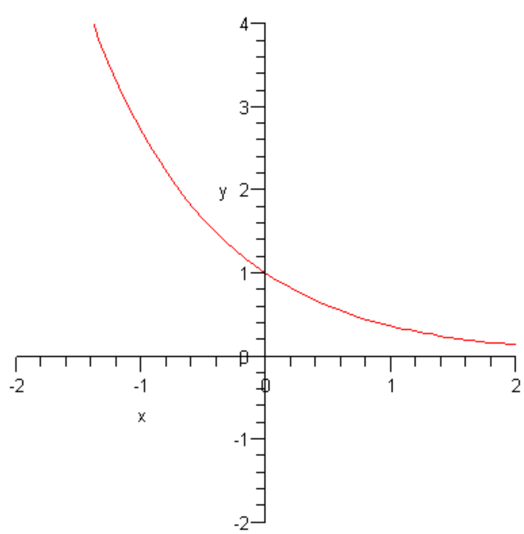
x	$f(x)$
-2	$f(-2) = e^{-2} = \frac{1}{e^2} = .14$
-1	$f(-1) = e^{-1} = \frac{1}{e^1} = .37$
0	$f(0) = e^0 = 1$
1	$f(1) = e^1 = 2.7$
2	$f(2) = e^2 = 7.4$



Example 5

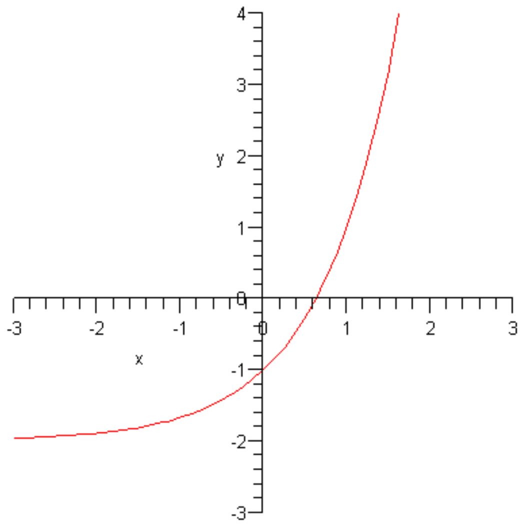
Graph $f(x) = e^{-x}$

x	$f(x)$
-2	$f(2) = e^{-(-2)} = e^{2} = 7.4$
-1	$f(1) = e^{-(-1)} = e^{1} = 2.7$
0	$f(0) = e^{0} = 1$
1	$f(1) = e^{-1} = \frac{1}{e^{1}} = .37$
2	$f(2) = e^{-2} = \frac{1}{e^{2}} = .14$



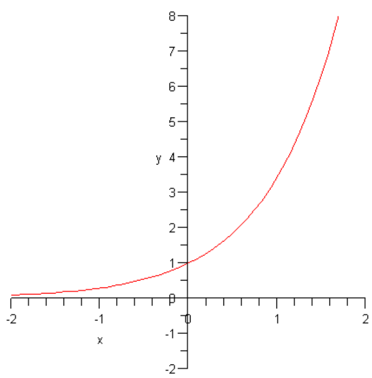
Example 5

Graph $f(x) = 3^x - 2$

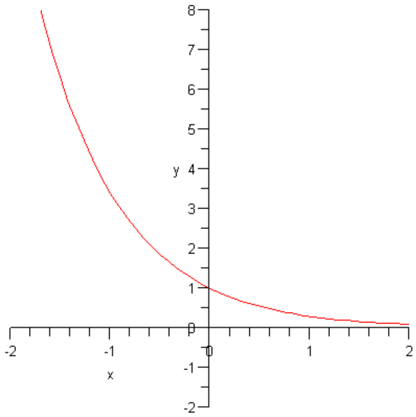


Exponential Modeling

Exponential Growth



Exponential Decay



Example 6

If a bacteria population starts with 200 bacteria and double every three hours, then the number of bacteria after t hours is $B(t) = 200(2)^{\frac{t}{3}}$

a) Find the population of the bacteria after 15 hours?

$$B(15) = 200(2)^{\frac{15}{3}} = 200(2^5) = 200(32) = 6400$$

b) Find the population of the bacteria after 25 hours?

$$B(15) = 200(2)^{\frac{25}{3}} = 200(322.54) \approx 64508$$

Example 7

The town of Blacksburg's population can be modeled by the equation $P = 40,000(1.015)^t$ over the next ten years assuming population grows at a steady rate.
Use the model to predict the population of Blacksburg in 10 years?

$$P = 40{,}000e^{.015t} = 40{,}000e^{.015(10)} = 40{,}000e^{.15} = 46473$$
