Section 1.1

Introduction to Functions

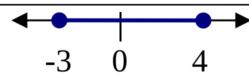
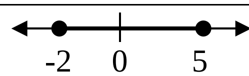
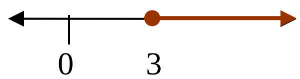
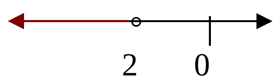
Introduction to Interval Notation

∞ - Positive Infinity

$-\infty$ - Negative Infinity

( ) –open interval

[ ] – closed interval

Inequality	Graph	Interval
$-3 \leq x \leq 4$		[-3,4]
$-2 < x < 5$		[-2,5]
$x \geq 3$		$[3, \infty)$
$x < 2$		$(-\infty, 2)$

Example 1

Write each inequality in interval notation

a) $-2 \leq x \leq 3$

Solution: [-2,3]

b) $x < -2$

Solution: $(-\infty, -2)$

Key Terms

A **relation** is a correspondence between two variables usually written in the form of an ordered pair.

Domain: The set of all the first coordinates in a relation.

Range: The set of all the second coordinates in a relation.

A **function** f is a rule that takes each value in the domain and assigns it to exactly one value in the range.

The most common method to visualize a function is its graph. If f is a function with a domain of A, then its graph is the set of ordered pairs.

$\{(x, f(x)), x \in A\}$

Example 1

Find the domain of the given function.

a) $f(x) = \dfrac{3}{x-2}$

The domain in set notation can be written as $\{x \mid x \neq 2\}$ or interval notation as $(-\infty, 2) \cup (2, \infty)$

b) $f(x) = \dfrac{1}{x^2 - 4}$

Notice that the denominator will factor as: $x^2 - 4 = (x-2)(x+2)$

Thus, the domain will be $\{x \mid x \neq -2 \ \ \textit{and} \ \ x \neq 2\}$ or $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

c) $f(x) = \sqrt{x-3}$

Notice that any value less than -3 will give a negative under the radical. Thus, only values of x greater than or equal to -3 will work in the function.
Therefore, the domain is $\{x \mid x \geq -3\}$ or $(-3, \infty)$

Example 2

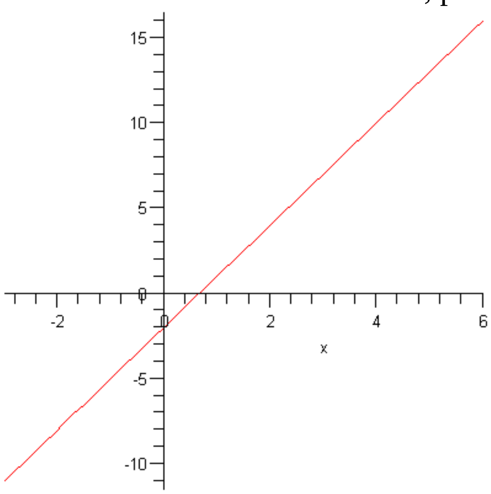
Sketch the graph and find the domain and range of the following function.

$f(x) = 3x - 2$

To make a sketch of the function, just find five points on the function and make a point plot. This can be done by choosing five values for x and substitution these value into the function to get the y value. For convenience, we will put the values in a table.

x	$f(x)$
-2	$f(-2) = 3(-2) - 2 = -8$
-1	$f(-1) = 3(-1) - 2 = -5$
0	$f(0) = 3(0) - 2 = -2$
1	$f(1) = 3(1) - 2 = 1$
2	$f(2) = 3(2) - 2 = 4$

To make a sketch of the function, plot the values in the table on the coordinate plane.



Domain: $(-\infty, \infty)$: Range $(-\infty, \infty)$

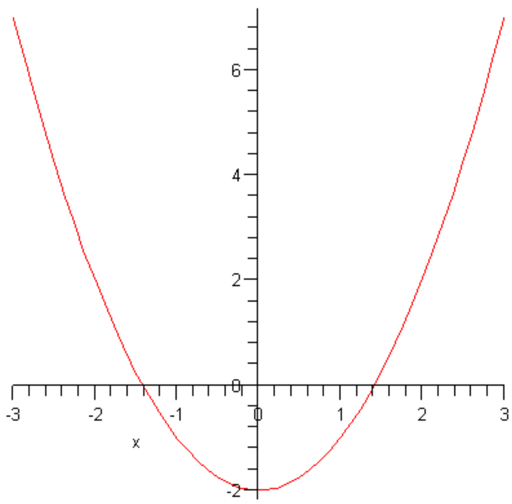
Example 3

Sketch the graph and find the domain and range of the following function.

$f(x) = x^2 - 2$

To make a sketch of the function, just find five points on the function and make a point plot. This can be done by choosing five values for x and substitution these value into the function to get the y value. For convenience, we will put the values in a table.

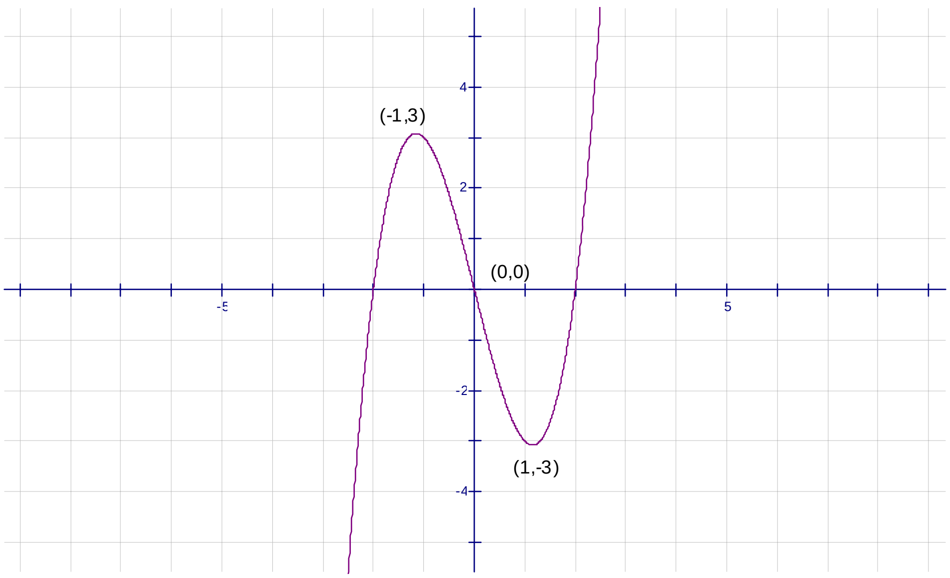
x	$f(x)$
-2	$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$
-1	$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$
0	$f(0) = 0^2 - 2 = -2$
1	$f(1) = (1)^2 - 2 = 1 - 2 = -1$
2	$f(2) = 2^2 - 2 = 2$



Domain: $(-\infty, \infty)$: Range $(-2, \infty)$

Example 4

Use the following graph to find each function value.



a) Find $f(-1)$
Solution: $f(-1) = 3$

b) Find $f(0)$
Solution: $f(0) = 0$

c) Find $f(1)$
Solution: $f(1) = -3$

Piecewise functions

Piecewise functions are divided up into separate smaller functions. Each separate function has its own domain. (See example below)

$$f(x) = \{ \begin{matrix} x - 2, x \geq 0 \\ 3 - x, x < 0 \end{matrix}$$

Example 5

Sketch a graph of the piecewise function shown in the previous illustration

$$f(x) = \{ \begin{matrix} x-2, x \geq 0 \\ 3-x, x < 0 \end{matrix}$$

Find the table values the first piece: $f(x) = x - 2$

Since the domain is $x \geq 0$, we must pick values equal to or greater that zero. Let's try $x = 0,1,2$

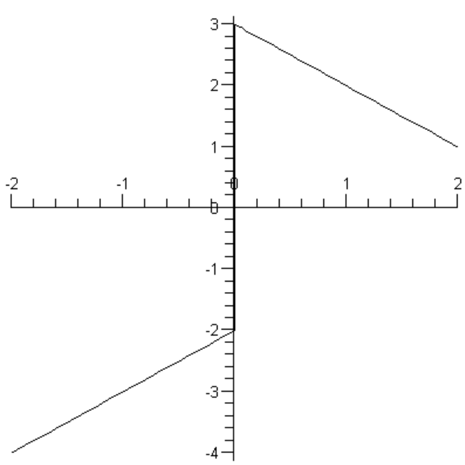
x	$f(x)$
0	$f(0) = 0 - 2 = -2$
1	$f(1) = 1 - 2 = -1$
2	$f(2) = 2 - 2 = -0$

Find the table values the second piece

Since the domain is $x < 0$, we must pick values equal less that zero. Let's try $x = 0,-1,-2$

x	$f(x)$
0	$f(0) = 3 - 0 = 3$
-1	$f(-1) = 3 - (-1) = 4$
-2	$f(-2) = 3 - (-2) = 5$

Now sketch a graph.



Example 6

Sketch a graph of the piecewise function.

$$f(x) = \{ \begin{matrix} x+1, x \geq 1 \\ x^2 - 1, x < 1 \end{matrix}$$

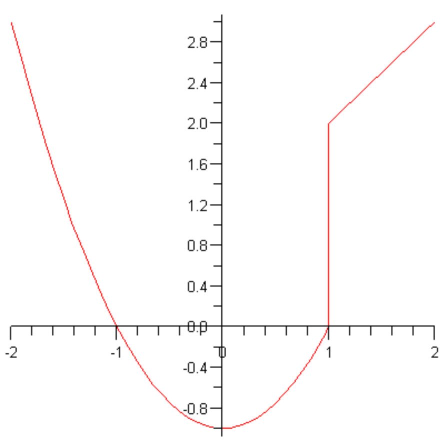
Since the domain is $x \geq 0$, we must pick values equal to or greater that zero. Let's try $x = 1,2,3$

x	$f(x)$
0	$f(1) = 1+1 = 2$
1	$f(2) = 2+1 = 3$
2	$f(3) = 3+1 = 4$

Find the table values the second piece

Since the domain is $x < 1$, we must pick values equal less that zero. Let's try $x = 0,-1,-2$

x	$f(x)$
0	$f(0) = 0^2 - 1 = -1$
-1	$f(-1) = (-1)^2 - 1 = 1 - 1 = 0$
-2	$f(-2) = (-2)^2 - 1 = 3$



Slope and intercept

Key Formulas

Slope: $m = \frac{y_2 - y_1}{x_2 - x_2}$

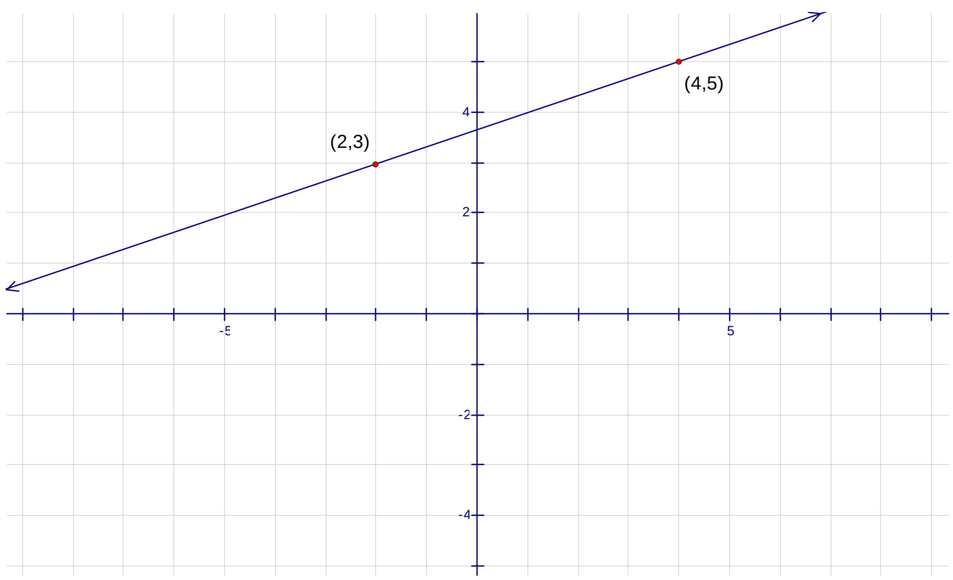
Slope-intercept formula: $y = mx + b$

Point-slope formula: $y - y_1 = m(x - x_1)$

Example 6

Find the slope of a line between the points (-2,3) and (4,5)

$$m = \frac{5-3}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$



Example 7

Find an expression for the function whose graph is the given curve.

The line segment joining the points (-1,1) and (2,4)

First find the slope between the points (-1,1) and (2,4)

$$m = \frac{4-1}{2-(-1)} = \frac{3}{3} = 1$$

Now find the equation using the point-slope formula

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 1(x - (-1))$$
$$y - 1 = 1(x + 1)$$
$$y - 1 = x + 1$$
$$y = x + 2$$
$$\Rightarrow f(x) = x + 2$$