

Unit 1

Math 116

Number Systems

Unit One Number Systems

Sections 1.1 - 1.2

Introduction to Number Systems

Through out history civilizations have keep records using their own number systems. This unit will introduce some basic number systems that were used by past civilizations.

Examples of Ancient Number Systems








- 1) Egyptian
- 2) Attic
- 3) Roman
- 4) Mayan
- 5) Traditional Chinese
- 6) Babylonian

D) Egyptian Number System

The Egyptian use symbols to represent the values that are multiples of ten.

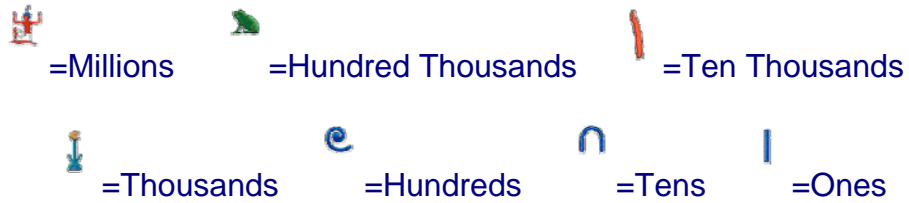
The symbols are written in figure 1-1

Figure 1.1

1 =		staff
10 =		heel bone
100 =		coil of rope
1000 =		lotus flower
10,000 =		pointing finger
100,000 =		tadpole
1,000,000 =		astonished man

(This chart is provided by the Department of Mathematics and Statistics at Wichita State University)

These symbols were provided by <http://eyelid.ukonline.co.uk/ancient/numbers.htm>



Examples

Write the following numbers as an Egyptian number.

1) 345



(Symbols courtesy <http://eyelid.ukonline.co.uk/ancient/numbers.htm>)

2) 456



(Symbols courtesy <http://eyelid.ukonline.co.uk/ancient/numbers.htm>)

3) 45623



(Symbols courtesy <http://eyelid.ukonline.co.uk/ancient/numbers.htm>)

II) Roman Numerals

Roman Numerals are very similar to the Egyptian system, but are based on 5 instead of 10.

The Roman Numerals

Symbol	Number Value
I	1
V	5
X	10
L	50
C	100
D	500
M	1000
\overline{V}	5000
\overline{X}	10000
\overline{L}	50000
\overline{C}	100000
\overline{D}	500000
\overline{M}	1000000

Examples

Convert the following decimal number to a Roman numeral.

1) 25

XXV

2) 246

200 = CC

40 = XL

6 = VI

So, the final answer would be
CCXLVI

3) 1989

1000 = M

900 = CM

80 = LXXX

9 = IX

Final Answer: MCMLXXXIX

4) 13020

\overline{X} = 10,000

MMM = 3,000

XX = 20

Final Answer is \overline{X} MMMXX

5) 1148

M = 1000

C = 100


XL = 40

VIII = 8


Final answer

MCXLVIII

III) The Mayan System

The Mayan system came into existence about 300 BC. This system is based on 18 and 20. The Mayans were the first to use the concept of zero. The number zero was denoted by the symbol 

Mayan symbols

Symbol	Number Value
	0
•	1
••	2
•••	3
••••	4
—	5
• —	6
• • —	7
• • • —	8
• • • • —	9
====	10
• ====	11
• • ====	12
• • • ====	13
• • • • ====	14
==== —	15

Number Systems

Section 1.3

Binary numbers

Babylonians system was based on 60

The Mayan system was based on 20

Our number system is based on 10

Computer use a number system based on 2 (Binary)

System	Base	Digits	Place Values
Binary	2	0,1	1,2,4,8,16,32
Quintary	5	0,1,2,3,4	1,5,25,125,625
Octal	8	0,1,2,3,4,5,6,7	1,8,64,512,4096

Binary numbers (Base Two)

Converting binary numbers to base numbers

1) Convert to base 10

1111_2

$$1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$8 + 4 + 2 + 1$$

15

2) Convert to base 10

101010_2

$$1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 \quad]$$

$$32 + 0 + 16 + 0 + 2$$

50

Convert base ten numbers to binary numbers

3) Convert 243 to a base 2 number (**Binary Number**)

First Check all power of 2 that divide 243

$$2^0 = 2$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256 \text{ which is too big, so use 128 or } 2^7$$

Write 243 as a difference (Note : $243 - 128 = 115$)

$$\text{Thus, } 243 = 128 + 115$$

$$\text{Then, } 243 = 2^7 + 115$$

Find the greatest power of two that divides 115 which is $2^6 = 64$

$$\text{So, } 243 = 2^7 + 64 + 51 \quad (\text{Note : } 115 = 64 + 51)$$

Keep repeating the process until the remainder is 1 or 0

$$\Rightarrow 243 = 2^7 + 2^6 + 32 + 19$$

$$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 19$$

$$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 16 + 3$$

$$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 2^4 + 3$$

$$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 2^4 + 2 + 1$$

$$\Rightarrow 243 = 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$\Rightarrow \text{The binary number is } 11110011_2$$

4) Convert 165 to a binary number

First Check all power of 2 that divide 165

$$2^0 = 2$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

Then, rewrite $165 = 128 + 37$

$$\Rightarrow 165 = 2^7 + 37$$

$$\Rightarrow 165 = 2^7 + 32 + 5$$

$$\Rightarrow 165 = 2^7 + 2^5 + 4 + 1$$

$$\Rightarrow 165 = 2^7 + 2^5 + 2^2 + 2^0$$

$$\Rightarrow 165 = 1 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$\Rightarrow 10100101_2$$

5) Convert 121 to a binary number

First Check all power of 2 that divide 121

$$2^0 = 2$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

Then, rewrite $121 = 64 + 57$

$$\Rightarrow 121 = 2^6 + 57$$

$$\Rightarrow 121 = 2^6 + 32 + 25$$

$$\Rightarrow 121 = 2^6 + 2^5 + 16 + 7$$

$$\Rightarrow 121 = 2^6 + 2^5 + 2^4 + 4 + 3$$

$$\Rightarrow 121 = 2^6 + 2^5 + 2^4 + 2^2 + 2 + 1$$

$$\Rightarrow 121 = 2^6 + 2^5 + 2^4 + 2^2 + 2^1 + 2^0$$

$$\Rightarrow 121 = 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$\Rightarrow 1110111_2$$

6) Convert 111_2 to a base ten number

$$111_2 \Rightarrow 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 4 + 2 + 1 = 7$$

7) Convert 11011_2 to a base 10 number

$$11011_2$$

$$\Rightarrow 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 16 + 8 + 0 + 2 + 1 = 27$$

8) Convert to base 10

$$1111_2$$

$$1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$8 + 4 + 2 + 1$$

$$15$$

9) Convert to base 10

101010_2

$$1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0]$$

$$32 + 0 + 16 + 0 + 2$$

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Optional Examples

10) Convert 245 into a base 5 number. (Optional)

List powers of 5

$$5^1 = 5$$

$$5^2 = 25$$

$$5^3 = 125$$

$$5^4 = 625$$

$$245 = 125 + 120$$

$$\Rightarrow 245 = 5^3 + 120$$

$$\Rightarrow 245 = 5^3 + 4 \cdot 25 + 20$$

$$\Rightarrow 245 = 5^3 + 4 \cdot 5^2 + 4 \cdot 5$$

$$\Rightarrow 245 = 1 \cdot 5^3 + 4 \cdot 5^2 + 4 \cdot 5 + 0 \cdot 5^0$$

This converts to 1440_5

11) Convert 49 into a base 6 number (Optional)

List powers of 6

$$6^0 = 1$$

$$6^1 = 6$$

$$6^2 = 36$$

$$6^3 = 216$$

$$49 = 36 + 13$$

$$\Rightarrow 49 = 6^2 + 13$$

$$\Rightarrow 49 = 6^2 + 2 \cdot 6 + 1$$

$$\Rightarrow 49 = 6^2 + 2 \cdot 6^1 + 6^0$$

$$\Rightarrow 49 = 1 \cdot 6^2 + 2 \cdot 6^1 + 1 \cdot 6^0$$

This converts to 121_6

Converting a based number other than base 10 to base 10

Write each of the following on a decimal numeral

12) Optional

$$253_7$$

$$2 \cdot 7^2 + 5 \cdot 7^1 + 3 \cdot 7^0$$

$$2 \cdot 49 + 5 \cdot 7 + 3 \cdot 1$$

$$98 + 35 + 3$$

$$136$$

13) Optional

$$1068_9$$

$$1 \cdot 9^3 + 0 \cdot 9^2 + 6 \cdot 9^1 + 8 \cdot 9^0$$

$$729 + 0 + 54 + 8$$

$$791$$