Unit 1

Math 116

Number Systems

Unit One
Number Systems

Sections 1.1 - 1.2

Introduction to Number Systems

Through out history civilizations have keep records using their own number systems. This unit will introduce some basic number systems that were used by past civilizations.

Examples of Ancient Number Systems

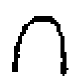
1) Egyptian
2) Attic
3) Roman
4) Mayan
5) Traditional Chinese
6) Babylonian

I) Egyptian Number System

The Egyptian use symbols to represent the values that are multiples of ten.

The symbols are written in figure 1-1

Figure 1.1

1 =		staff
10 =		heel bone
100 =		coil of rope
1000 =		lotus flower
10,000 =		pointing finger
100,000 =		tadpole
1,000,000 =		astonished man

(This chart is provided by the Department of Mathematics and Statistics at Wichita State University)

These symbols were provided by http://eyelid.ukonline.co.uk/ancient/numbers.htm

 =Millions  =Hundred Thousands  =Ten Thousands

 =Thousands  =Hundreds  =Tens =Ones

Examples

Write the following numbers as an Egyptian number.

1) 345

(Symbols courtesy http://eyelid.ukonline.co.uk/ancient/numbers.htm)

2) 456

(Symbols courtesy http://eyelid.ukonline.co.uk/ancient/numbers.htm)

3) 45623

(Symbols courtesy http://eyelid.ukonline.co.uk/ancient/numbers.htm)

Example 2

Write the following Egyptian Numbers as a decimal number.

a)

Answer: 134

b)

Answer: 2105

c)

Answer: 30400

II) Roman Numerals

Roman Numerals are very similar to the Egyptian system, but are based on 5 instead of 10.

The Roman Numerals

Symbol	Number Value
I	1
V	5
X	10
L	50
C	100
D	500
M	1000
$\overline{V}$	5000
$\overline{X}$	10000
$\overline{L}$	50000
$\overline{C}$	100000
$\overline{D}$	500000
$\overline{M}$	1000000

Example 3

Convert the following decimal number to a Roman numeral.

1) 25

XXV

2) 246

200 = CC
40 = XL
6 = VI

So, the final answer would be
CCXLVI

3) 1989

1000 = M
900 = CM
80 = LXXX
9 = IX

Final Answer: MCMLXXXIX

4) 13020

$\overline{X} = 10{,}000$

$MMM = 3{,}000$

$XX = 20$

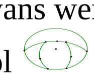
$Final\ \ Answer\ \ is\ \ \overline{X}MMMXX$

5) 1148

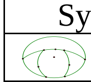
M = 1000
C = 100
XL = 40
VIII = 8

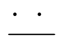
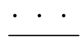
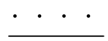
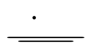
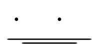
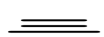
Final answer: MCXLVIII

III) The Mayan System

The Mayan system came into existence about 300 BC. This system is based on 18 and 20. The Mayans were the first to use the concept of zero. The number zero was denoted by the symbol 

Mayan symbols

Symbol	Number Value
	0
•	1
••	2
•••	3

••••	4
	5
	6
	7
	8
	9
	10
	11
	12
	13
	14
	15

Number Systems

Section 1.3

Binary numbers

Babylonians system was based on 60

The Mayan system was based on 20

Our number system is based on 10

Computer use a number system based on 2 (Binary)

System	Base	Digits	Place Values
Binary	2	0,1	1,2,4,8,16,32

Quintary	5	0,1,2,3,4	1,5,25,125,625
Octal	8	0,1,2,3,4,5,6,7	1,8,64,512,4096

Converting a based number other than base 10 to base 10

Write each of the following on a decimal numeral

1) **Optional**

253_7

$2\cdot 7^2 + 5\cdot 7^1 + 3\cdot 7^0$

$2\cdot 49 + 5\cdot 7 + 3\cdot 1$

$98 + 35 + 3$

136

2) **Optional**

1068_9

$1\cdot 9^3 + 0\cdot 9^2 + 6\cdot 9^1 + 8\cdot 9^0$

$729 + 0 + 54 + 8$

791

3) Convert to base 10

1111_2

$1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$

$8 + 4 + 2 + 1$

15

4) Convert to base 10

101010_2

$1\cdot 2^5 + 0\cdot 2^4 + 1\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0$]

$32 + 0 + 16 + 0 + 2$

50

Binary Numbers (Base Two)

5) Convert 243 to a base 2 number **(Binary Number)**

$First\ Check\ all\ power\ of\ 2\ that\ divide\ 243$

$2^0 = 2$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256\ \ which\ is\ too\ big,\ so\ use\ 128\ or\ 2^7$

$Write\ 243\ as\ a\ difference\ (Note: 243 - 128 = 115)$

$Thus,\ \ 243 = 128 + 115$

$Then,\ 243 = 2^7 + 115$

$Find\ the\ greatest\ power\ of\ two\ that\ divides\ 115\ which\ is\ 2^6 = 64$

$So,\ 243 = 2^7 + 64 + 51 \quad (Note: 115 = 64 + 51)$

$Keep\ repeating\ the\ process\ until\ the\ remainder\ is\ 1\ or\ 0$

$\Rightarrow 243 = 2^7 + 2^6 + 32 + 19$

$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 19$

$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 16 + 3$

$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 2^4 + 3$

$\Rightarrow 243 = 2^7 + 2^6 + 2^5 + 2^4 + 2 + 1$

$\Rightarrow 243 = 1\cdot 2^7 + 1\cdot 2^6 + 1\cdot 2^5 + 1\cdot 2^4 + 0\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$

$\Rightarrow The\ binary\ number\ is\ 11110011_2$

3) Convert 165 to a binary number

$First\ Check\ all\ power\ of\ 2\ that\ divide\ 165$

$2^0 = 2$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$Then, rewrite\ 165 = 128 + 37$

$\Rightarrow 165 = 2^7 + 37$

$\Rightarrow 165 = 2^7 + 32 + 5$

$\Rightarrow 165 = 2^7 + 2^5 + 4 + 1$

$\Rightarrow 165 = 2^7 + 2^5 + 2^2 + 2^0$

$\Rightarrow 165 = 1 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$

$\Rightarrow 10100101_2$

3) Convert 121 to a binary number

$First\ Check\ all\ power\ of\ 2\ that\ divide\ 121$

$2^0 = 2$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$Then, rewrite\ 121 = 64 + 57$

$\Rightarrow 121 = 2^6 + 57$

$\Rightarrow 121 = 2^6 + 32 + 25$

$\Rightarrow 121 = 2^6 + 2^5 + 16 + 7$

$\Rightarrow 121 = 2^6 + 2^5 + 2^4 + 4 + 3$

$\Rightarrow 121 = 2^6 + 2^5 + 2^4 + 2^2 + 2 + 1$

$\Rightarrow 121 = 2^6 + 2^5 + 2^4 + 2^2 + 2^1 + 2^0$

$\Rightarrow 165 = 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

$\Rightarrow 1110111_2$

5) Convert 111_2 to a base ten number

$111_2 \Rightarrow 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 4 + 2 + 1 = 7$

6) Convert 11011_2 to a base 10 number

11011_2

$\Rightarrow 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^{0\backslash} = 16 + 8 + 0 + 2 + 1 = 27$

7) Convert 245 into a base 5 number (**Optional)**

List powers of 5

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$5^4 = 625$

$245 = 125 + 120$

$\Rightarrow 245 = 5^3 + 120$

$\Rightarrow 245 = 5^3 + 4 \cdot 25 + 20$

$\Rightarrow 245 = 5^3 + 4 \cdot 5^2 + 4 \cdot 5$

$\Rightarrow 245 = 1 \cdot 5^3 + 4 \cdot 5^2 + 4 \cdot 5 + 0 \cdot 5^0$

This converts to 1440_5

8) Convert 49 into a base 6 number (**Optional)**

List powers of 6

$6^0 = 1$

$6^1 = 6$

$6^2 = 36$

$6^3 = 216$

$49 = 36 + 13$

$\Rightarrow 49 = 6^2 + 13$

$\Rightarrow 49 = 6^2 + 2 \cdot 6 + 1$

$\Rightarrow 49 = 6^2 + 2 \cdot 6^1 + 6^0$

$\Rightarrow 49 = 1 \cdot 6^2 + 2 \cdot 6^1 + 1 \cdot 6^0$

This converts to 121_6

Prime Numbers

A **prime numb**er is a number that is only divisible by the number itself and one.

The Prime factorization of a number

If a number is written as the product of it prime factors, this is called the **prime factorization**.

Examples of Prime Factorizations of Numbers

Example 1

Write the prime factorization of the following number in canonical form

60

$$\begin{aligned}&60\\&6\cdot 10\\&2\cdot 3\cdot 2\cdot 5\\&2^2\cdot 3\cdot 5 \Rightarrow \mathit{Canonical\ \ Form}\end{aligned}$$

Example 2

Write the prime factorization of the following number in canonical form

90

$$\begin{aligned}&90\\&9\cdot 10\\&3\cdot 3\cdot 2\cdot 5\\&2\cdot 3^2\cdot 5 \Rightarrow \mathit{Canonical\ \ Form}\end{aligned}$$