Networks and graphs

Key terms

Vertex (Vertices)

Each point of a graph

Edge

An edge is a segment that connects two vertices.

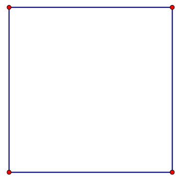
Region

A region is each individual area or separate piece of the plane that is divided up by the network.

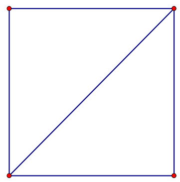
Example 1

Complete a table for the following networks.

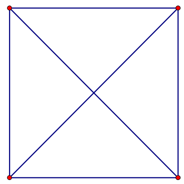
a)



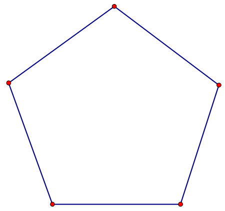
b)



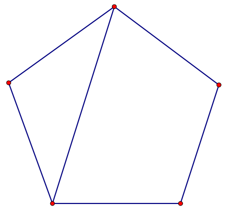
c)



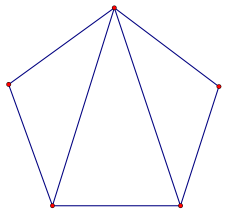
d)



e)



f)



Graph	Edges	Vertices	Regions	$V+R-2$
A	4	4	2	$4+2-2=4$
B	5	4	3	$4+3-2=5$
C	8	5	5	$5+5-2=8$
D	5	5	2	$5+2-2=5$
E	6	5	3	$5+3-2=6$
F	7	5	4	$5+4-2=7$

A network is said to **traversable** if it can be traced in one sweep without lifting the pencil from the paper and without tracing the same edge more than once.

The **degree of a vertex** is the number of edges that meet at that vertex.

Graph	Number of edges	Degree of each vertex	Sum	Traversable
A	4	2,2,2,2	8	Yes
b	5	3,2,3,2	10	Yes
c	8	3,3,3,3,4	16	No
d	5	2,2,2,2,2	10	Yes
e	6	3,2,2,3,2	12	Yes
f	7	4,2,3,3,2	14	Yes

Rules for the number odd vertices

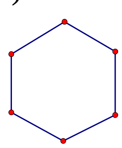
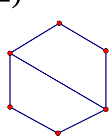
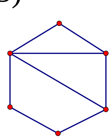
1) If the network has no odd vertices, then the network is traversable and any point is a starting point. The starting point will also turn out to be the ending point.

2) If the network has exactly one odd vertex, then the network is not traversable. A network cannot have only one starting point or ending point without the other.

3) If the network has two odd vertices, then the network is traversable. One odd vertex must be the starting point and the other odd vertex must be the ending point.

4) If the network has more than two odd vertices, then the network is not traversable. A network cannot have more than one starting point and one ending point.

Assignment (Complete the following table)

Graph	Edges	Vertices	Regions	Degree of each vertex	Sum	Traversable (Yes/No)
1) 						
2) 						
3) 						

Euler Circuits

Definition

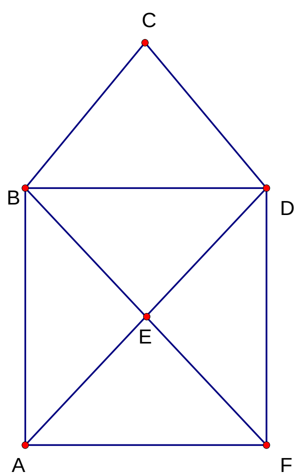
A network is an **Euler circuit** if you can start at one vertex and return to that vertex in one sweep without lifting your pencil and without tracing over the same edge more than once.

An Euler Circuit is a special case of being traversable where you start and finish at the same vertex.

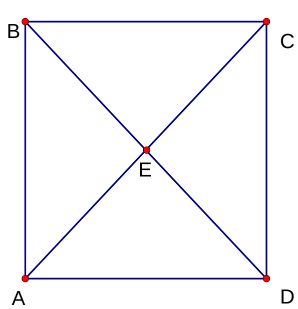
Example 2

Which the following networks have an Euler circuit?

Network 1



Network 2



Solution

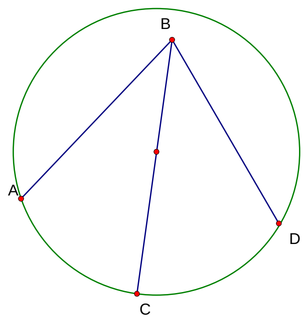
Network 1 is traversable since the graph has two odd vertices and four even vertices. (See rule above) Vertices A and F are odd and vertices B, C, D, and E are even. However the network does not have an Euler circuit because the path that is traversable has different starting and ending points.
Network 2 is not even traversable because it has four odd vertices which are A, B, C, and D. Thus, the network will not have an Euler circuit.

Hamiltonian cycles

In a Hamilton cycle, you must start from a given vertex and visit each vertex only once, then return to the original vertex.

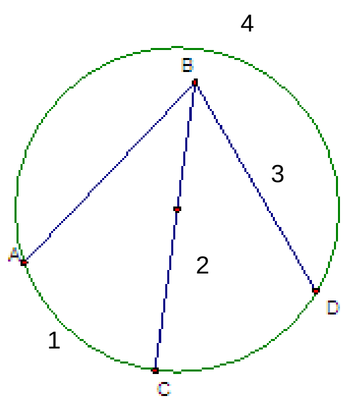
Example 3

Find the Hamiltonian cycle for the given network



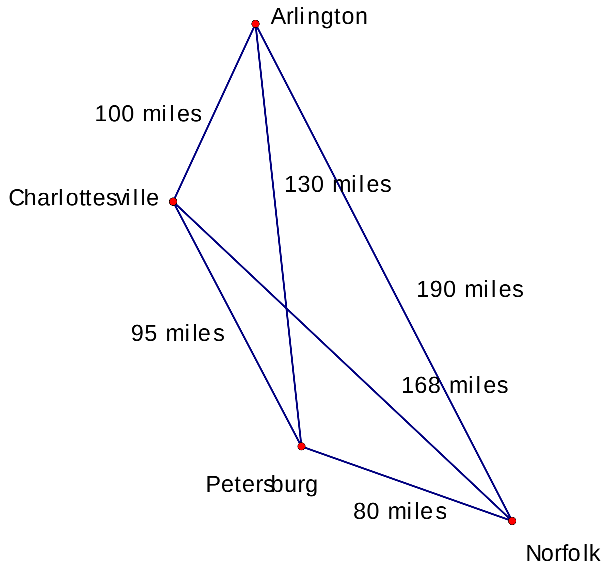
Solution: One Hamiltonian cycle would be $A \to C \to B \to D \to A$

This pattern is shown below:



Example 4

A salesman wants to visit four Virginia cities, Arlington, Charlottesville, Petersburg, and Norfolk. Driving distances are shown in figure 1. What is the shortest trip starting and ending in Arlington?



A=Arlington
N=Norfolk
P=Petersburg
C=Charlottesville

Different Paths

$$A \xrightarrow{100} C \xrightarrow{95} P \xrightarrow{80} N \xrightarrow{190} A \quad Total \;\; Miles = 465$$

$$A \xrightarrow{130} P \xrightarrow{80} N \xrightarrow{168} C \xrightarrow{100} A \quad Total \;\; Miles = 478$$

$$A \xrightarrow{190} N \xrightarrow{168} C \xrightarrow{95} P \xrightarrow{130} A \; Total \;\; Miles = 583$$

The shortest path would be
$Arlington \rightarrow Charlottesville \rightarrow Petersburg \rightarrow Norfolk \rightarrow Arlington$

Sort-edge method
(Nearest Neighbor)

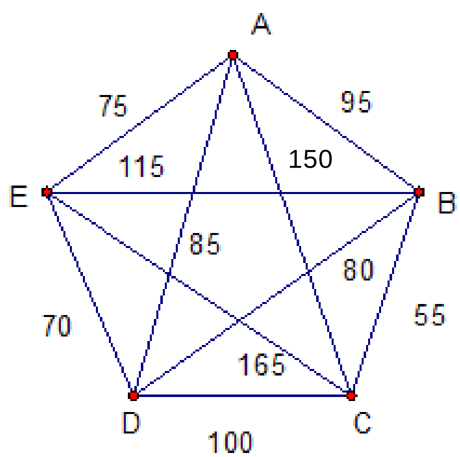
1) Choose the edge attached to the starting vertex that has the shortest distance or the lowest cost. Travel along this edge to the next vertex.

2) At the next vertex travel along the edge with the shortest distance or lowest cost. Do not choose a vertex that would lead to a vertex already visited.

3) Continue until all vertices are visited until arriving back at the original vertex.

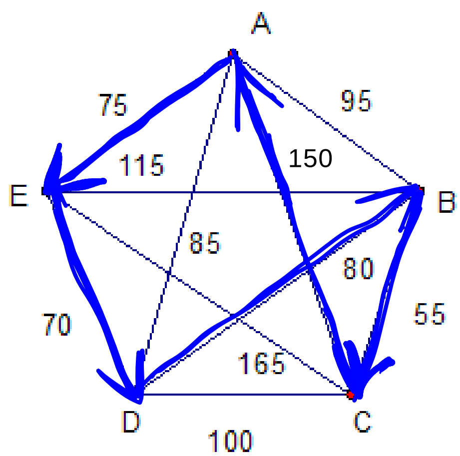
Example 5

The graph shows the mileage between various towns labeled A,B,C,D, and E. Tom, who lives in A, wishes to visit all the towns the exactly once and the return home. If the sorted–edge (nearest neighbor) algorithm is used to determine an approximate solution for his best route, what is the total distance along that route?



Solution: $A \to E \to D \to B \to C \to A$

The path is shown in the following diagram



Total Distance: $75 + 70 + 80 + 55 + 150 = 420 \ miles$

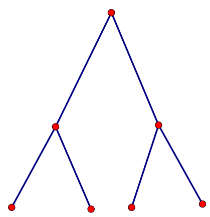
Section 6.2

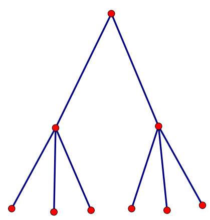
Trees and Minimum Spanning Trees

A **tree** is a graph that is connected and has no circuits.

Example 1

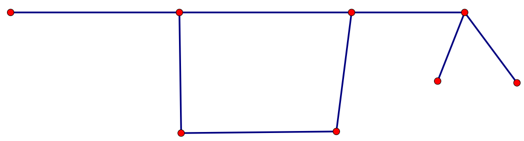
Examples of trees



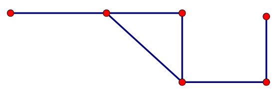


Example 2

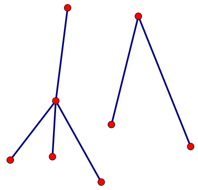
Examples of thing that are not trees



This object contains a circuit.



This object contains a circuit.



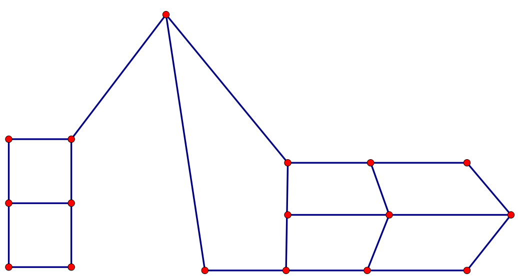
This object is not connected

Spanning Trees

A tree that is created from another graph by removing edges but keeping the path to each vertex is called a **spanning tree**.

Example 4

Original Graph



Remove all edges without eliminating any paths to each vertex. This will result in the tree in the diagram 1

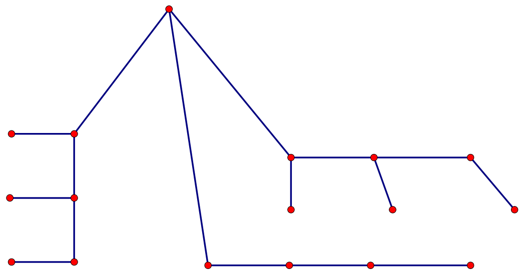
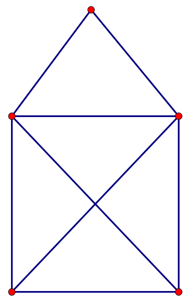


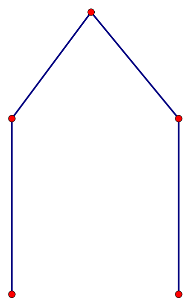
Diagram 1

Example 5

Find a spanning for the given graph.

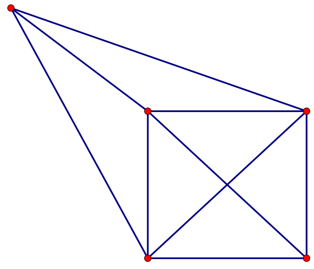


Example of a Spanning Tree

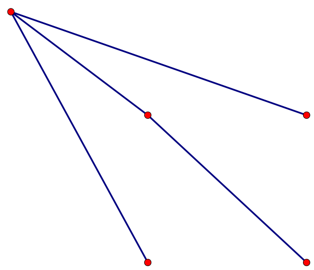


Example 6

Given the following graph, produce a spanning tree by removing edges from the graph.

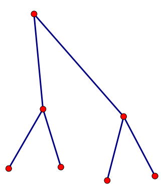


Spanning tree



If a graph is a tree with n vertices, then the number of edges is $n-1$.

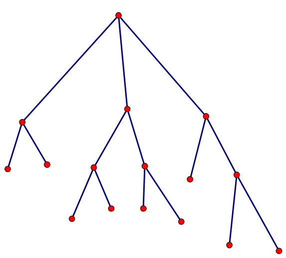
Example 7 Find the number of vertices and edge in the following graph



The above graph has 7 vertices

The number of edges: $n-1=7-1=6$ *edges*

Example 8 Find the number of vertices and edge in the following graph



The above object has 16 vertices

Number of edges: $n-1=16-1=15$ *edges*
