

Math 126
Final Review

Functions

1) Given $f(x) = x^2 - 3x$ and $g(x) = 4x$, find the following functions.

a) Find $f(3)$

$$f(3) = 3^2 - 3(3) = 9 - 9 = 0$$

b) Find $g(f(1))$

$$f(1) = 1^2 - 3(1) = 1 - 3 = -2$$

$$g(f(1)) = g(-2) = 4(-2)$$

2) Find the inverse of $f(x) = 2x + 1$

$$f(x) = 2x + 1$$

$$y = 2x + 1$$

$$x = 2y + 1$$

$$x - 1 = 2y + 1 - 1$$

$$2y = x - 1$$

$$\frac{2y}{2} = \frac{x-1}{2}$$

$$y = \frac{x-1}{2}$$

3) Find the inverse of $f(x) = x^3 - 4$

$$f(x) = x^3 - 4$$

$$y = x^3 - 4$$

$$x = y^3 - 4$$

$$x - 4 = y^3$$

$$\sqrt[3]{x-4} = \sqrt[3]{y}$$

$$\sqrt[3]{x-4} = y$$

$$f^{-1}(x) = \sqrt[3]{x-4}$$

Limits

4) Evaluate $\lim_{x \rightarrow 2} x^2 - 7$

$$\lim_{x \rightarrow 2} x^2 - 7 = 2^2 - 7 = 4 - 7 = -3$$

5) Evaluate $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x - 1}$

$$\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 3)(x - 1)}{x - 1} = \lim_{x \rightarrow 1} x - 3 = 1 - 3 = -2$$

6) Evaluate $\lim_{x \rightarrow 5} \frac{3x}{x - 5}$

$$\lim_{x \rightarrow 5} \frac{3x}{x - 5} = \frac{3(5)}{5 - 5} = \frac{15}{0} \text{ DNE}$$

Derivatives

Find the derivative of each function.

7) $f(x) = 5x^3 - 2x^2 + 4x - 2$

$$f(x) = 5x^3 - 2x^2 + 4x - 2$$

$$f'(x) = 15x^{3-1} - 4x + 4$$

$$f'(x) = 15x^2 - 4x + 4$$

9) $f(x) = x^3 e^{4x}$

$$f(x) = 2x^2 e^{6x}$$

$$f'(x) = (2x^2)' e^{6x} + (e^{6x})' (2x^2)$$

$$f'(x) = 4xe^{6x} + 6e^{6x}(2x^2)$$

$$10) f(x) = (x^2 + 2)(x^2 + 3x + 2)$$

$$f(x) = (x^2 + 2)(x^2 + 3x + 2)$$

$$f'(x) = (2x)(x^2 + 3x + 2) + (2x + 3)(x^2 + 2)$$

$$11) f(x) = 2x^4 \ln(x^2 - 5)$$

$$f(x) = 2x^4 \ln(x^2 - 5)$$

$$f'(x) = (2x^4)'(x^2 - 5) + (\ln(x^2 - 5))'(2x^4)$$

$$f'(x) = 4x^3(x^2 - 5) + \frac{2x}{x^2 - 5}(2x^4)$$

$$f'(x) = 4x^3(x^2 - 5) + \frac{4x^5}{x^2 - 5}$$

$$12)$$

$$y = \frac{5}{x^2}$$

$$y = 5x^{-2}$$

$$y' = -10x^{-2-1} = -10x^{-3} = -\frac{10}{x^3}$$

$$13) f(x) = \sqrt{x^2 - 2x}$$

$$f(x) = \sqrt{x^2 - 2x} = \sqrt{u} = u^{\frac{1}{2}}$$

$$u = x^2 - 2x$$

$$du = 2x - 2$$

$$f(x) = \frac{1}{2} u^{\frac{1}{2}-1} du = \frac{1}{2} u^{-\frac{1}{2}} = \frac{1}{2} (x^2 - 2x)^{\frac{1}{2}} = \frac{2x - 2}{2\sqrt{x^2 - 2x}} = \frac{x - 1}{\sqrt{x^2 - 2x}}$$

$$14) f(x) = (x^2 - 3x)^6$$

$$f(x) = (x^2 - 3x)^6$$

$$f(x) = u^6 \text{ where } u = x^2 - 3x \Rightarrow du = 2x - 3$$

$$f'(x) = 6u^{6-1} du = 6u^5 du = 6(2x - 3)(x^2 - 3x)^5$$

$$15) f(x) = e^{x^2}$$

$$f'(x) = 2xe^{x^2}$$

$$16) f(x) = \ln(x^4)$$

$$f(x) = \ln(x^4)$$

$$f'(x) = \frac{4x^3}{x^4} = \frac{4}{x}$$

Discuss the continuity of the following functions.

$$18) f(x) = 6x^2 + 7x + 3$$

Continuous on the interval $(-\infty, \infty)$

$$19) f(x) = \frac{1}{x^2 - 9}$$

$$f(x) = \frac{1}{x^2 - 9} = \frac{1}{(x-3)(x+3)}$$

Continuous on the interval $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Logarithms

20) Write the following expression as a single logarithm. $4\ln x + 4\ln y - 5\ln t$

$$4\ln x + 4\ln y - 5\ln t = \ln x^4 + \ln y^4 - \ln t^5 = \ln(x^4 y^4) - \ln t^5 = \ln\left(\frac{x^4 y^4}{t^5}\right)$$

21) Write $\log_8 64 = 2$ as an exponent expression.

$$8^2 = 64$$

22) Write $5^3 = 125$ as a logarithm expression.

$$\log_5 125 = 3$$

Solve the following exponential functions.

23) Solve $4^{2x} = \frac{1}{64}$

$$4^{2x} = \frac{1}{64}$$

$$4^{2x} = \frac{1}{4^3}$$

$$4^{2x} = 4^{-3}$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

24) $e^x = 15$

$$e^x = 15$$

$$\ln e^x = \ln(15)$$

$$x = \ln(15)$$

$$25) \ln e^{3x} = 4$$

$$\ln e^{3x} = 4$$

$$3x = 4$$

$$x = \frac{4}{3}$$