

Section 1.3

Function Notation

Writing functions in an equation form.

Example 1

Let $f(x) = x^2 - 3x + 4$

a) Find $f(2)$

$$f(2) = 2^2 - 3(2) + 4 = 4 - 6 + 4 = 2$$

b) $f(-3)$

$$f(-3) = (-3)^2 - 3(-3) + 4 = 9 + 9 + 4 = 22$$

c) $f(2d)$

$$f(2d) = (2d)^2 - 3(2d) + 4 = 4d^2 - 6d + 4$$

Composition of two functions

$$f \circ g(x) = f(g(x))$$

$$g \circ f(x) = g(f(x))$$

Example 2

Given $f(x) = x^2 + 4x$ and $g(x) = 3x - 4$, find the following functions.

a) Find $f \circ g(x)$

$$f \circ g(x) = f(g(x))$$

$$f \circ g(x) = f(3x - 4)$$

$$f \circ g(x) = (3x - 4)^2 + 4(3x - 4)$$

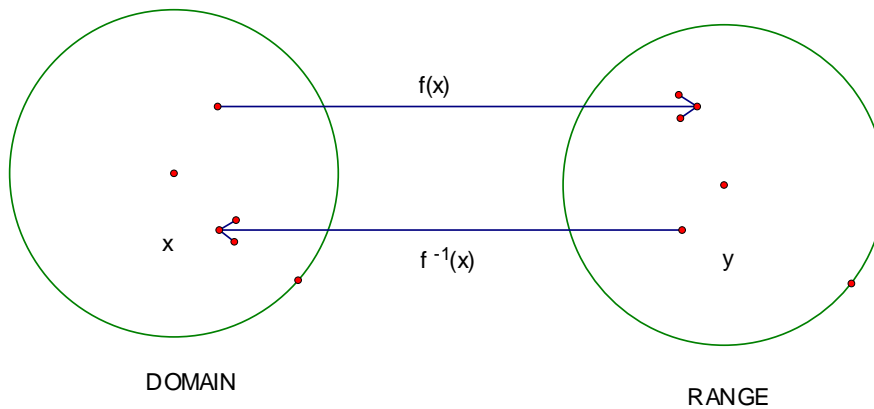
$$f \circ g(x) = (3x - 4)(3x - 4) + 12x - 16$$

$$f \circ g(x) = 9x^2 - 12x - 12x + 12x - 16$$

$$f \circ g(x) = 9x^2 - 12x - 16$$

Inverse Functions

An **inverse function** $f^{-1}(x)$ of a function $f(x)$ is function that maps each point range back to the domain where $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$



Find the inverse function of the function $f = \{(1,4), (2,6), (3,4), (4,5)\}$

Solution: Just switch the x and y components.

Thus, the inverse function would be $f^{-1} = \{(4,1), (6,2), (4,3), (5,4)\}$

Example 3 Find the inverse function of $f(x) = 3x - 4$

Solution: Replace $f(x)$ with y, switch x and y, and then solve for y:

$$f(x) = 3x - 4 \Rightarrow y = 3x - 4$$

$$\text{Switch } x \text{ and } y \Rightarrow x = 3y - 4$$

Solve for y

$$x = 3y - 4$$

$$x + 4 = 3y - 4 + 4$$

$$x + 4 = 3y$$

$$\frac{x + 4}{3} = \frac{3y}{3}$$

$$\frac{x + 4}{3} = y \Rightarrow f^{-1}(x) = \frac{x + 4}{3}$$

Example 4

Find the inverse function of $f(x) = x^3 - 1$ and then sketch a graph of $f(x)$ and $f^{-1}(x)$

$$f(x) = x^3 - 1 \Rightarrow y = x^3 - 1$$

$$\text{Switch } x \text{ and } y \Rightarrow x = y^3 - 1$$

Solve for y

$$x = y^3 - 1$$

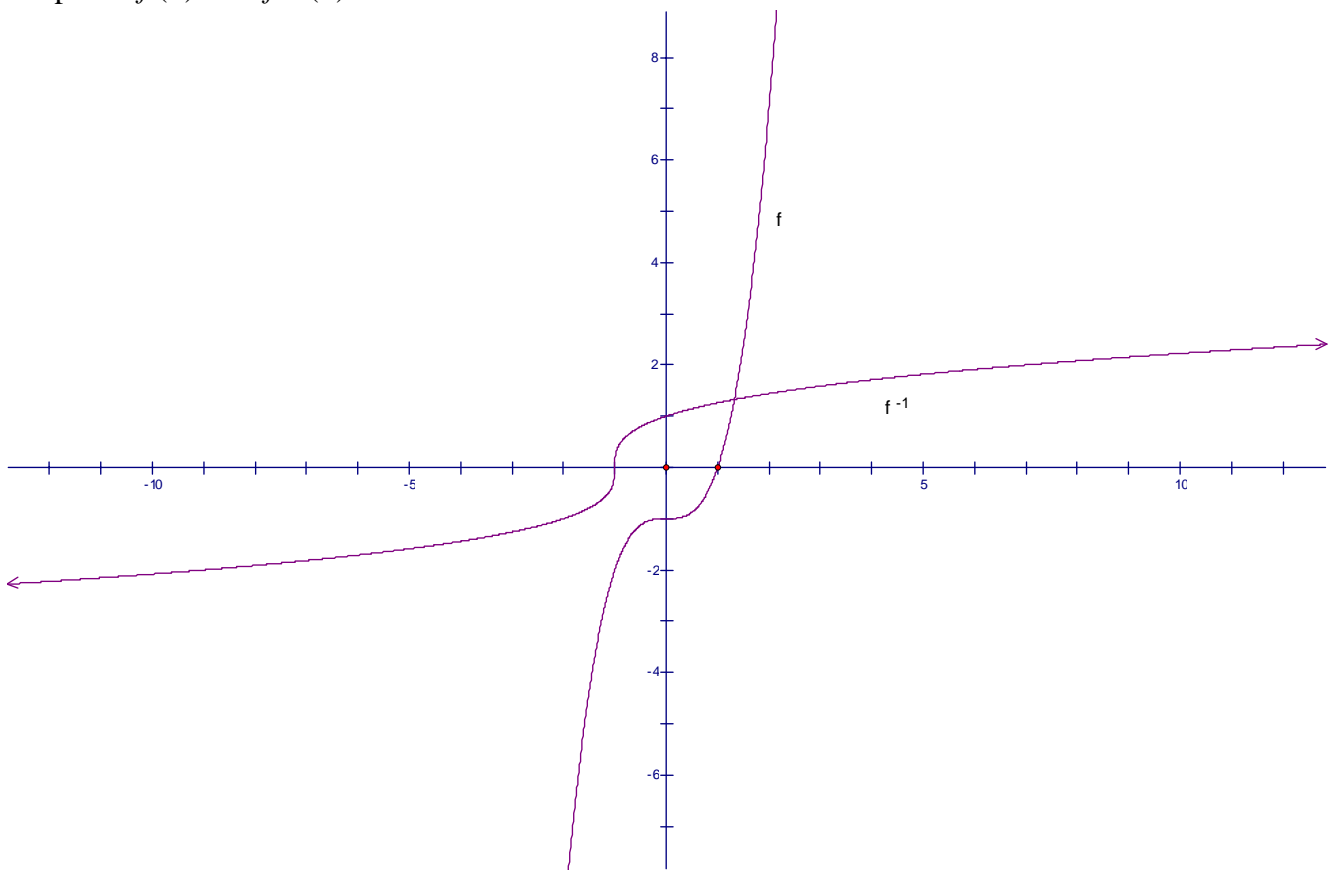
$$x + 1 = y^3 - 1 + 1$$

$$x + 1 = y^3$$

$$\sqrt[3]{x+1} = \sqrt[3]{y^3}$$

$$\sqrt[3]{x+1} = y \Rightarrow f^{-1}(x) = \sqrt[3]{x+1}$$

Graph of $f(x)$ and $f^{-1}(x)$



Example 5

Find the inverse function of $f(x) = x^2 + 3$

$$f(x) = x^2 + 3 \Rightarrow y = x^2 + 3$$

$$\text{Switch } x \text{ and } y \Rightarrow x = y^2 + 3$$

Solve for y

$$x = y^2 + 3$$

$$x - 3 = y^2 + 3 - 3$$

$$x - 3 = y^2$$

$$\sqrt{x-3} = \sqrt{y^2}$$

$$\Rightarrow \pm\sqrt{x-3} = y \text{ However, } \pm\sqrt{x-3} \text{ is not a function}$$

Therefore, f does not have an inverse function

Graphing Equations

Sketching Graphs

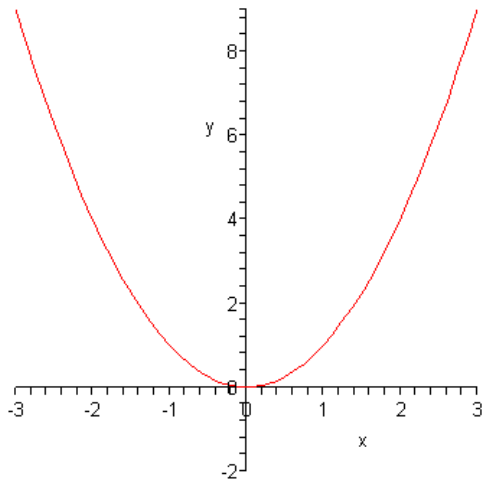
Basic Families of Graphs

Example 1 (The Standard Parabola)

Graph $y = x^2$

x	y
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$(0)^2 = 0$
1	$(1)^2 = 1$
2	$(2)^2 = 4$

Plot the following values $(-2,4), (-1,1), (0,0), (1,1), (2,4)$ from the table will give the following graph.

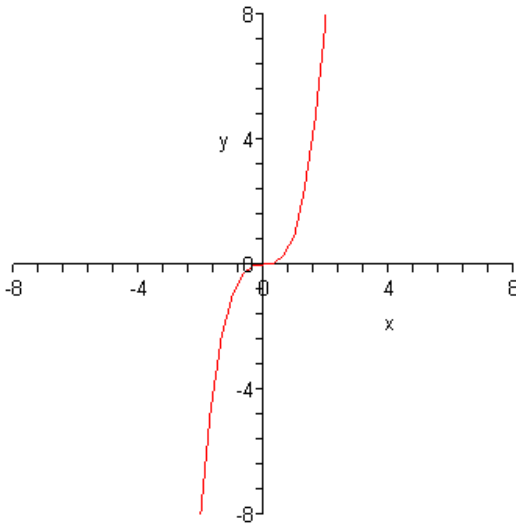


Example 2 (The standard “cubic” graph)

Graph $y = x^3$

x	$y = x^3$
-2	$(-2)^3 = -8$
-1	$(-1)^3 = -1$
0	$0^3 = 0$
1	$1^3 = 1$
2	$2^3 = 8$

Plot the values from the table will result in the following graph

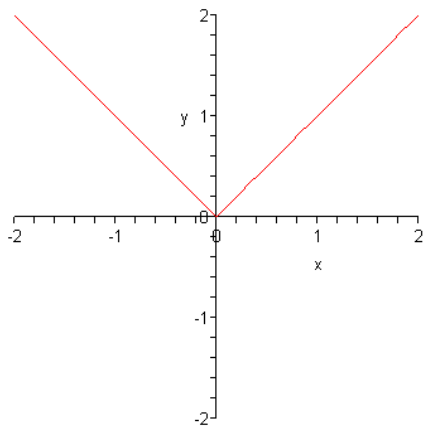


Example 3 (The Standard Absolute Value Graph)

Graph $y = |x|$

x	$y = x $
-2	$ -2 = 2$
-1	$ -1 = 1$
0	$ 0 = 0$
1	$ 1 = 1$
2	$ 2 = 2$

Plot the values from the table will give you a v-shaped graph



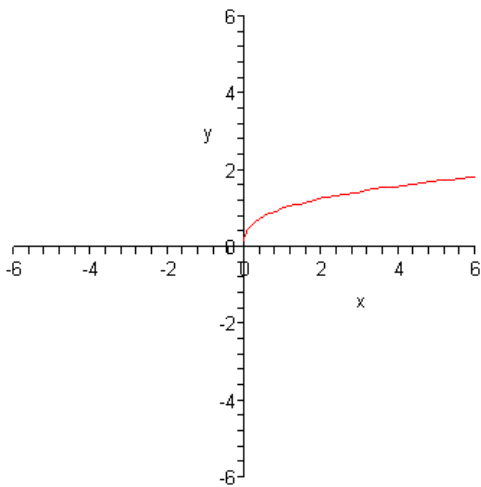
Example 4 (Standard Square Root Graph)

Graph $y = \sqrt{x}$

Again, use a table of values to make a graph of the equation

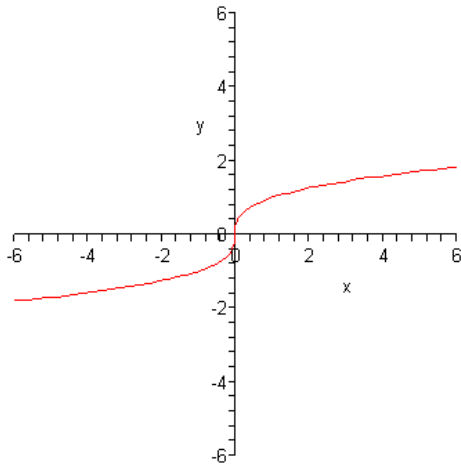
x	$y = \sqrt{x}$
0	$\sqrt{0} = 0$
1	$\sqrt{1} = 1$
4	$\sqrt{4} = 2$
9	$\sqrt{9} = 3$

Resulting Graph



Example 5

The graph of $y = \sqrt[3]{x}$

**Horizontal and Vertical Translations (Shifts)**

Horizontal Translation: An operation that moves the graph of an equation to the left or right while at the same time preserves the shape of the graph.

Vertical Translation: An operation that moves the graph of an equation to the left or right while at the same time preserves the shape of the graph.

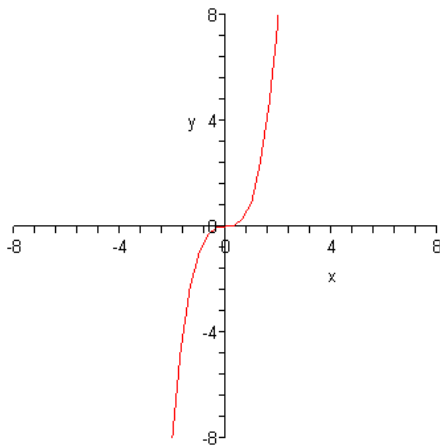
Example 6

Example of a Vertical Translation

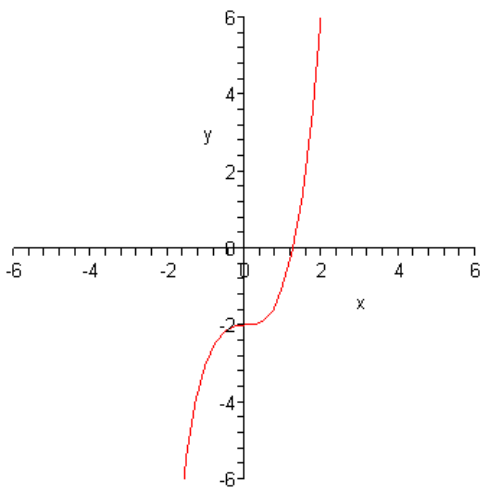
$$y = x^3 - 2$$

Since the -2 lies outside the x^3 term, the value -2 indicates a vertical translation of 2 units. The negative sign in value of -2 indicates that the translation will move the graph of $y = x^3$ down two units as shown below:

The graph of $y = x^3 - 2$



The graph of $y = x^3 - 2$ shifted down to units



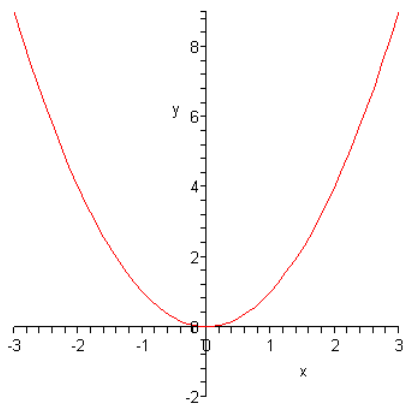
Example 7

Example of a Horizontal Translation

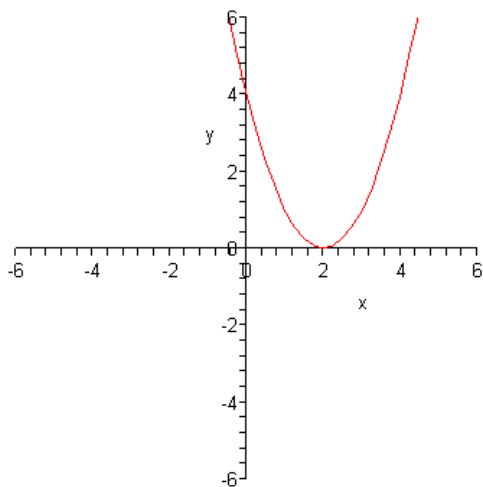
$$y = (x - 2)^2$$

In this example, the -2 inside the parentheses indicates that there is a horizontal translation of two units to the right. A negative sign inside the parentheses will always result in a shift to the right.

The original graph of $y = x^2$

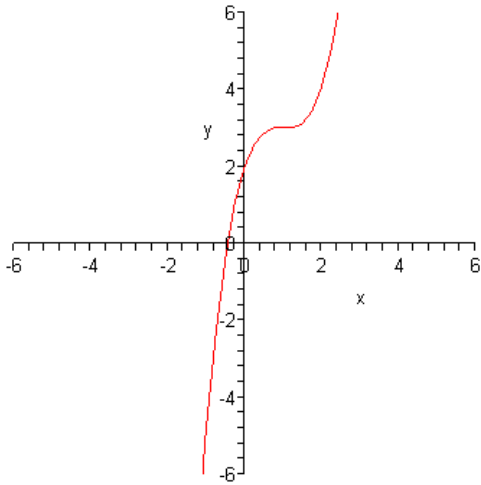


The graph of $y = x^2$ after a horizontal translation of 2 units to the right



Example 8

The graph of $y = (x-1)^3 + 3$



Example 9 The graph of $y = -x^2$

The graph of $y = -x^2$ is the inverted graph of $y = x^2$. The negative sign in front of the x^2 term simply turns the graph of $y = x^2$ upside down.

