

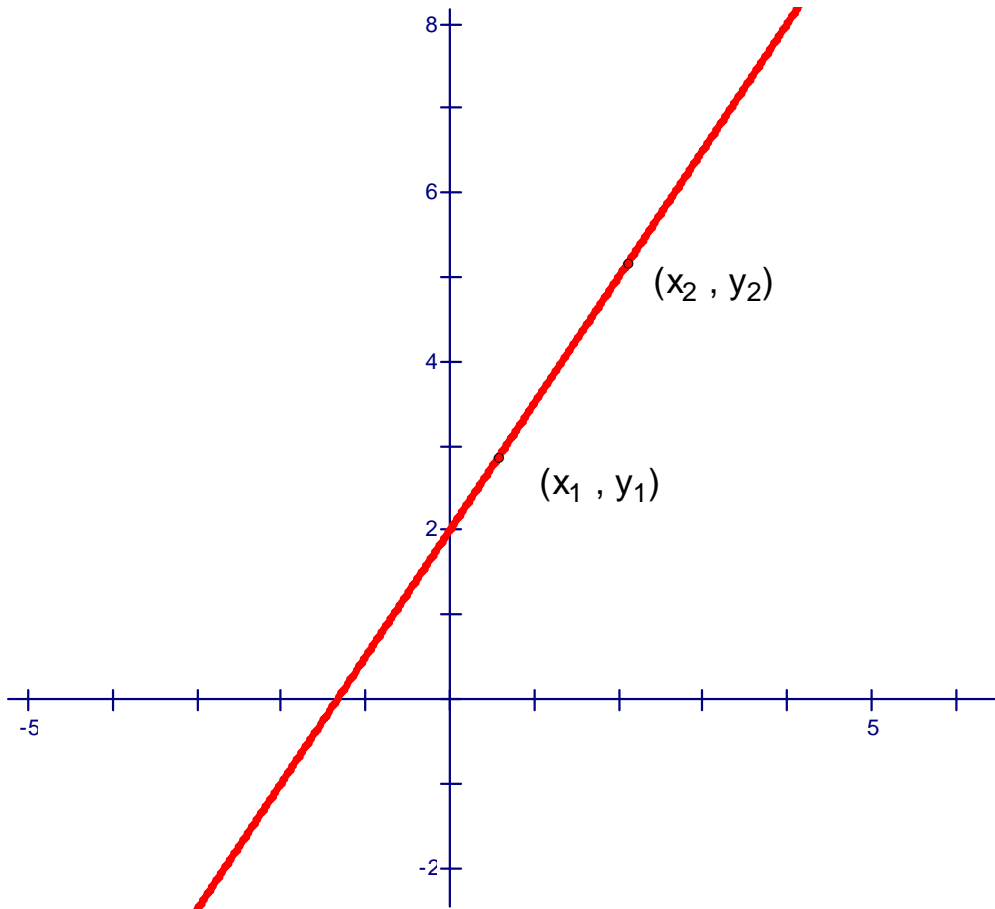
Math 151
Section 1.1
Slope and Intercept

Slope

The slope of a non-vertical line is measure of the number of units the line rises (or falls) for each unit of horizontal change from left to right.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Verticals lines do not have a slope

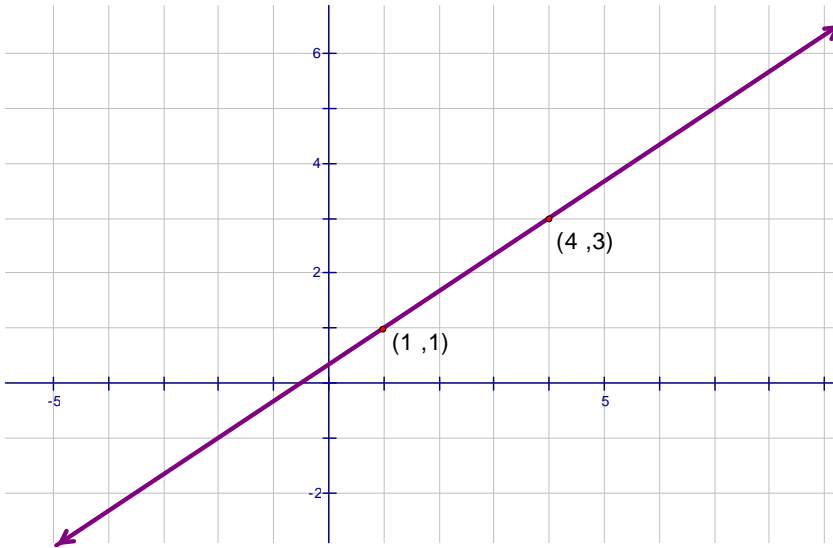


Example 1

Find the slope of a line passing through the given points.

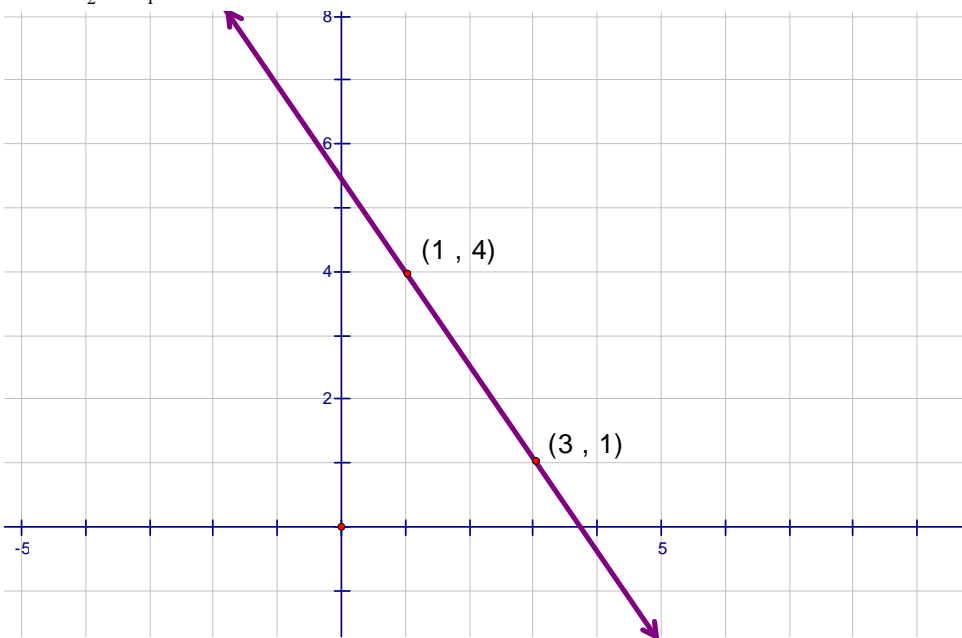
A) (1,1) and (4,3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{4 - 1} = \frac{2}{3}$$



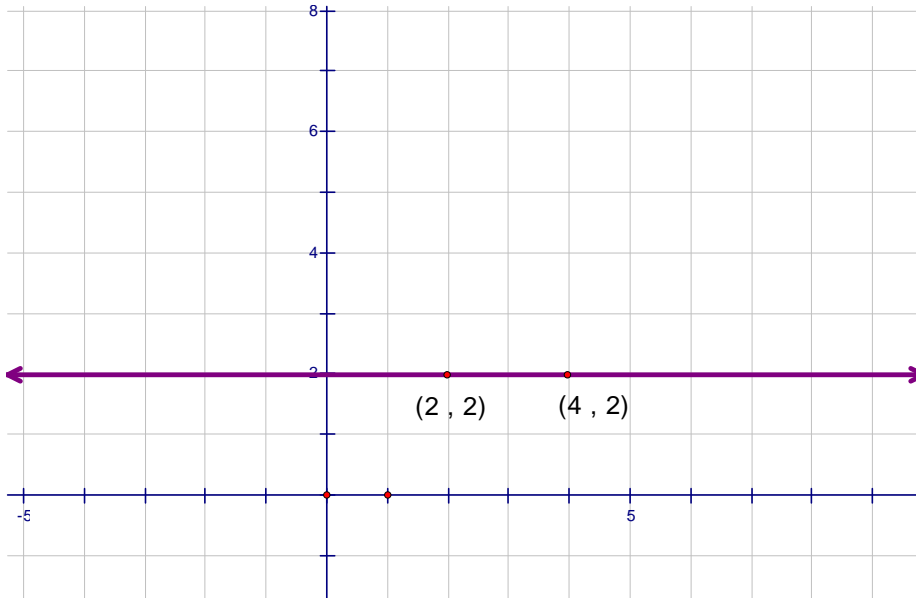
B) (1,4) and (3,1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{3 - 1} = -\frac{3}{2}$$



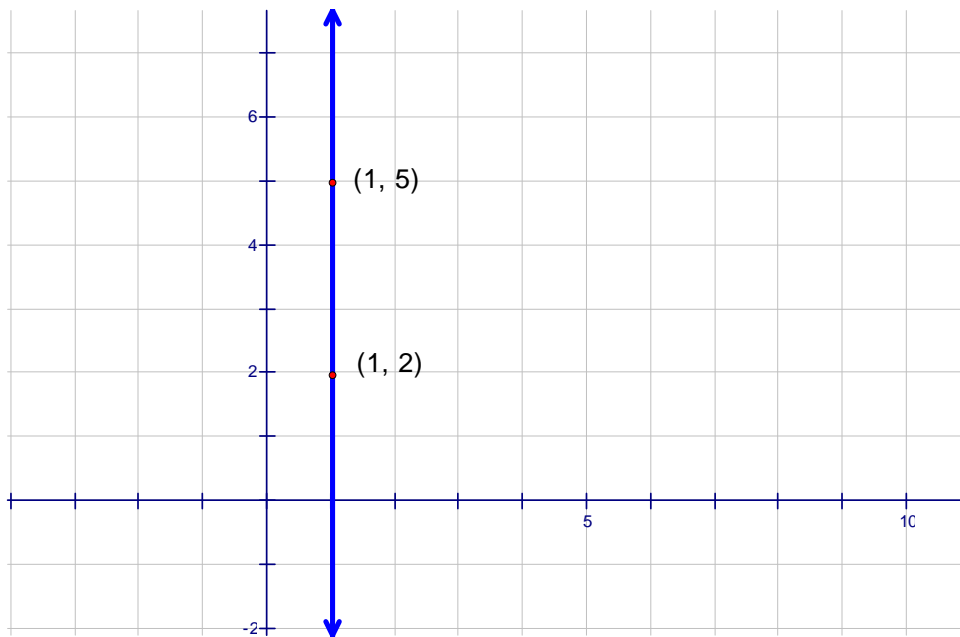
C) (2,3) and (4,3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{4 - 2} = \frac{0}{2} = 0$$



D) (1,2) and (1,5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{1 - 1} = \frac{3}{0} \text{ Slope is undefined}$$



Slope-intercept form of a linear equation

The slope intercept form of a linear equation is $y = mx + b$ where m is the slope of the line and b is the y-intercept.

Point-slope form of a linear equation

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$ where m is the slope of a line and (x_1, y_1) is a point on the line.

Example 3

Use the slope of the line and given point to find three addition points that the line passes through.

Point: $(2,5)$ Slope: $m = 2$

First find the equation of the line using the point-slope formula.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 2(x - 2)$$

$$y - 5 = 2x - 4$$

$$y - 5 + 5 = 2x - 4 + 5$$

$$y = 2x + 1$$

Next, chose any 3 values for x and substitute in the equation $y = 2x + 1$ to get the y value.

In this example we will use the values of -1,0, and 1 for x.

x	$y = 2x + 1$
-1	$y = 2(-1) + 1 = -2 + 1 = -1$
0	$y = 2(0) + 1 = 0 + 1 = 1$
1	$y = 2(1) + 1 = 2 + 1 = 3$

Three points that lie on the line are $(-1,-1)$, $(0,1)$, and $(1,3)$

Example 4

Find the slope and y-intercept of the given line (if they exist)

$$y = 2x + 1$$

Solution:

$$m = 2$$

$$b = 1$$

Example 5

Find the slope and y-intercept of the given line (if they exist)

$$2y + 3x = 2$$

In this example, we have to put the line in slope-intercept form first before we find the slope and y-intercept

$$2y + 3x = 2$$

$$2y + 3x - 3x = -3x + 2$$

$$2y = -3x + 2$$

$$\frac{2y}{2} = \frac{-3x}{2} + \frac{2}{2}$$

$$y = -\frac{3}{2}x + 1$$

$$m = -\frac{3}{2}$$

$$b = 1$$

Example 6

Find the slope and y-intercept of the given line (if they exist)

$$x = -3$$

Since this equation does not have a variable of y, it can not be put in slope-intercept form. Thus, the equation has a graph that is a vertical line which means that the slope is undefined and the line has no y-intercept. (See example 2 B)

Example 7

Find the equation of a line that passes through the given point that has the indicated slope.

$$(2,1): m = \frac{2}{3}$$

Simply substitute the values of the x and y coordinates of the point along with the slope into the point-slope formula.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{2}{3}(x - 2)$$

$$y - 1 = \frac{2}{3}x - \frac{2}{3} \cdot 2$$

$$y - 1 = \frac{2}{3}x - \frac{4}{3}$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

Example 8

Find the equation of a line that passes through the given points.

$$(2,4) \text{ and } (-2,-1)$$

First find the slope of the line using the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 4}{-2 - 2} = \frac{-5}{-4} = \frac{5}{4}$$

Next, find the equation of the line by choosing a point on the line and substituting that point along with the slope into the point-slope formula.

$$y - y_1 = m(x - x_1)$$

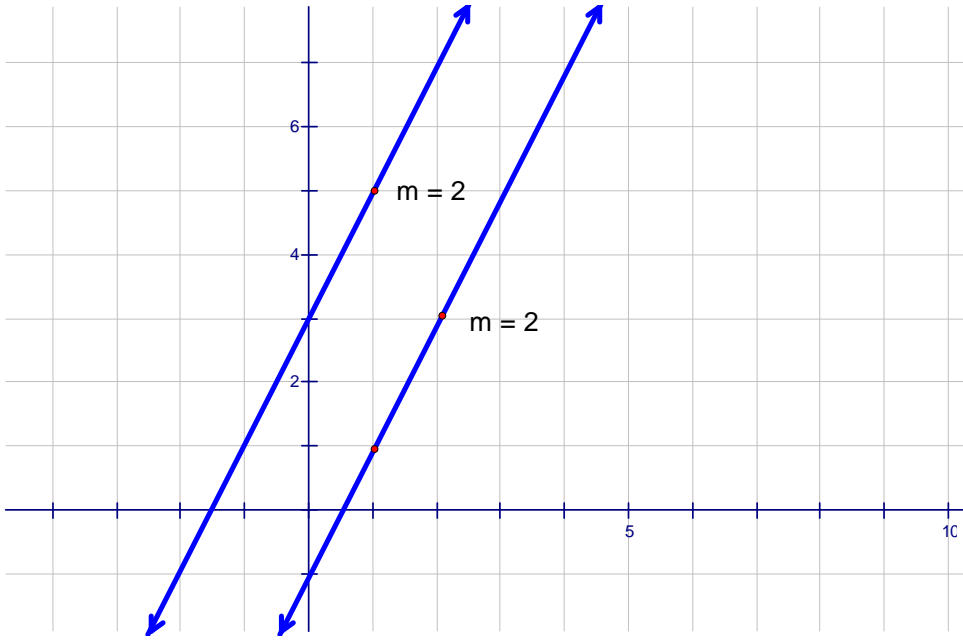
$$y - 4 = \frac{5}{4}(x - 2)$$

$$y - 4 = \frac{5}{4}x - \frac{5}{4} \cdot 2$$

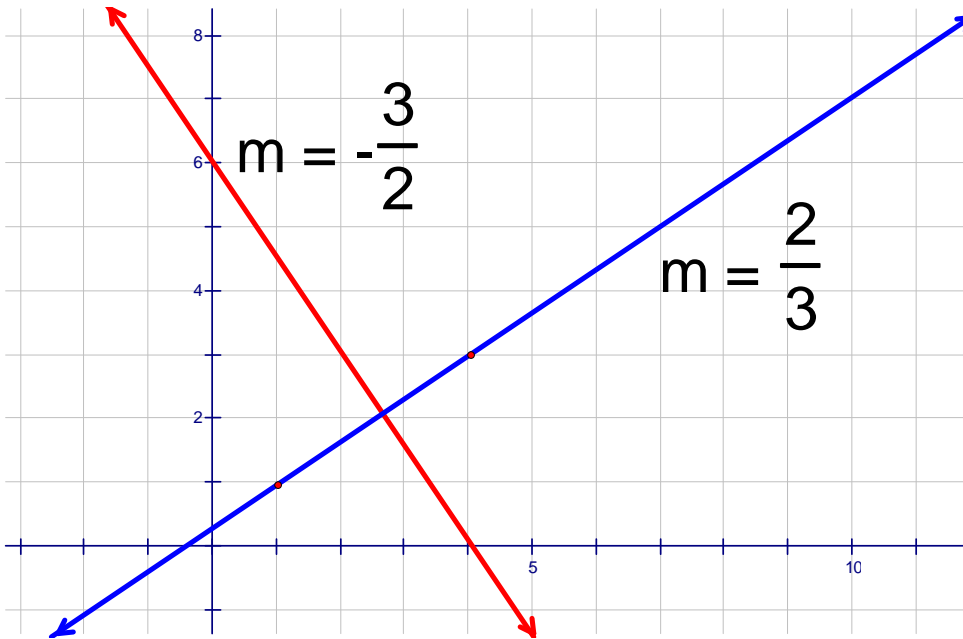
$$y - 4 = \frac{5}{4}x - \frac{5}{2} \Rightarrow y = \frac{5}{4}x + \frac{3}{2}$$

Parallel and Perpendicular Lines

Parallel lines have the same slope.



Perpendicular lines have slope that are negative reciprocals of each other.



Example 9

Find the equation of a line that passes through the given point and is:

- parallel to the given line
- perpendicular to the given line

$$\text{line : } 4x - 2y = 3$$

Point on the line: (2,1)

Find the slope the line.

$$4x - 2y = 3$$

$$4x - 4x - 2y = -4x + 3$$

$$-2y = -4x + 3$$

$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{3}{2}$$

$$y = 2x + \frac{3}{2}$$

$$m = 2, b = \frac{3}{2}$$

Slope of parallel line is the same as the line $\Rightarrow m_{\parallel} = 2$

Slope of the perpendicular line would be $m_{\perp} = -\frac{1}{2}$

Find the equations

Parallel Line

Use $m = 2$ and the point (2,1)

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x - 2)$$

$$y - 1 = 2x - 4$$

$$y = 2x - 3$$

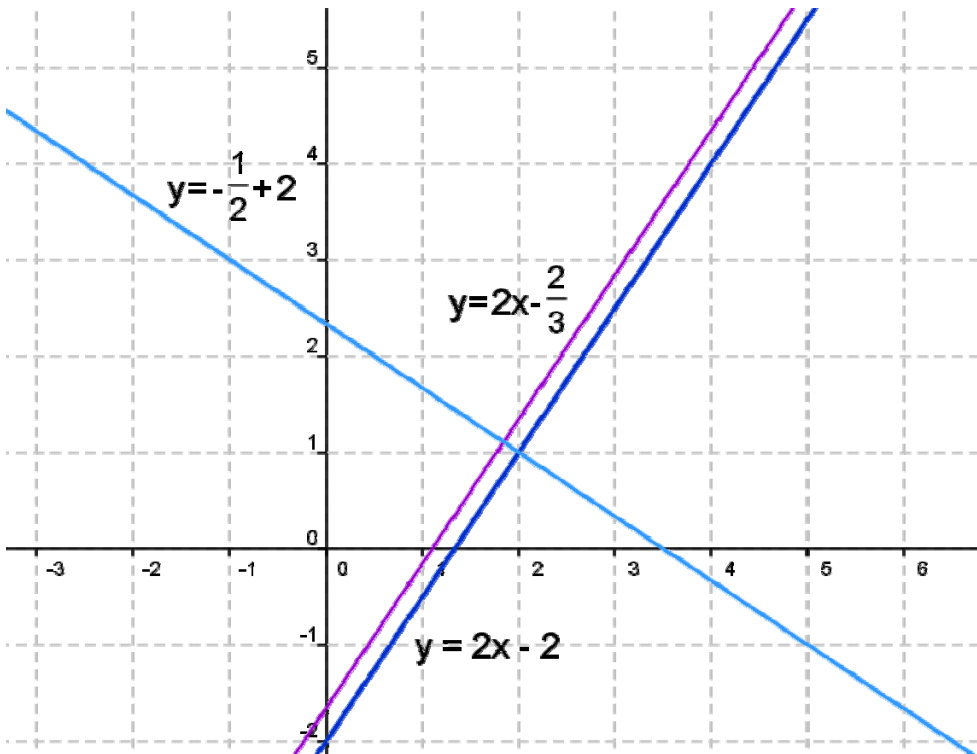
Perpendicular Line

Use $m = -\frac{1}{2}$ and the point (2,1)

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y - 1 = -\frac{1}{2}x + 1 \Rightarrow y = -\frac{1}{2}x + 2$$



Example 10

Sketch a graph of the equation $y - 3 = 2(x - 1)$

First solve the equation for y so it's in slope intercept form

$$y - 3 = 2(x - 1)$$

$$y - 3 = 2x - 2$$

$$y - 3 + 3 = 2x - 2 + 3$$

$$y = 2x + 1$$

Slope: $m = 2$

Intercept: $b = 1$

Graph:

