Math 151
Section 1.1
Slope and Intercept

Slope

The slope of a non-vertical line is measure of the number of units the line rises (or falls) for each unit of horizontal change from left to right.

\[ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \]

Note: Vertical lines do not have a slope
Example 1

Find the slope of a line passing through the given points.

A) \((1,1)\) and \((4,3)\)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{4 - 1} = \frac{2}{2} = 1
\]

B) \((1,4)\) and \((3,1)\)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{3 - 1} = \frac{-3}{2}
\]
C) \((2,3)\) and \((4,3)\)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{4 - 2} = \frac{0}{2} = 0 \]

D) \((1,2)\) and \((1,5)\)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{1 - 1} = \frac{3}{0} \quad \text{Slope is undefined} \]
Slope-intercept form of a linear equation

The slope intercept form of a linear equation is \( y = mx + b \) where \( m \) is the slope of the line and \( b \) is the y-intercept.

Point-slope form of a linear equation

The point-slope form of a linear equation is \( y - y_1 = m(x - x_1) \) where \( m \) is the slope of a line and \((x_1, y_1)\) is a point on the line.

Example 3

Use the slope of the line and given point to find three addition points that the line passes through.

Point: \((2,5)\)  Slope: \( m = 2 \)

First find the equation of the line using the point-slope formula.

\[
y - y_1 = m(x - x_1) \\
y - 5 = 2(x - 2) \\
y - 5 = 2x - 4 \\
y - 5 + 5 = 2x - 4 + 5 \\
y = 2x + 1
\]

Next, chose any 3 values for \( x \) and substitute in the equation \( y = 2x + 1 \) to get the \( y \) value.

In this example we will use the values of -1,0, and 1 for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2x + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>( y = 2(-1) + 1 = -2 + 1 = -1 )</td>
</tr>
<tr>
<td>0</td>
<td>( y = 2(0) + 1 = 0 + 1 = 1 )</td>
</tr>
<tr>
<td>1</td>
<td>( y = 2(1) + 1 = 2 + 1 = 3 )</td>
</tr>
</tbody>
</table>

Three points that lie on the line are \((-1,-1),(0,1),\) and \((1,3)\)
Example 4

Find the slope and y-intercept of the given line (if they exist)

\[ y = 2x + 1 \]

**Solution:**

\[ m = 2 \]
\[ b = 1 \]

Example 5

Find the slope and y-intercept of the given line (if they exist)

\[ 2y + 3x = 2 \]

In this example, we have to put the line in slope-intercept form first before we find the slope and y-intercept

\[ 2y + 3x = 2 \]
\[ 2y + 3x - 3x = -3x + 2 \]
\[ 2y = -3x + 2 \]
\[ \frac{2y}{2} = \frac{-3x}{2} + \frac{2}{2} \]
\[ y = -\frac{3}{2}x + 1 \]
\[ m = -\frac{3}{2} \]
\[ b = \frac{1}{2} \]

Example 6

Find the slope and y-intercept of the given line (if they exist)

\[ x = -3 \]

Since this equation does not have a variable of y, it cannot be put in slope-intercept form. Thus, the equation has a graph that is a vertical line which means that the slope is undefined and the line has no y-intercept. (See example 2 B)
Example 7

Find the equation of a line that passes through the given point that has the indicated slope.

\[(2,1): m = \frac{2}{3}\]

Simply substitute the values of the x and y coordinates of the point along with the slope into the point-slope formula.

\[y - y_1 = m(x - x_1)\]
\[y - 1 = \frac{2}{3}(x - 2)\]
\[y - 1 = \frac{2}{3}x - \frac{2}{3} \cdot 2\]
\[y - 1 = \frac{2}{3}x - \frac{4}{3}\]
\[y = \frac{2}{3}x - \frac{1}{3}\]

Example 8

Find the equation of a line that passes through the given points.

\[(2,4) \text{ and } (-2,-1)\]

First find the slope of the line using the slope formula

\[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 4}{-2 - 2} = \frac{-5}{-4} = \frac{5}{4}\]

Next, find the equation of the line by choosing a point on the line and substituting that point along with the slope into the point-slope formula.

\[y - y_1 = m(x - x_1)\]
\[y - 4 = \frac{5}{4}(x - 2)\]
\[y - 4 = \frac{5}{4}x - \frac{5}{4} \cdot 2\]
\[y - 4 = \frac{5}{4}x - \frac{5}{2} \Rightarrow y = \frac{5}{4}x + \frac{3}{2}\]
Parallel and Perpendicular Lines

Parallel lines have the same slope.

Perpendicular lines have slope that are negative reciprocals of each other.
Example 9

Find the equation of a line that passes through the given point and is:
  a) parallel to the given line
  b) perpendicular to the given line

Line: $4x - 2y = 3$
Point on the line: (2,1)

Find the slope the line.

$4x - 2y = 3$
$4x - 4x - 2y = -4x + 3$
$-2y = -4x + 3$
$\frac{-2y}{-2} = \frac{-4x + 3}{-2}$
$y = 2x + \frac{3}{2}$

$m = 2, \ b = \frac{3}{2}$

Slope of parallel line is the same as the line $\Rightarrow m_\parallel = 2$

Slope of the perpendicular line would be $m_\perp = \frac{-1}{2}$

Find the equations

**Parallel Line**

*Use $m = 2$ and the point (2,1)*

$y - y_1 = m(x - x_1)$
$y - 1 = 2(x - 2)$
$y - 1 = 2x - 4$
$y = 2x - 3$

**Perpendicular Line**

*Use $m = \frac{-1}{2}$ and the point (2,1)*

$y - y_1 = m(x - x_1)$
$y - 1 = \frac{-1}{2}(x - 2)$
$y - 1 = \frac{-1}{2}x + 1$
$y = \frac{-1}{2}x + 2$
$y = -\frac{1}{2}x + 2$

$y = 2x - \frac{3}{2}$

$y = 2x - 3$