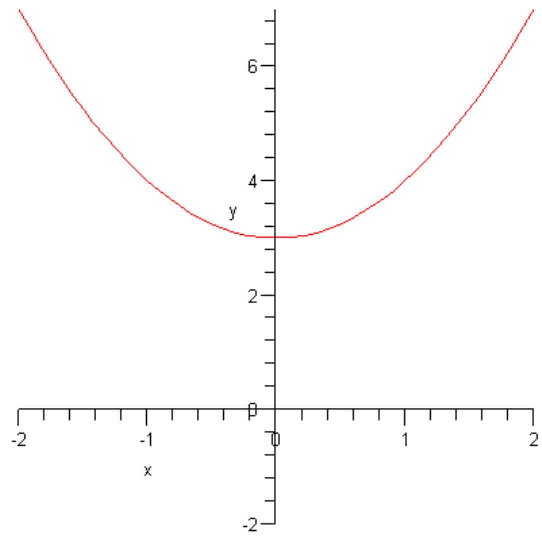
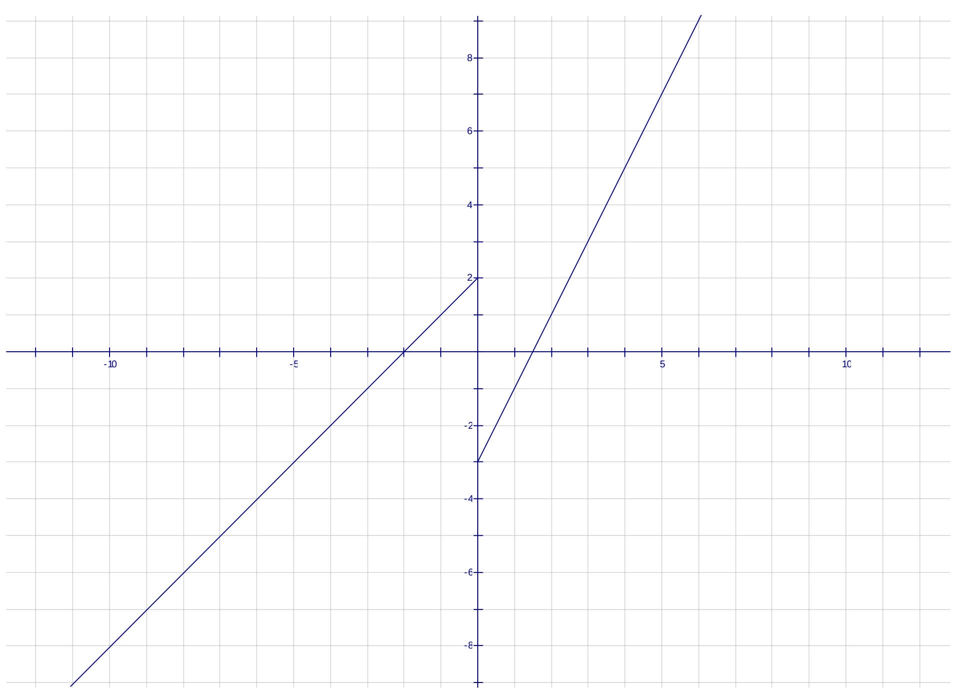
Math 151
Problem Set 2 Solutions

1) Graph the following equations

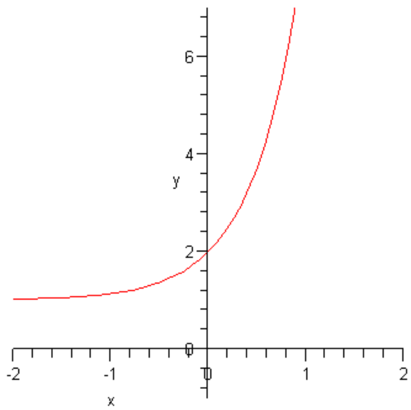
a) $f(x) = x^2 + 3$



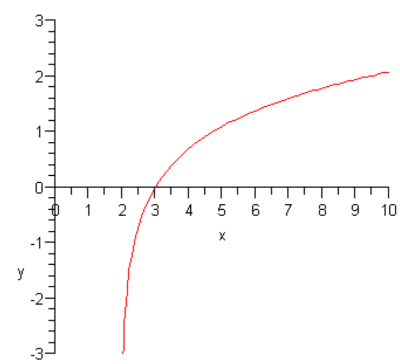
b) $f(x) = \begin{cases} 2x - 3, & x \geq 0 \\ x + 2, & x < 0 \end{cases}$



c) $f(x) = e^{2x} + 1$



d) $f(x) = \ln(x - 2)$



2) What is the domain of $f(x) = \ln(5x - 15)$

Domain: $(3, \infty)$

3) Find the inverse function of $f(x) = 3x - 4$

$$f(x) = 3x - 4$$
$$y = 3x - 4$$
$$x = 3y - 4$$
$$x + 4 = 3y$$
$$\frac{x+4}{3} = \frac{3y}{3}$$
$$y = \frac{x+4}{3}$$
$$f'(x) = \frac{x+4}{3}$$

4) Find the inverse function of $f(x) = \frac{x+2}{x-3}$

$$f(x) = \frac{x+2}{x-3}$$
$$y = \frac{x+2}{x-3}$$
$$x = \frac{y+2}{y-3}$$
$$x(y-3) = y + 2$$
$$xy - 3x = y + 2$$
$$xy - y = 3x + 2$$
$$y(x-1) = 3x + 2$$
$$y = \frac{3x+2}{x-1}$$

5) The growth of Blacksburg, Virginia is modeled by the function $P(t) = 40{,}000e^{.02t}$ where t is the time in years.

a) Find the population of Blacksburg in 10 years?

$$P(t) = 40{,}000e^{.02t}$$
$$P(10) = 40{,}000e^{.02(10)}$$
$$P(10) = 40{,}000e^{.2}$$
$$P(10) = 48856$$

b) Using this model how long will it take the population to reach 50,000?

$$P(t) = 40{,}000e^{.02t}$$
$$50{,}000 = 40{,}000e^{.02t}$$
$$\frac{50{,}000}{40{,}000} = \frac{40{,}000}{40{,}000}e^{.02t}$$
$$1.25 = e^{.02t}$$
$$\ln(1.25) = \ln e^{.02t}$$
$$\ln(1.25) = .02t \ln e$$
$$\ln(1.25) = .02t \Rightarrow t = \frac{\ln(1.25)}{.02} \Rightarrow t \approx 11 \; years$$

6) Find $\log_6 216$

$$\log_6 216 \Rightarrow 6^x = 216 \Rightarrow 6^x = 6^3 \Rightarrow x = 3$$

7) Find $\log_{11} 1000$

$$\log_{11} 1000$$
$$x = \frac{\log(1000)}{\log(11)} = 2.88$$

8) Solve $3^{2x-3} = 81$

$$3^{2x-3} = 81$$
$$3^{2x-3} = 3^4$$
$$2x - 3 = 4$$
$$2x = 3 + 4$$
$$2x = 7$$
$$\frac{2x}{2} = \frac{7}{2}$$
$$x = \frac{7}{2}$$

9) Solve $e^{3x+4} = 12$

$$e^{3x+4} = 12$$
$$\ln e^{3x+4} = \ln 12$$
$$(3x+4)\ln e = \ln 12$$
$$3x + 4 = \ln 12$$
$$3x = \ln 12 - 4$$
$$x = \frac{\ln 12 - 4}{3}$$
$$x \approx -.5$$

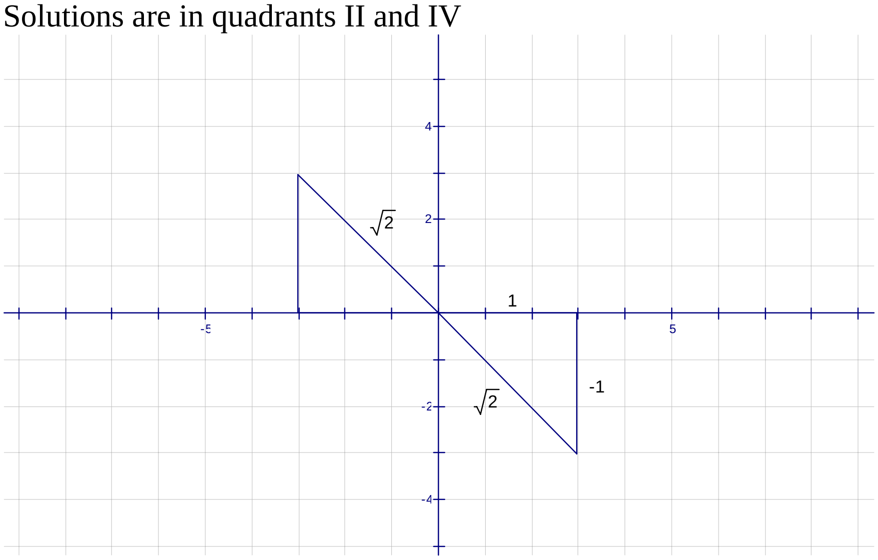
10) Solve $\ln(2x-1)=2$

$$\ln(2x-1)=2$$
$$e^{\ln(2x-1)}=e^2$$
$$2x-1=7.39$$
$$2x=7.39+1$$
$$2x=8.39$$
$$\frac{2x}{2}=\frac{8.39}{2}$$
$$x=4.2$$

11) Find $\cot^{-1}(-1)$

$$\cot A=-1$$
$$x=-y$$

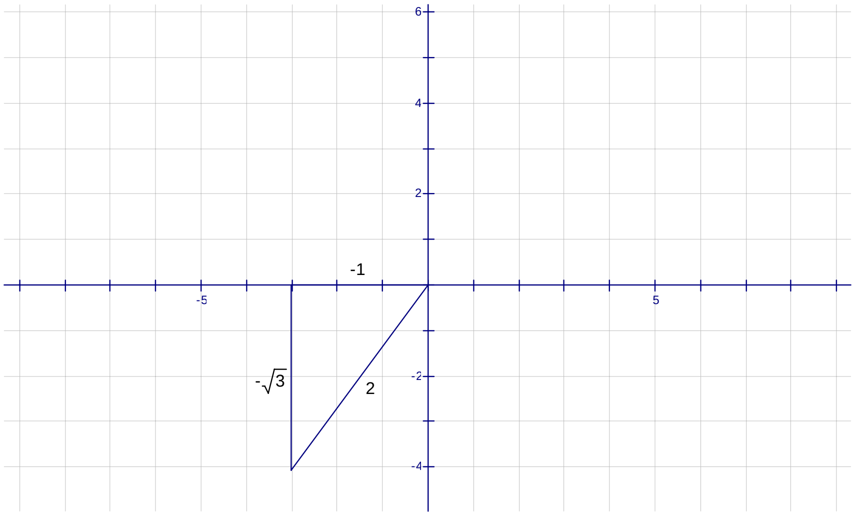
Solutions are in quadrants II and IV



So, use $x=-1$ *and* $y=1$ *and* $x=1$ *and* $y=-1$

$$A=135^0,\ 315^0$$
$$A=\frac{3\pi}{4}, A=\frac{7\pi}{4}$$

12) Find $\sin\left(\frac{4\pi}{3}\right)$



$$\sin\left(\frac{4\pi}{3}\right) = \frac{y}{r} = -\frac{\sqrt{3}}{2}$$

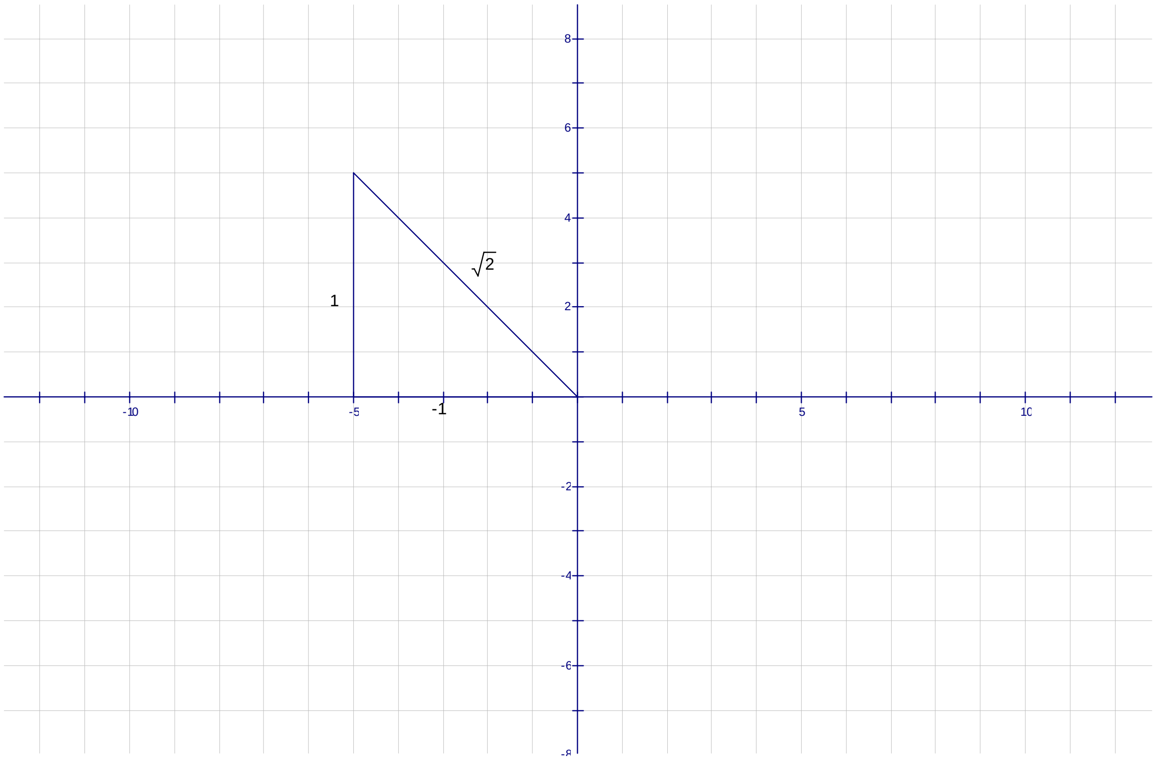
13) Convert $\frac{7\pi}{4}$ to degrees

$$\frac{7\pi}{4} \cdot \frac{180}{\pi} = 7(45) = 315$$

14) Convert $330°$ to radians

$$330° \cdot \frac{\pi}{180} = \frac{11\pi}{6}$$

15) Given the radian measure of $\frac{3\pi}{4}$ in standard position, find the values of the six trigonometric values.



$$\sin\left(\frac{3\pi}{4}\right)=\frac{y}{r}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{3\pi}{4}\right)=\frac{x}{r}=\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$

$$\tan\left(\frac{3\pi}{4}\right)=\frac{1}{-1}=-1$$

$$\csc\left(\frac{3\pi}{4}\right)=\frac{\sqrt{2}}{1}=\sqrt{2}$$

$$\sec\left(\frac{3\pi}{4}\right)=\frac{\sqrt{2}}{-1}=-\sqrt{2}$$

$$\cot\left(\frac{3\pi}{4}\right)=\frac{-1}{1}=-1$$

16) Given that $\sin\theta = \dfrac{12}{13}$ and the angle θ lies in Quadrant II, find the value of the other five trigonometric functions.

$$\sin\theta = \frac{12}{13} \Rightarrow y = 12 \ and \ r = 13$$
$$r^2 = x^2 + y^2$$
$$13^2 = 12^2 + y^2$$
$$169 = 144 + y^2$$
$$169 - 144 = y^2$$
$$y^2 = 25$$
$$\sqrt{y^2} = \sqrt{25}$$
$$y = \pm 5 \Rightarrow y = -5 \ (Quadrant \ II)$$

$$\cos\theta = -\frac{5}{13}, \ \tan\theta = -\frac{12}{5}, \csc\theta = \frac{13}{12}, \sec\theta = -\frac{13}{5}, \cot\theta = -\frac{5}{12}$$

17) Solve $4\cos^2\theta - 1 = 0$. **Limit solutions to Quadrants I and II (**$0 \le \theta \le \pi$**)**

$$4\cos^2\theta - 1 = 0$$
$$4\cos^2\theta = 1$$
$$\sqrt{4\cos^2\theta} = \sqrt{1}$$
$$2\cos\theta = \pm 1$$
$$\cos\theta = \pm\frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

Class Notes

1) A certain bacteria is modeled by the function $P(x) = 300e^{\frac{t}{2}}$ where t represents time in months. Use the model to answer the following questions.

a) How bacteria would there be in 6 months?

$$P(6) = 300e^{\frac{6}{2}} = 300e^{3} \approx 6026$$

b) How long would it take the bacteria to double in size?

$$P(x) = 300e^{\frac{t}{2}}$$

$$600 = 300e^{\frac{t}{3}}$$

$$\frac{600}{300} = \frac{300e^{\frac{t}{3}}}{300}$$

$$2 = e^{\frac{t}{3}}$$

$$\ln 2 = \ln e^{\frac{t}{3}}$$

$$\ln 2 = \frac{t}{3} \cdot \ln e$$

$$\ln 2 = \frac{t}{3}$$

$$t = 3\ln 2 \approx 1.4 \; months$$

c) How long will take to reach 20,000?

$$P(x) = 300e^{\frac{t}{2}}$$

$$2000 = 300e^{\frac{t}{2}}$$

$$\frac{20000}{300} = \frac{300e^{\frac{t}{3}}}{300}$$

$$66.7 = e^{\frac{t}{2}}$$

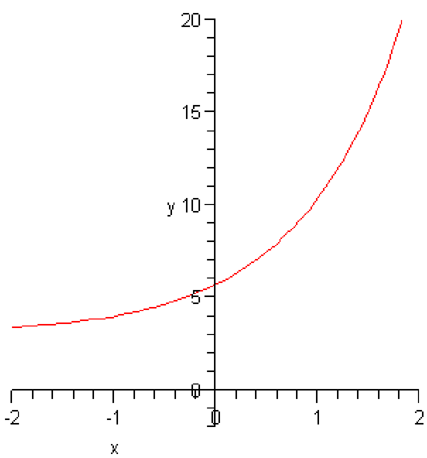
$$\ln(66.7) = \ln e^{\frac{t}{2}}$$

$$\ln(66.7) = \frac{t}{2} \cdot \ln e$$

$$\ln(66.7) = \frac{t}{2}$$

$$t = 2\ln(66.7) \approx 8.4 \ months$$

2) Graph $y = e^{x-1} + 3$



3) Find the inverse of $f(x) = \frac{x-3}{2x-1}$

$$f(x) = \frac{x-3}{2x-1}$$

$$y = \frac{x-3}{2x-1}$$

$$x = \frac{y-3}{2y-1}$$

$$x(2y-1) = \frac{y-3}{2y-1}(2y-1)$$

$$2xy - x = y - 3$$

$$2xy - y = x - 3$$

$$2y(x-1) = x - 3$$

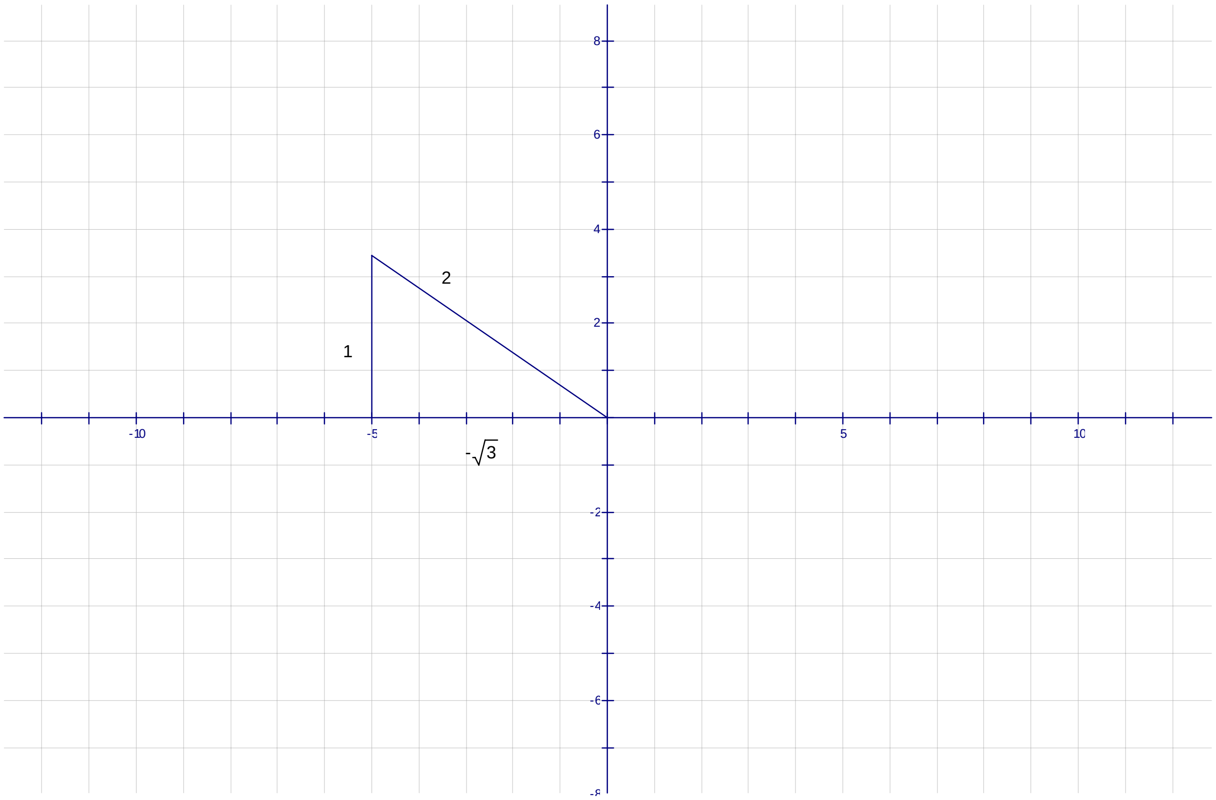
$$\frac{2y(x-1)}{x-1} = \frac{x-3}{x-1}$$

$$2y = \frac{x-3}{x-1}$$

$$y = \frac{x-3}{2(x-1)}$$

$$f^{-1}(x) = \frac{x-3}{2x-2}$$

4) Given the radian measure of $\frac{5\pi}{6}$ in standard position, find the values of the six trigonometric values.



$$\sin\left(\frac{5\pi}{6}\right) = \frac{y}{r} = \frac{1}{2}$$

$$\cos\left(\frac{5\pi}{6}\right) = \frac{x}{r} = -\frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{5\pi}{6}\right) = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\csc\left(\frac{5\pi}{6}\right) = \frac{2}{1} = 2$$

$$\sec\left(\frac{5\pi}{6}\right) = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\cot\left(\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$