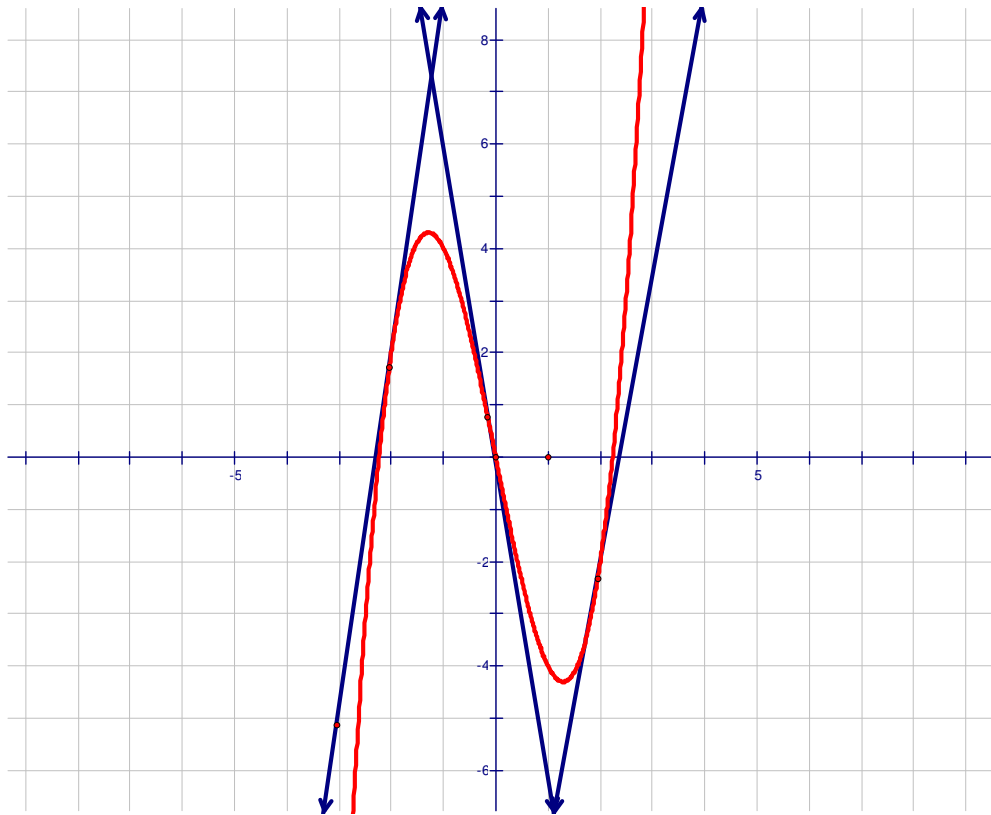


Math 151

Increasing and Decreasing Functions



Test for increasing and decreasing functions

Let f be a differentiable function on the interval

- 1) If $f'(x) > 0$ for all x in (a,b) , then f is increasing on (a,b)
 - 2) If $f'(x) < 0$ for all x in (a,b) , then f is decreasing on (a,b)
 - 3) If $f'(x) = 0$ for all x in (a,b) , then f is constant on (a,b)
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Example 1

Give the intervals where the function is either increasing or decreasing

$$f(x) = x^2 - 8x$$

$$f'(x) = 2x - 8$$

$$2x - 8 = 0$$

$$2x - 8 + 8 = 0 + 8$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

$$f'(2) = 2(2) - 8 = 4 - 8 = -4$$

$$f'(3) = 2(5) - 8 = 10 - 8 = 2$$

| | | |
|-----------------|----------------|---------------|
| Interval | $(-\infty, 4)$ | $(4, \infty)$ |
| Test Value | $x = 2$ | $x = 5$ |
| Sign of $f'(x)$ | Negative | Positive |
| Conclusion | Decreasing | Increasing |

Example 2

Find the intervals where the function is either decreasing or increasing

$$f(x) = x^3 - 4$$

$$f'(x) = 3x^2$$

$$3x^2 = 0$$

$$\frac{3x^2}{3} = \frac{0}{3}$$

$$x^2 = 0$$

$$x = 0$$

$$f'(-1) = 3(-1)^2 = 3$$

$$f'(1) = 3(1)^2 = 3$$

| | | |
|-----------------|----------------|---------------|
| Interval | $(-\infty, 0)$ | $(0, \infty)$ |
| Test Value | $x = -1$ | $x = 1$ |
| Sign of $f'(x)$ | Positive | Positive |
| Conclusion | Increasing | Increasing |

Example 3

Find the intervals where the function is either increasing or decreasing.

$$f(x) = x^3 - 3x^2$$

$$f'(x) = 3x^2 - 6x$$

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$3x = 0 \text{ or } x - 2 = 0$$

$$\frac{3x}{3} = \frac{0}{3} \text{ or } x - 2 + 2 = 0 + 2$$

$$x = 0 \qquad x = 2$$

$$f'(-1) = 3(-1)^2 - 6(-1) = 3 + 6 = 9$$

$$f'(1) = 3(1)^2 - 6(1) = 3 - 6 = -3$$

$$f'(3) = 3(3)^2 - 6(3) = 27 - 18 = 9$$

| | | | |
|-----------------|----------------|------------|---------------|
| Interval | $(-\infty, 0)$ | $(0, 2)$ | $(2, \infty)$ |
| Test Value | $x = -1$ | $x = 1$ | $x = 3$ |
| Sign of $f'(x)$ | Positive | Negative | Positive |
| Conclusion | Increasing | Decreasing | Increasing |

Example 4

Find the intervals where the function is either increasing or decreasing on the interval $[0, 2\pi]$

$$f(x) = \cos(x) + 3$$

$$f(x) = \cos(x) + 3$$

$$f'(x) = -\sin(x)$$

$$-\sin(x) = 0$$

$$\sin(x) = 0$$

$$x = \arcsin(x)$$

$$x = 0, \pi, 2\pi$$

$$f'\left(\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -1$$

| | | |
|-----------------|---------------------|----------------------|
| Interval | $(0, \pi)$ | $(\pi, 2\pi)$ |
| Test Value | $x = \frac{\pi}{2}$ | $x = \frac{3\pi}{2}$ |
| Sign of $f'(x)$ | Negative | Negative |
| Conclusion | Decreasing | Increasing |

Example 5

Determine the intervals where the functions is either increasing or decreasing.

$$f(x) = e^{2x} + 1$$

$$f(x) = e^{2x} + 1$$

$$f'(x) = 2e^{2x}$$

Notice that $f'(x) = 2e^{2x}$ is never zero

In fact, $f'(x) = 2e^{2x} > 0$

Therefore, f is increasing for all x .