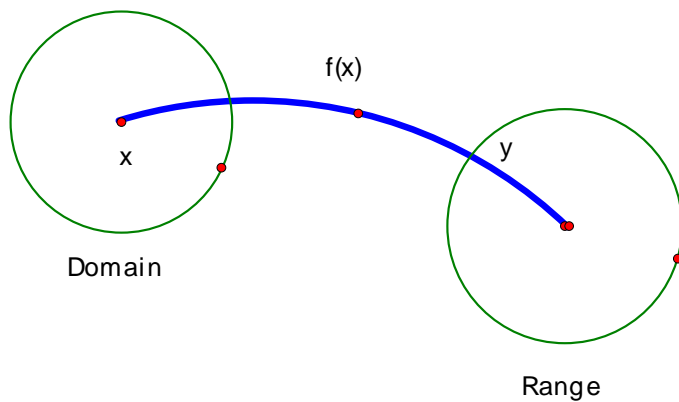


Section 1.2 Functions and Their Graphs

A **relation** between two sets X and Y is a set of ordered pairs in the form (x, y) where $x \in X$ and $y \in Y$

The **domain** of relation is the set of the first coordinates or x-values in the relation.

The **range** of relation is the set of the second coordinates or y-values in the relation.



A **real-value function f of a real variable x** from X to Y is a correspondence that assigns to each number x in X exactly one number y in Y .

Example 1 (Evaluating a Function)

Given the function, $f(x) = x^2 + 3x + 4$, evaluate the following functions.

A) Find $f(2)$

$$f(2) = 2^2 + 3 \cdot 2 + 4 = 4 + 6 + 4 = 14$$

B) Find $f(3b)$

$$f(3b) = (3b)^2 + 3 \cdot 3b + 4 = 9b^2 + 9b + 4$$

C) Find $f(x+h)$

$$f(x+h) = (x+h)^2 + 3(x+h) + 4 = (x+h)(x+h) + 3x + 3h + 4 = x^2 + 2xh + h^2 + 3x + 3h + 4$$

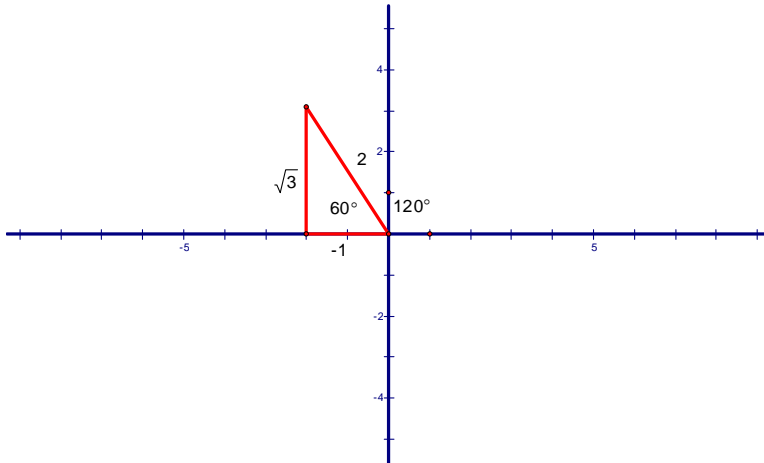
Rearrange the terms to get the following function: $f(x) = x^2 + h^2 + 2xh + 3x + 3h + 4$

Example 2

Given the following function, $g(x) = \sin 2x$, evaluate the following functions.

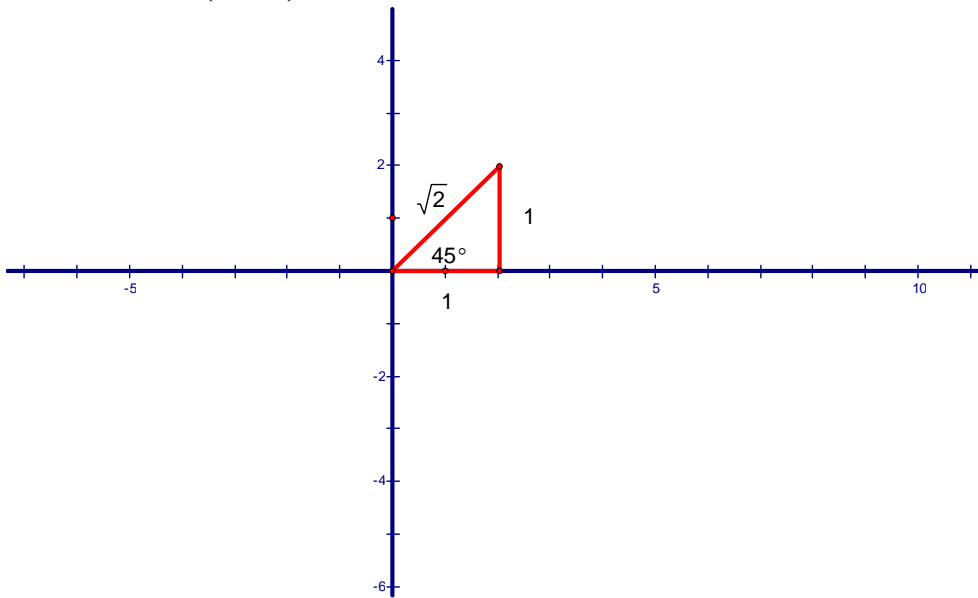
A) Evaluate $g\left(\frac{\pi}{3}\right)$

$$g\left(\frac{\pi}{3}\right) = \sin\left(2\left(\frac{\pi}{3}\right)\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$



B) Evaluate $g\left(\frac{\pi}{8}\right)$

$$g\left(\frac{\pi}{8}\right) = \sin\left(2\left(\frac{\pi}{8}\right)\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$



C) Evaluate $g(0)$

$$g(0) = \sin(2 \cdot 0) = \sin(0) = 0$$

Implicit form of a function: $x^2 + y = 3$

Explicit form of a function: $y = 3 - x^2$ (The equation is solve for y in terms)

How to find the domain and range of a function

Example 3

Find the domain and range of each function.

1) $f(x) = x^2 + 5$

Domain: $(-\infty, \infty)$ or $\{x : -\infty < x < \infty\}$

Range: $(5, \infty)$ or $\{y : y \geq 5\}$

2) $f(x) = -\sqrt{2x + 5}$

Domain: $\left[\frac{5}{2}, \infty\right)$ or $\left\{x : x \geq \frac{5}{2}\right\}$

Range: $[0, \infty)$ or $\{y : y \geq 0\}$

3) $f(x) = \sin 2x$

Domain: $(-\infty, \infty)$ or $\{x : -\infty < x < \infty\}$

Range: $[-1, 1]$ or $\{y : -1 \geq y \geq 1\}$

Composition of two functions

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

Example 4

Given the following functions:

$$f(x) = 2x + 3$$

$$g(x) = x^2 + 4$$

Evaluate each of the following functions.

1) Evaluate $f(g(2))$

$$f(g(2)) = f(2^2 + 4) = f(4 + 4) = f(8) = 2(8) + 3 = 16 + 3 = 19$$

2) Evaluate $(g \circ f)(-1)$

$$(g \circ f)(-1) = g(f(-1)) = g(2(-1) + 3) = g(1) = 1^2 + 4 = 1 + 4 = 5$$

3) Evaluate $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 4) = 2(x^2 + 4) = 2x^2 + 8$$

4) Evaluate $(g \circ f)(x)$

$$(g \circ f)(x)$$

$$= g(f(x))$$

$$= g(2x + 3)$$

$$= (2x + 3)^2 + 4$$

$$= (2x + 3)(2x + 3) + 4$$

$$= 4x^2 + 6x + 6x + 9 + 4$$

$$= 4x^2 + 12x + 13$$

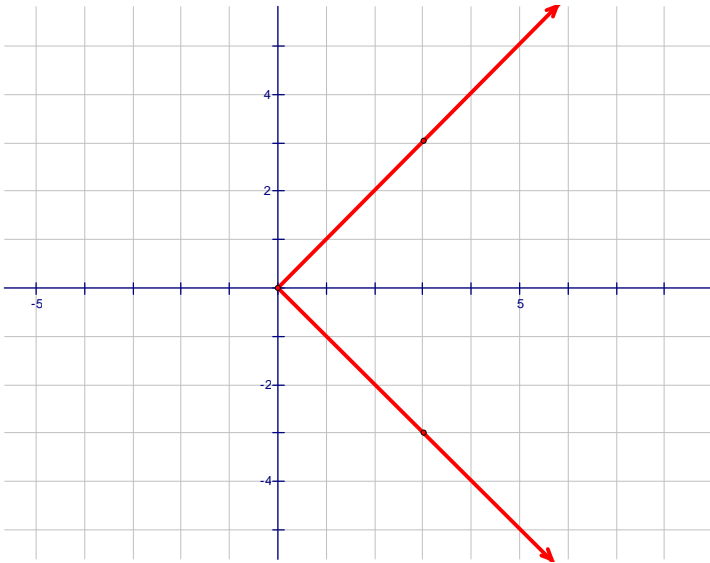
5) Evaluate $(f \circ f)(x)$

$$(f \circ f)(x) = f(f(x)) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 6 + 3 = 4x + 9$$

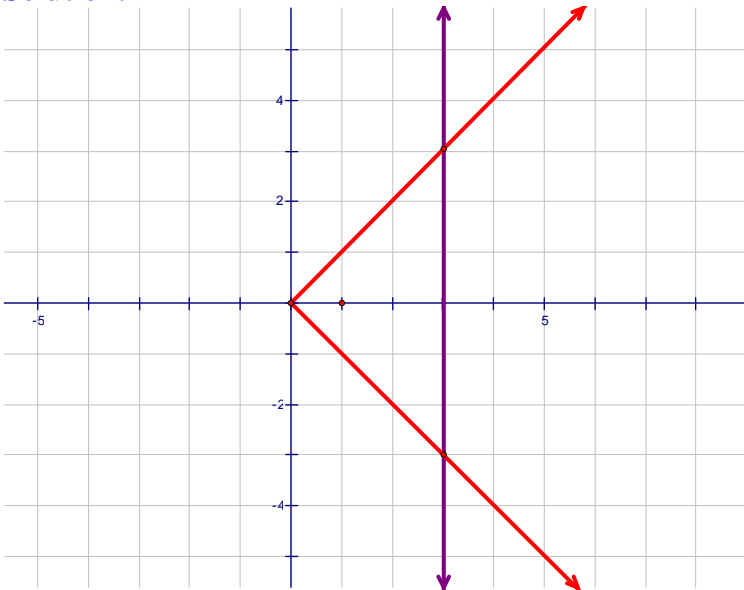
Vertical Line Test

If a vertical line can be drawn so that it intersects the graph a relation 2 or more times, then the relation is not a function.

Examples 5 Use the graph to determine if the relation is a function, and give the domain and range.



Solution:

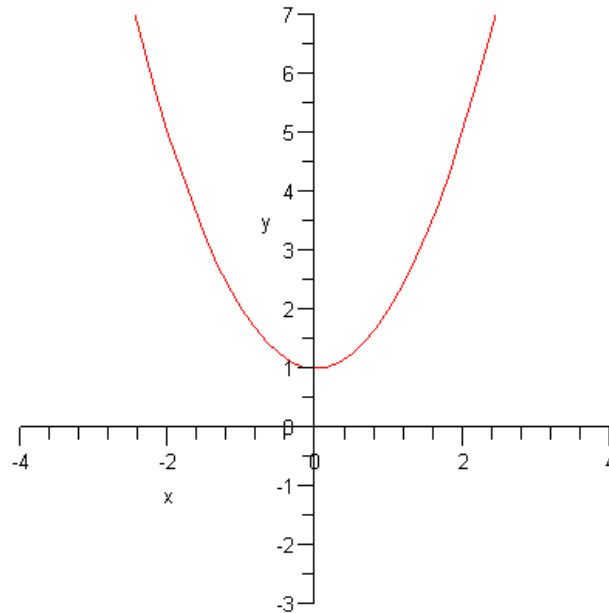


This is not a function, since the relation fails the vertical line test.

Domain: $(0, \infty)$, **Range:** $(-\infty, \infty)$

Example 6

Use the graph to determine if the relation is a function, and give the domain and range.



$$f(x) = x^2 + 1$$

This is a function since every vertical line on the graph would intersect the relation only once. Domain: $(-\infty, \infty)$; Range: $[1, \infty)$

Example 7

Determine whether y is a function of x .

$$y + x^2 = x + 5$$

Solution: Solve for y :

$$y = x^x - x^2 = -x^2 + x + 5$$

$$y = -x^2 + x + 5$$

Yes, this is a function of x .

Example 8

Determine if y is a function of x .

$$x^2 + y^2 = 9$$

Solution: Solve for y

$$x^2 + y^2 = 9$$

$$y^2 = 9 - x^2$$

$$y = \pm\sqrt{9 - x^2}$$

This is not a function. Therefore, y is not a function of x .
