

**Math 151**  
**Section 2.2**

**Derivatives of polynomials**

**Review of the limit definition of a derivative.**

$f(x)$	$f'(x)$
$f(x) = 4x + 5$	$f'(x) = 4$
$f(x) = x^2 - 2x$	$f'(x) = 2x - 2$
$f(x) = x^3 + 4$	$f'(x) = 3x^2$
$f(x) = 2x^4 - 4x^3$	$f'(x) = 8x^3 - 12x^2$

**Proof of the power rule**

Let  $f(x) = cx^n$ , and find  $f'(x)$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{c(x+h)^n - cx^n}{h} \\
 &= \lim_{h \rightarrow 0} \frac{c \left[ x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots + nxh^{n-1} + h^n \right] - cx^n}{h} \\
 &= c \cdot \lim_{h \rightarrow 0} \frac{nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots + h^n}{h} \\
 &= c \cdot \lim_{h \rightarrow 0} \frac{h \left( nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \dots + h^{n-1} \right)}{h} \\
 &= c \cdot \lim_{h \rightarrow 0} \left( nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \dots + h^{n-1} \right) \\
 &= c \left( nx^{n-1} + 0 + 0 + \dots + 0 \right) \\
 &= cnx^{n-1}
 \end{aligned}$$


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**Rule 1 (Power Rule):**  $\frac{d}{dx}(cx^n) = cnx^{n-1}$

**Rule 2 (Constant Rule):**  $\frac{d}{dx}(c) = 0$  where **c** is a constant

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**Examples using the power rule.**

**Example 1**

If  $f(x) = 2x^3$ , then find  $f'(x)$  using the power rule.

$$f'(x) = 3 \cdot 2x^{3-1} = 6x^2$$

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**Example 2**

If  $f(x) = x^4 + 2x^3 + 3x$ , then find  $f'(x)$  using the power rule.

$$f'(x) = 4x^{4-1} + 3 \cdot 2x^{3-1} + 3 = 4x^3 + 6x^2 + 3$$

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**Example 3**

If  $f(x) = \frac{1}{x^2}$ , then find  $f'(x)$  using the power rule.

$$f(x) = \frac{1}{x^2} = x^{-2}$$

$$f'(x) = -2x^{-2-1} = -2x^{-3} = -\frac{2}{x^3}$$

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**Example 4**

If  $f(x) = \frac{1}{x^3} + \frac{4}{x^2}$ , then find  $f'(x)$  using the power rule.

$$f(x) = \frac{1}{x^3} + \frac{4}{x^2} = x^{-3} + 4x^{-2}$$

$$f'(x) = -3x^{-3-1} + (-2)4x^{-2-1} = -3x^{-4} - 8x^{-3} = -\frac{3}{x^4} - \frac{8}{x^3}$$

**Example 5**

If  $f(x) = \sqrt{x}$ , then find  $f'(x)$  using the power rule.

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

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**Example 6**

If  $f(x) = 6\sqrt{x}$ , then find  $f'(x)$  using the power rule.

$$f(x) = 6\sqrt{x} = 6x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(6)x^{\frac{1}{2}-1} = 3x^{-\frac{1}{2}} = \frac{3}{x^{\frac{1}{2}}} = \frac{3}{\sqrt{x}}$$

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**Example 7**

If  $f(x) = 2\sqrt[3]{x}$ , then find  $f'(x)$  using the power rule.

$$f(x) = 2\sqrt[3]{x} = 2x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} \cdot 2x^{\frac{1}{3}-1} = \frac{2}{3} x^{-\frac{2}{3}} = \frac{2}{3x^{\frac{2}{3}}} = \frac{2}{3\sqrt[3]{x^2}}$$

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**Example 8**

Find the equation of a tangent line to the function  $f(x) = x^2 + x$  through the point (1,2).

Find the derivative of  $f(x) = x^2 + x$

$$f'(x) = 2x + 1$$

Find the slope of the tangent line:  $m = f'(1) = 2(1) + 1 = 2 + 1 = 3$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 3(x - 1)$$

$$y - 2 = 3x - 3$$

$$y - 2 + 2 = 3x - 3 + 2$$

$$y = 3x - 1$$

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**Example 9**

Find the equation of a tangent line to the function  $f(x) = x^3 + e^x$  through the point (0,1).

Find the derivative of  $f(x) = x^3 + e^x$

$$f'(x) = 3x^2 + e^x$$

Find the slope of the tangent line:  $m = f'(0) = 3(0)^2 + e^0 = 0 + 1 = 1$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 1(x - 0)$$

$$y - 1 = x$$

$$y - 1 + 1 = x + 1$$

$$y = x + 1$$

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**Example 10**

The equation of motion of a particle is  $s(t) = t^3 - 4t$  where  $s$  is in meters and  $t$  is seconds.

- a) Find the velocity and acceleration as a function of  $t$ .

$$v(t) = s'(t) = 3t^{3-1} - 4 = 3t^2 - 4$$

$$a(t) = s''(t) = 6t$$

- b) Find the velocity after 2 seconds.

$$v(t) = 3t^{3-1} - 4 = 3t^2 - 4$$

$$v(2) = 3(2)^2 - 4 = 12 - 4 = 8 \frac{m}{s}$$

- c) Find the acceleration after 2 seconds.

$$a(t) = s''(t) = 6t$$

$$a(2) = 6(2) = 12 \frac{m}{s^2}$$

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