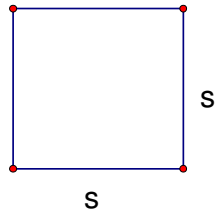
Area and Perimeter Unit

Key Terms

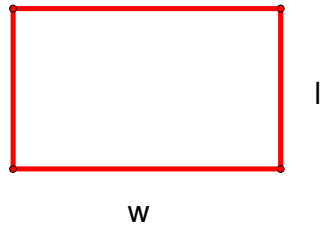
The Perimeter of a Polygon is the sum of the lengths of its sides.

Perimeter Formulas

Square: $P = 4s$

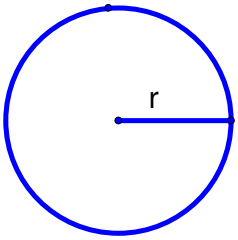


Rectangle: $P = 2l + 2w$



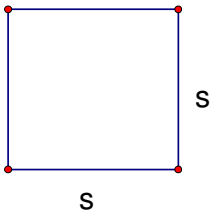
The perimeter of a circle is the product of Pi and the diameter of the circle

Circle: $A = \pi d = 2\pi r$

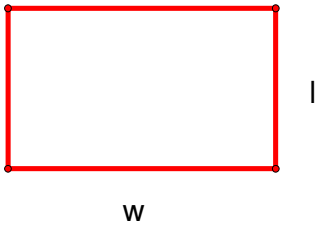


Area

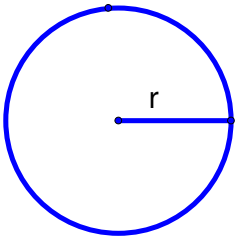
Area of Square: $A = s^2$



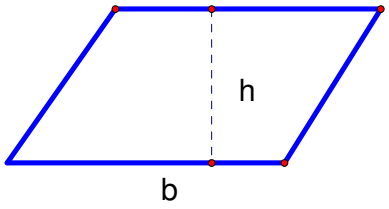
Area of Rectangle: $A = l \cdot w$



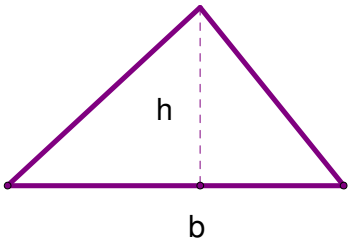
Area of a Circle: $A = \pi \cdot r^2$



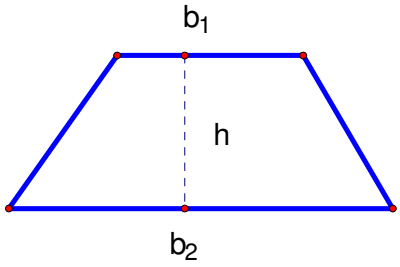
Area of a Parallelogram: $A = b \cdot h$



Area of a Triangle: $A = \frac{1}{2}bh$

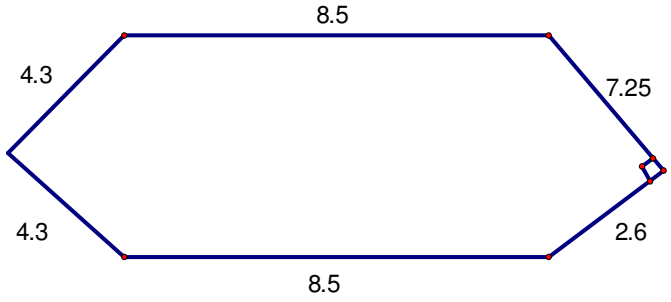


Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$



Example 1

Find the area and perimeter of the following Object. All measurements are in feet.



Perimeter: $P = 3.1 + 3.1 + 8.5 + 2.6 + 7.25 + 8.5 = 33.05\ ft$

Area:

$c^2 = a^2 + b^2$

$c^2 = 2.6^2 + 7.25^2$

$c^2 = 6.76 + 52.5625$

$c^2 = 59.3225$

$c = 7.70$

Area of the rectangle: $A = 7.70 \ x \ 8.5 = 65.45 \ ft^2$

Area of triangle #1: $A = \frac{1}{2}(2.6)(7.25) = 9.425 \ ft^2$

Height of a triangle #2:

$c^2 = a^2 + b^2$

$4.3^2 = a^2 + 3.35^2$

$18.49 = a^2 + 11.2225$

$18.49 - 11.2225 = a^2 + 11.225 - 11.2225$

$7.2675 = a^2$

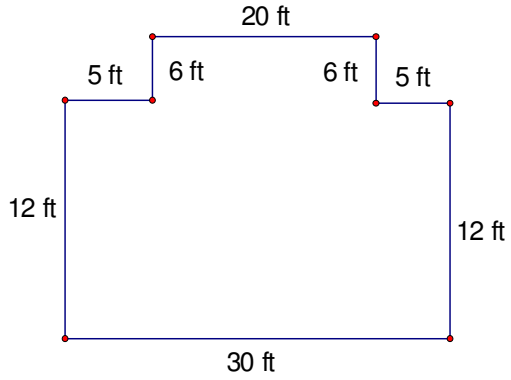
$a = \sqrt{7.2675} \approx 2.70$

Area of triangle #2: $A = \frac{1}{2}(3.35)(2.70) = 4.5225 \ ft^2$

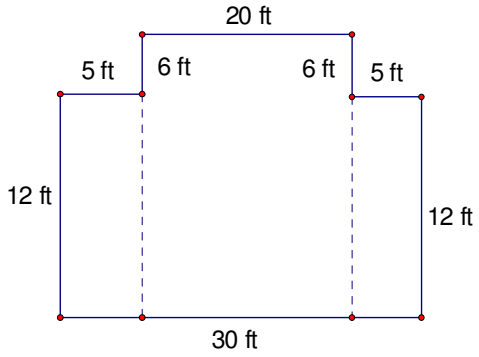
$Area = 65.45 + 9.425 + 4.5225 = 79.3975 \ ft^2$

Example 2

Given the following floor plans of the first level of a house, below find the area of the first floor of the house



Divide into three rectangles:



$A_1 = (12\ ft)(5\ ft) = 60\ ft^2$

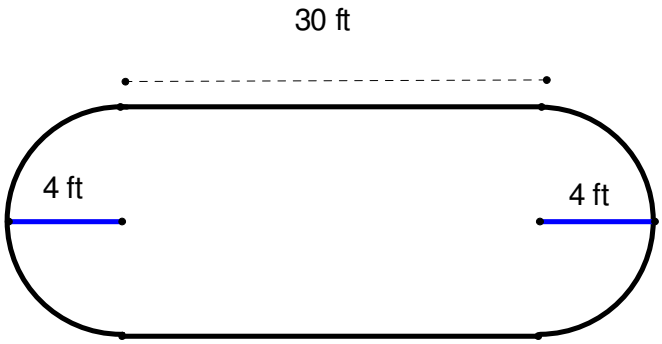
$A_2 = (12\ ft)(5\ ft) = 60\ ft^2$

$A_1 = (18\ ft)(20\ ft) = 360\ ft^2$

$Area = 60 + 60 + 360 = 480\ ft^2$

Example 3

Find the perimeter and area of the following region.



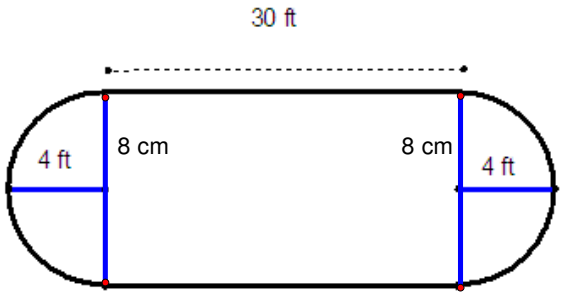
Perimeter of the region:

$$P = 2(30\,ft) + 2\pi(4\ ft) = 60\ ft + 8\pi\ ft = 85.12\ ft$$

Area of region:

$$Area\ of\ \ cirlce = \pi(4)^2 = 16\pi$$

Area of rectangle:

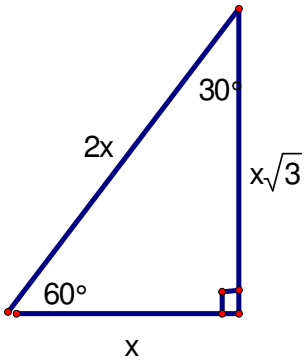


$$A = 30\,ft\ \ x\ 8\ ft = 240\ \ ft^2$$

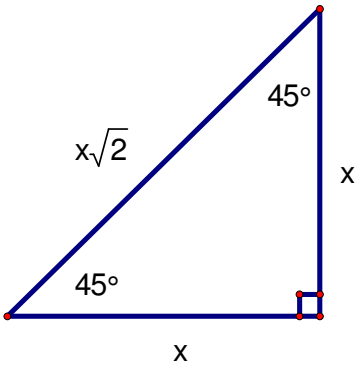
$$A = 240 + 16\pi = 290.24$$

Special Right Triangles

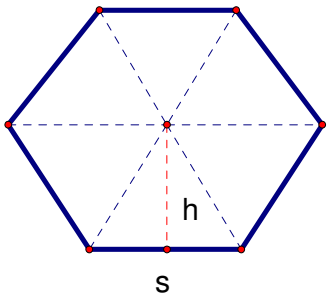
$30° - 60° - 90°$ **Triangle**



$45° - 45° - 90°$ **Triangle**



Area of Regular Polygon

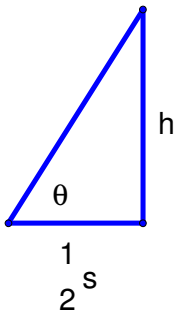


$A = \frac{1}{2}Ph$

$P = Perimeter$

$h = height$

Recall that in Trigonometry the sine ratio is opposite over hypotenuse. Thus, we can find the height by the following formula.



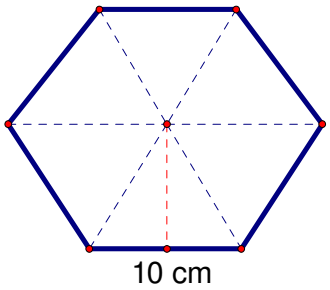
$$\tan\theta = \frac{opp.}{adj.} = \frac{h}{\frac{1}{2}s}$$

$$\tan\theta = \frac{h}{\frac{1}{2}s}$$

$$\Rightarrow h = \frac{1}{2}s(\tan\theta)$$

Example 4

Find the area of a regular hexagon with sides that have a length of 10 cm.



Step 1: Find the height

Find the measure of the interior angle of the hexagon.

$$I = \frac{(n-2)\cdot 180}{n} = \frac{(6-2)\cdot 180}{6} = \frac{4\cdot 180}{6} = \frac{720}{6} = 120°$$

The angle θ is half the measure of the interior angle.

$$\theta = \frac{1}{2}(120°) = 60°$$

Now, we can find the height

$$h = \frac{1}{2}(10)(\tan(60)) = 5\cdot\frac{\sqrt{3}}{1} = 5\sqrt{3}$$

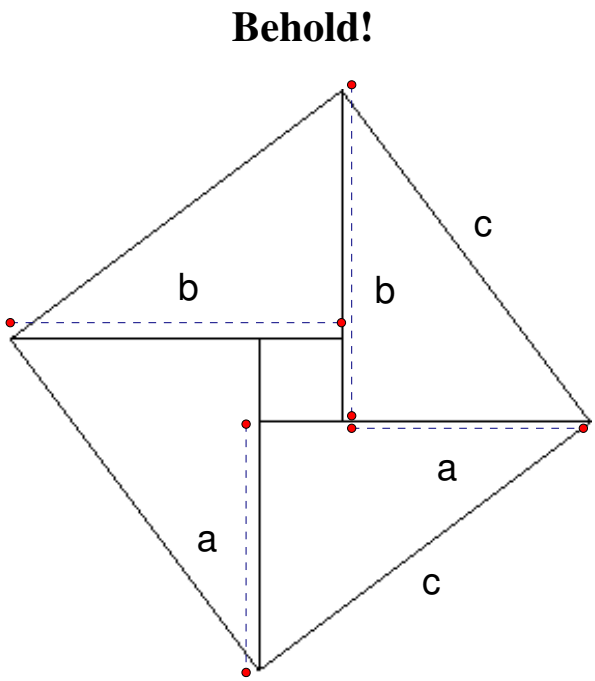
The perimeter of the hexagon is: $P = 6\cdot 10\ cm = 60\ cm$

Thus, the area would be:

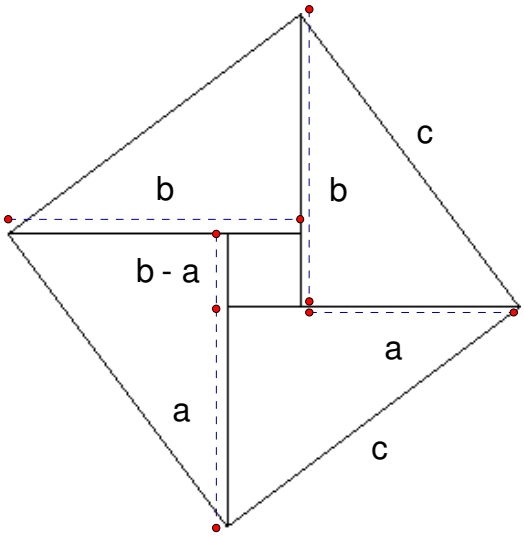
$$A = \frac{1}{2}Ph = \frac{1}{2}(60)\left(5\sqrt{3}\right) = 30\left(5\sqrt{3}\right) = 150\sqrt{3} \approx 259.8\ cm^2$$

Example 5

Use the following picture to derive the Pythagorean Theorem. $\left(c^2 = a^2 + b^2\right)$



The length of the middle square is b –a

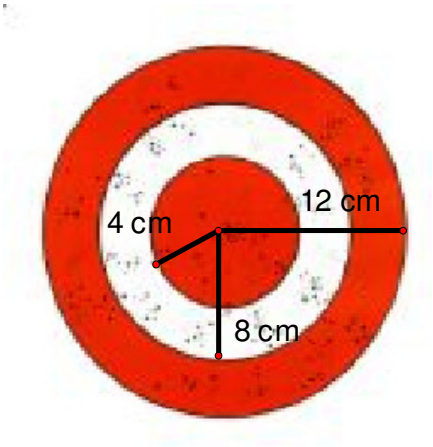


Area of big Square = Area of 4 right triangles + Area of smaller square

$$c^2 = 4\left(\frac{1}{2}\cdot a\cdot b\right)+(b-a)^2$$
$$c^2 = 2ab+(b-a)(b-a)$$
$$c^2 = 2ab+b^2-2ab+a^2$$
$$c^2 = a^2+b^2$$

Example 6

Find the shaded area



$$Area\ of\ inner\ circle = \pi(4\ cm)^4 = 16\pi\ cm^2$$

Area of outer ring = Area of circle with radius of 12 cm – Area of circle with radius of 8 cm

$$Area\ of\ outer\ ring = \pi(12\ cm)^2 - \pi(8)^2 = 144\pi - 64\pi = 80\pi \approx 251.2\ cm^2$$