Math 135

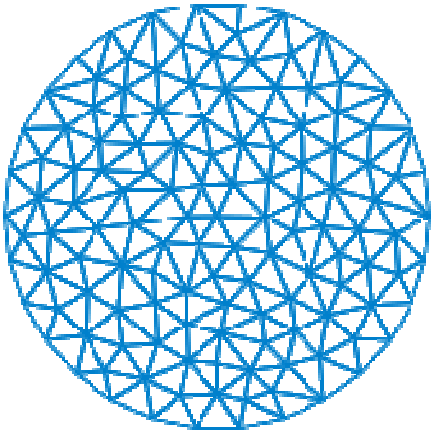
Tessellations

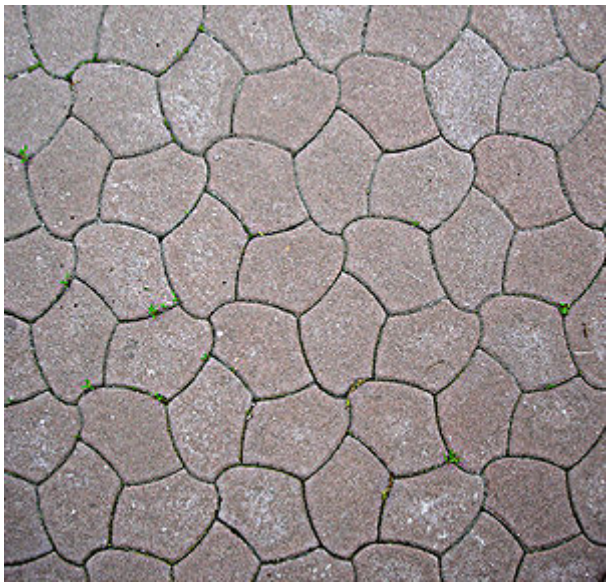
Definition:

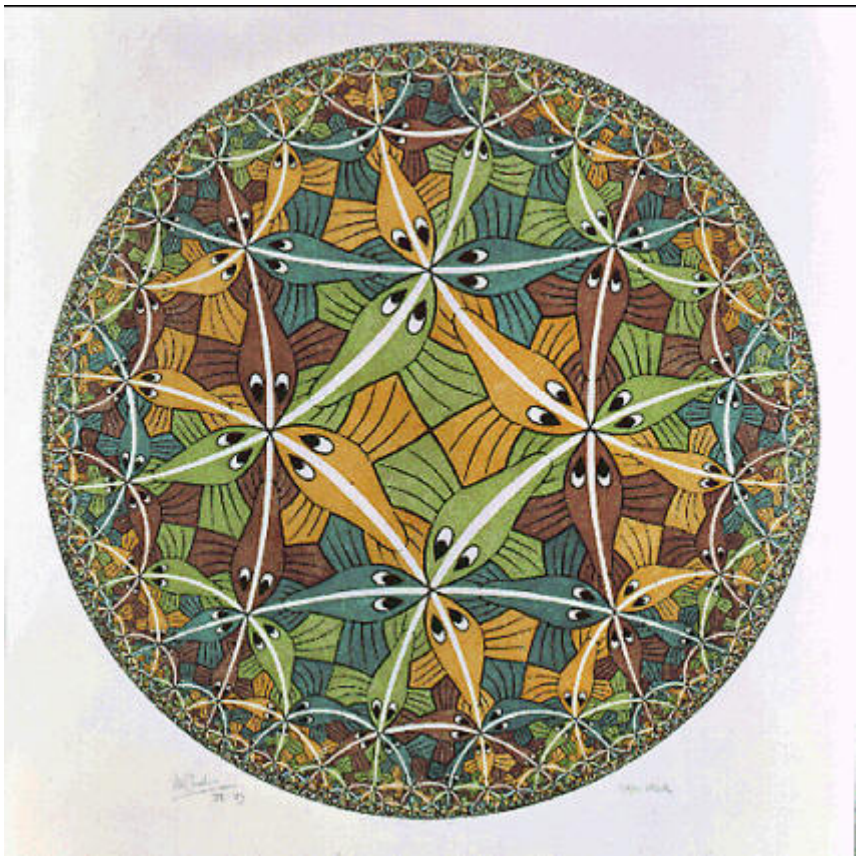
A tessellation or tilling of a plane is a collection of plane figures that fills the plane with no overlaps and gaps.

A **regular tessellation** is a highly symmetric tessellation made up of congruent regular polygons.

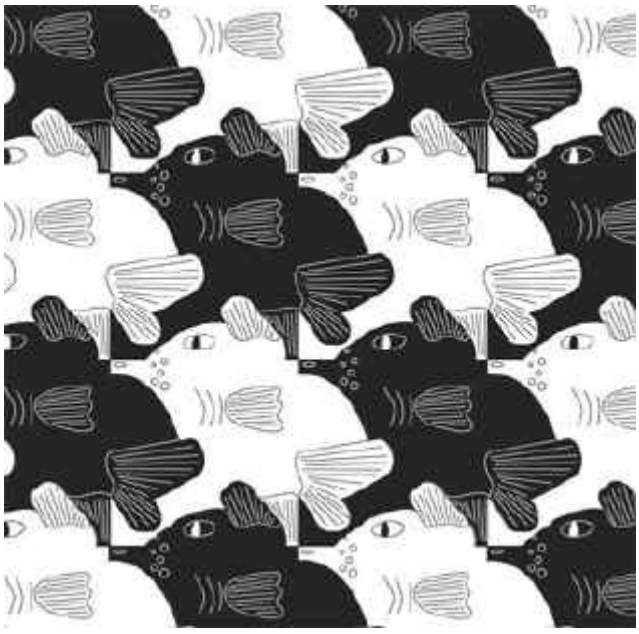
Examples of tessellations







Courtesy of Wikipedia.com



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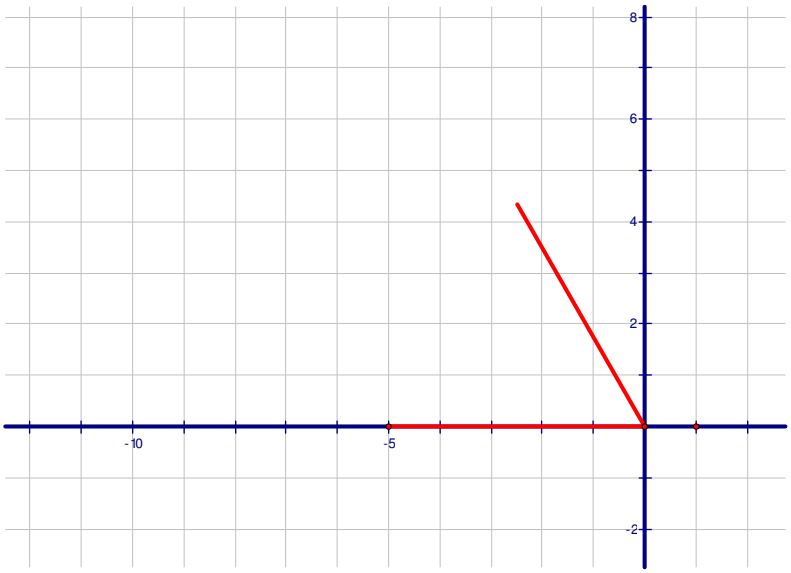
Example 1

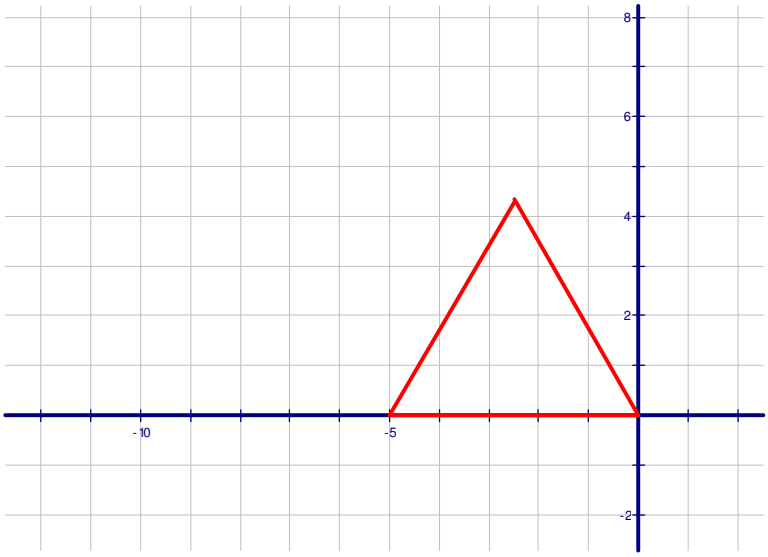
Creating tessellation with Geometer's Sketchpad (GSP)

The first tessellation we are going create is formed by equilateral triangles.

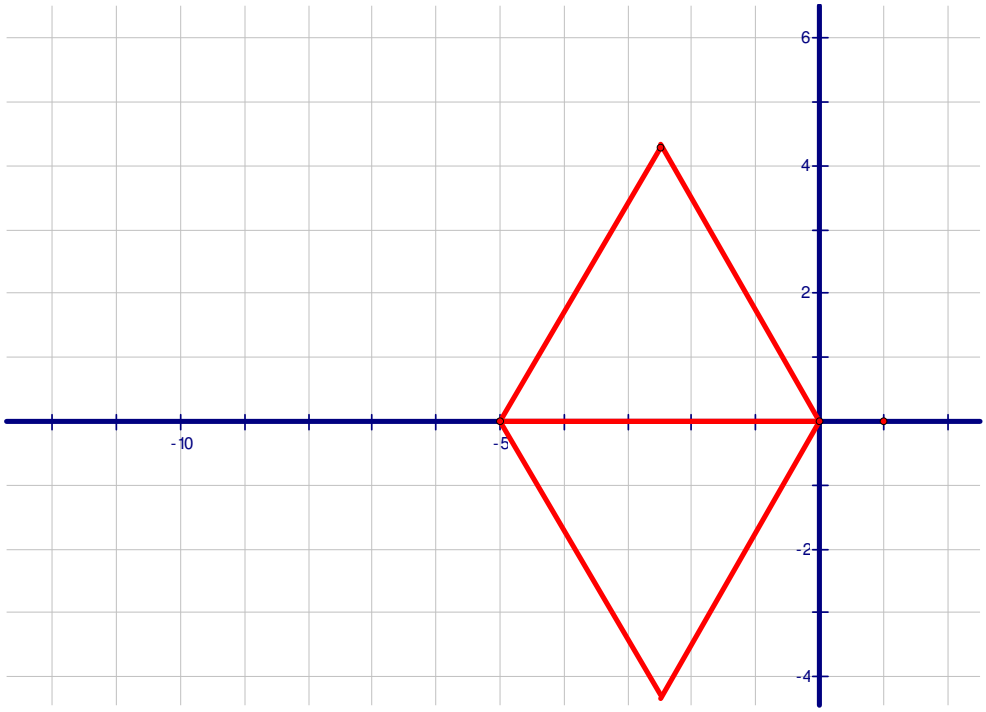
Using GSP, make a segment with the segment tool that is 5 cm long and rotate the segment $-60°$ about the origin.



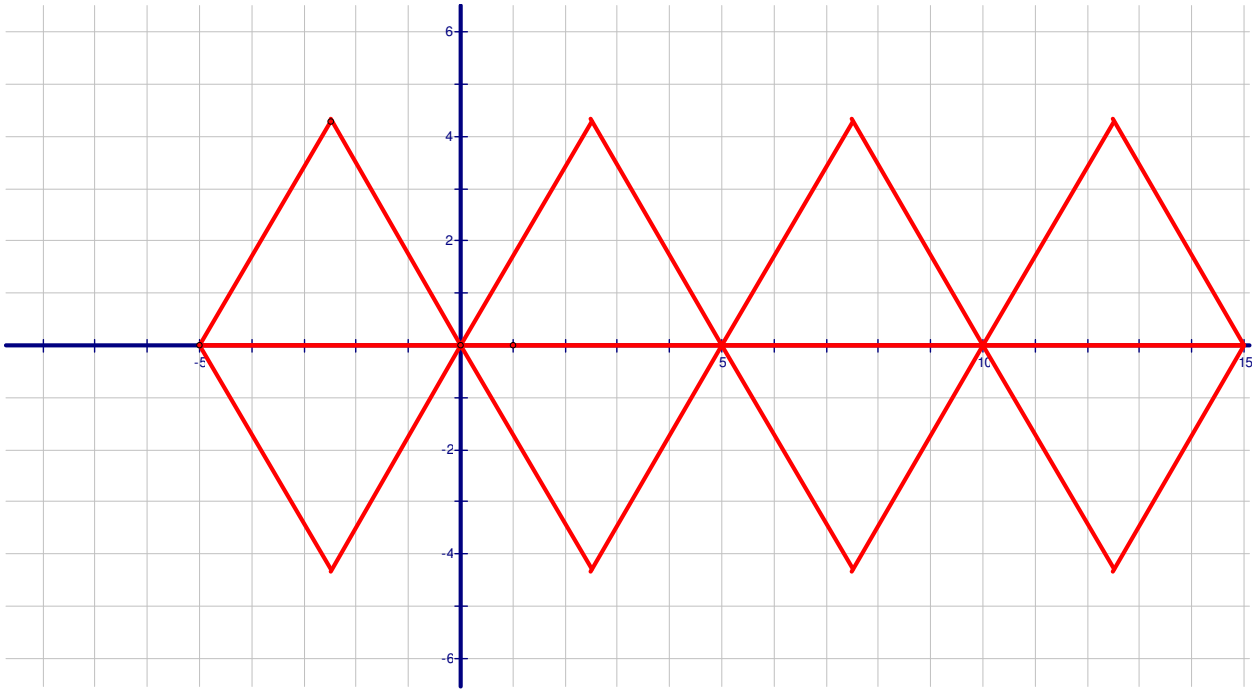
Next, form an equilateral triangle by connecting the endpoints using the segment tool.



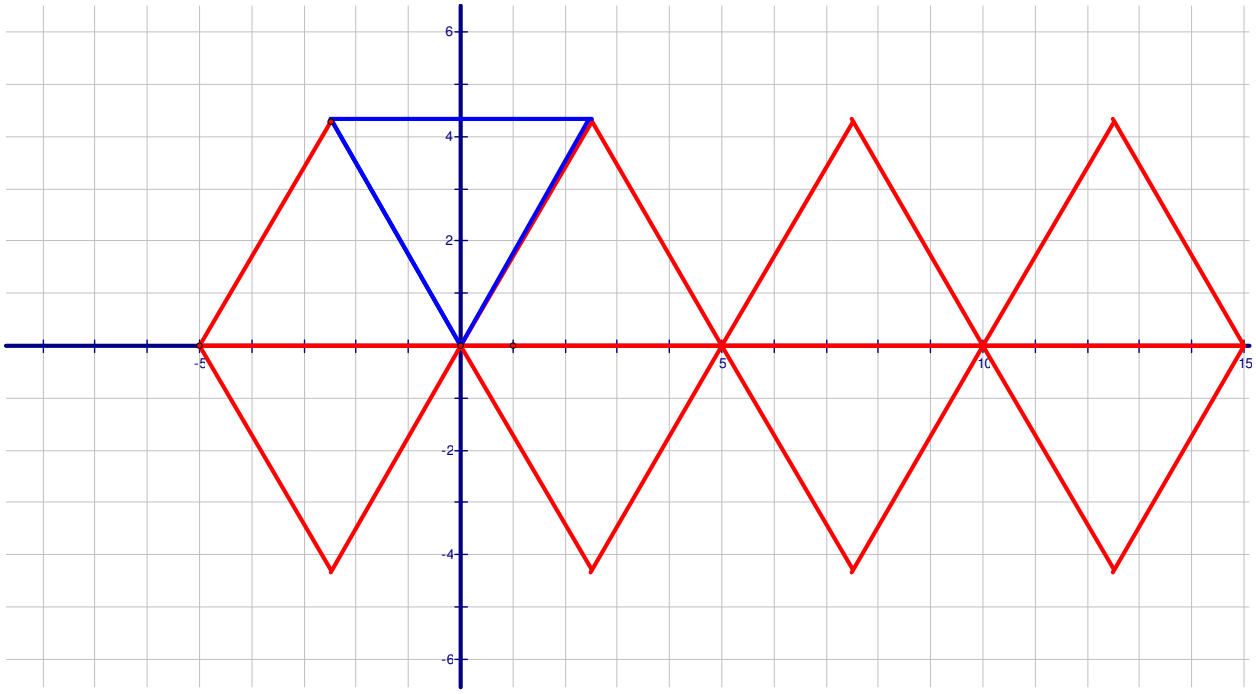
Now, reflect the equilateral triangle about the x-axis



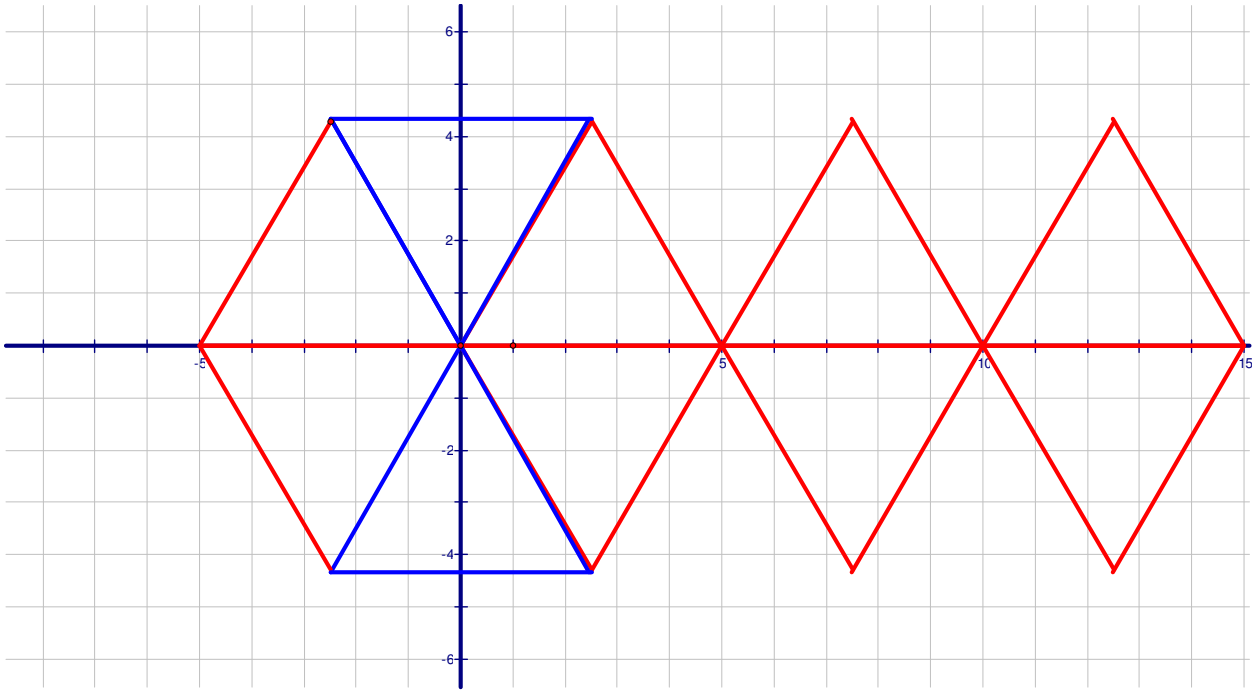
Using GSP you can translate the above triangles with the vector $(5\,cm, 0\,cm)$



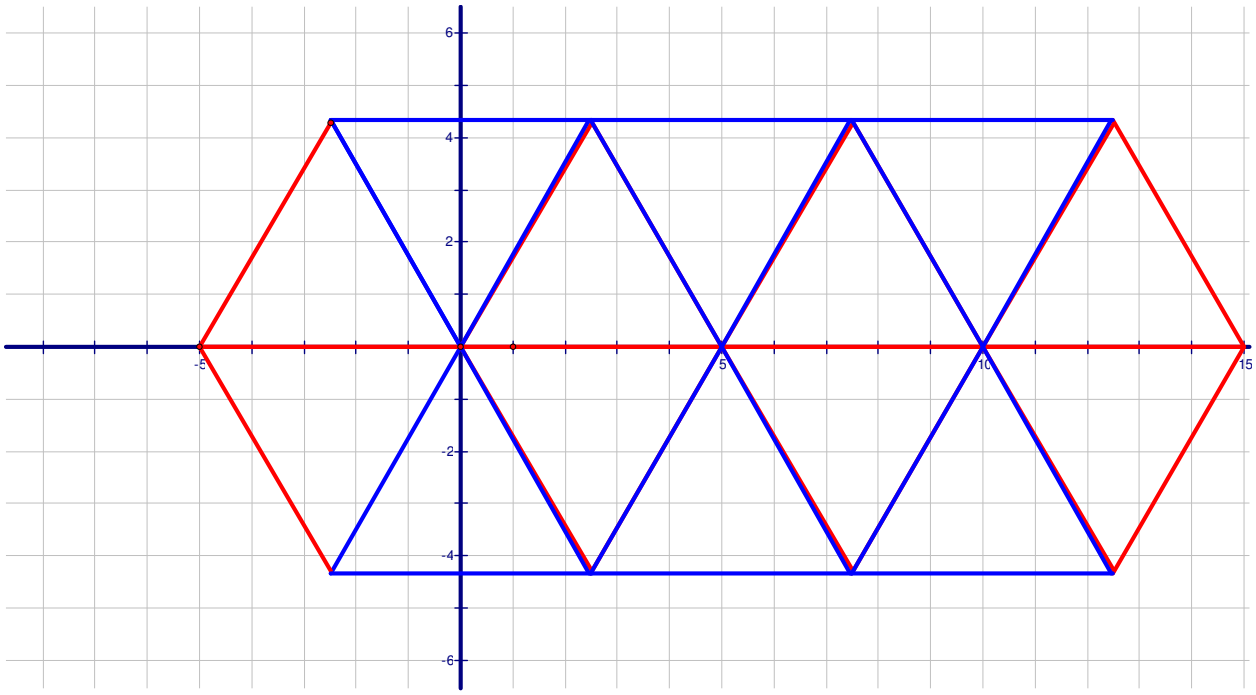
If you rotate one of the bottom triangles 120 degrees, then you place a triangle in between two of the upper triangles.



Reflect this new triangle about the x-axis will help fill in the gaps between the lower triangles.

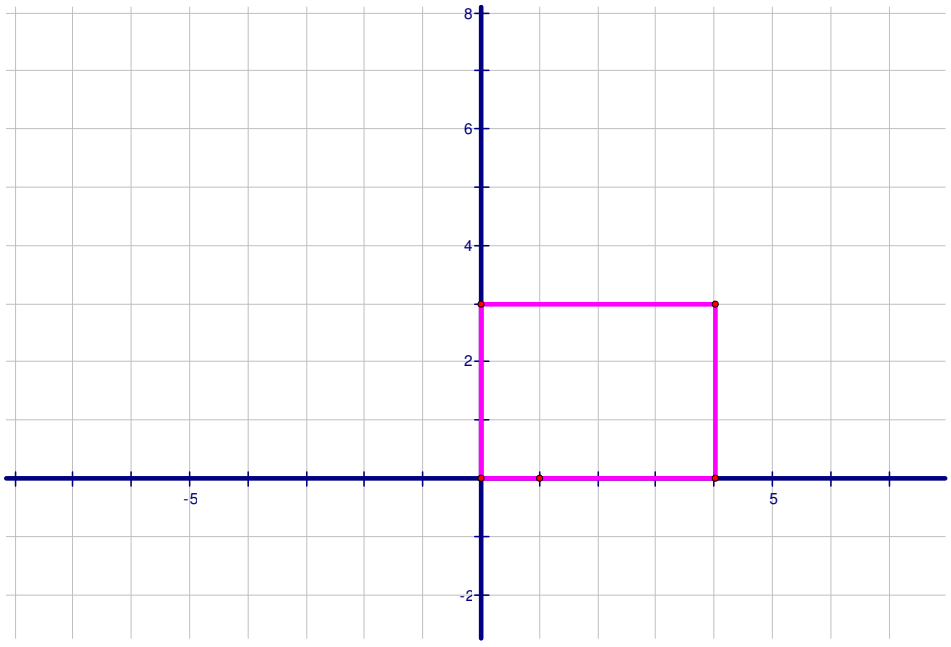


Now, we can use the translation $(5\,cm, 0\,cm)$ with blue triangle above to fill in the rest of the gaps.

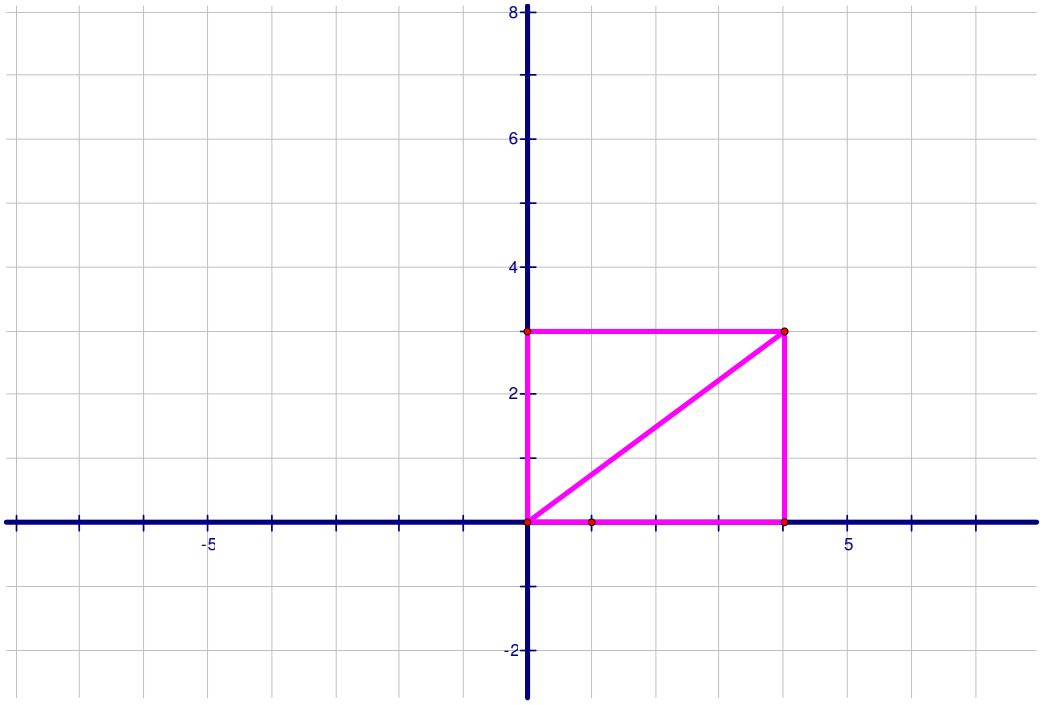


Example 2

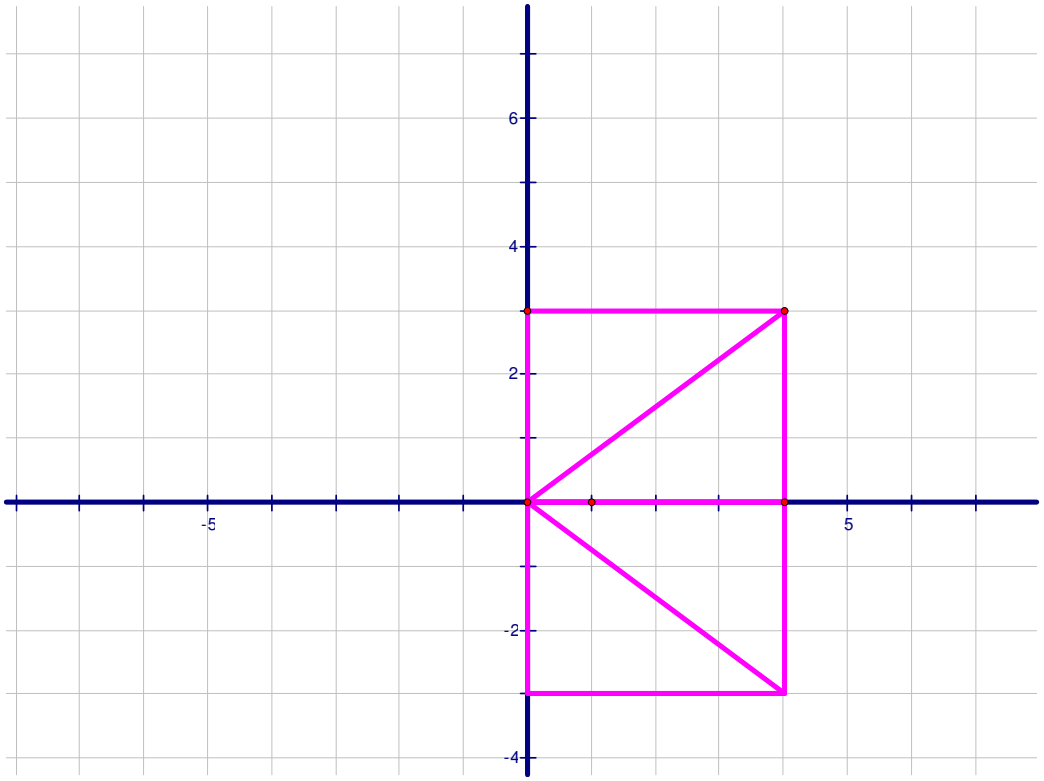
Using GSP display the grid, and then construct a rectangle with a vertex at the origin and a width of 3 cm and height of 2 cm.



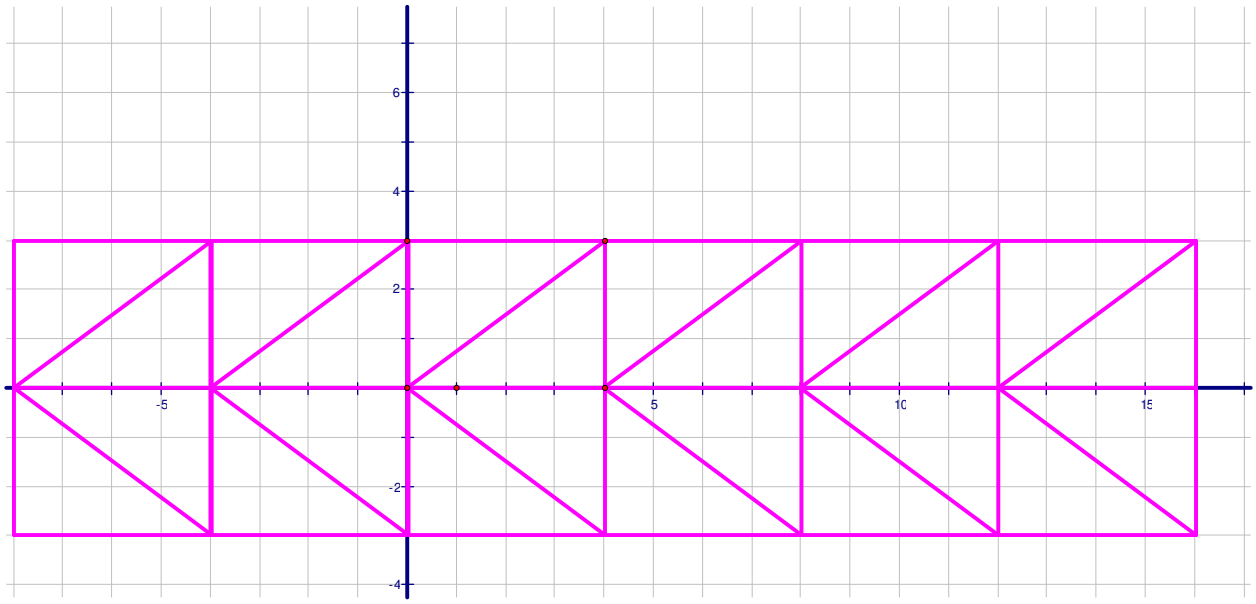
Using the segment tool, construct a diagonal as shown in next figure.



Now, reflect the rectangle with the diagonal about the x-axis.

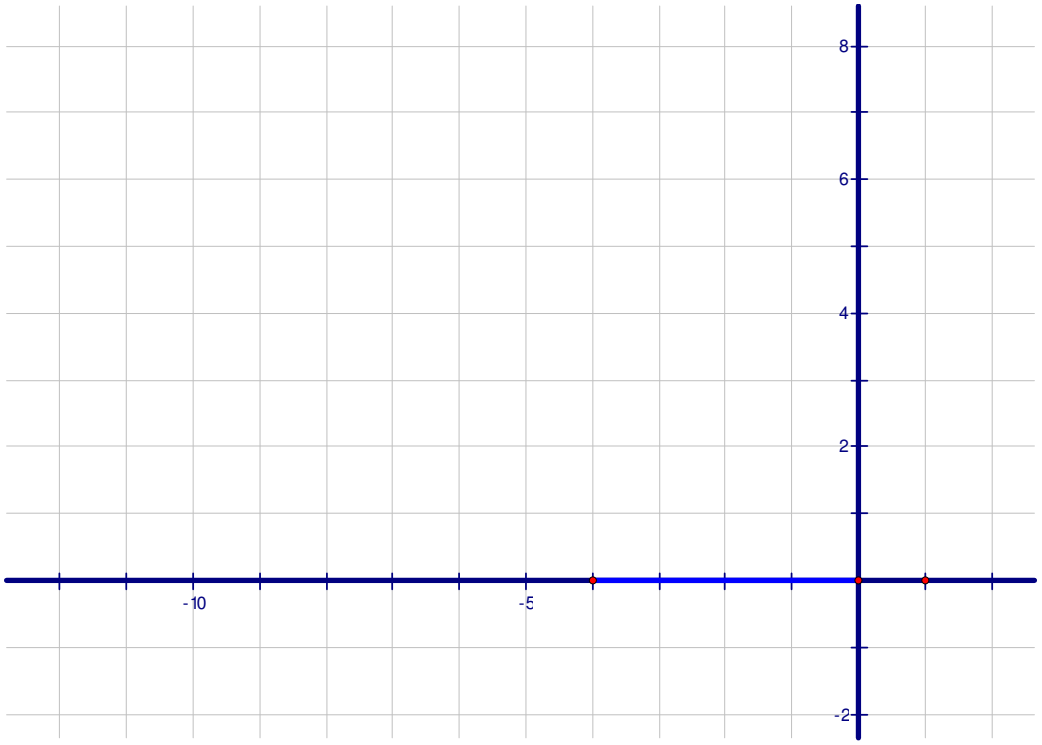


Using translations of $(4\,cm, 0\,cm)$ and $(-4\,cm, 0\,cm)$, you and create the following tessellation.

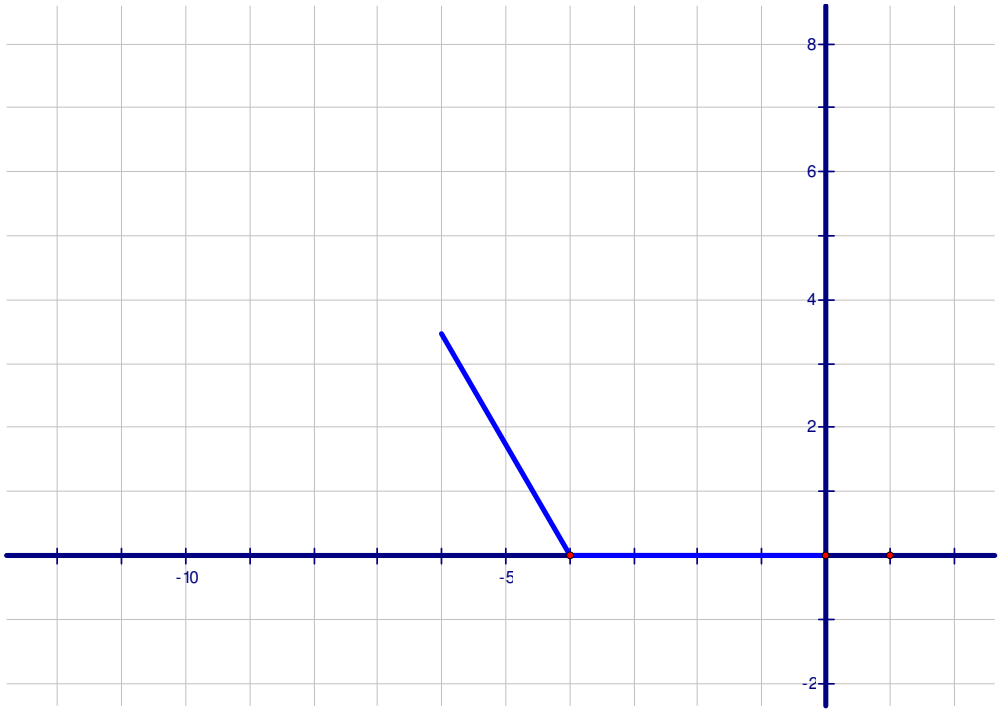


Example 3

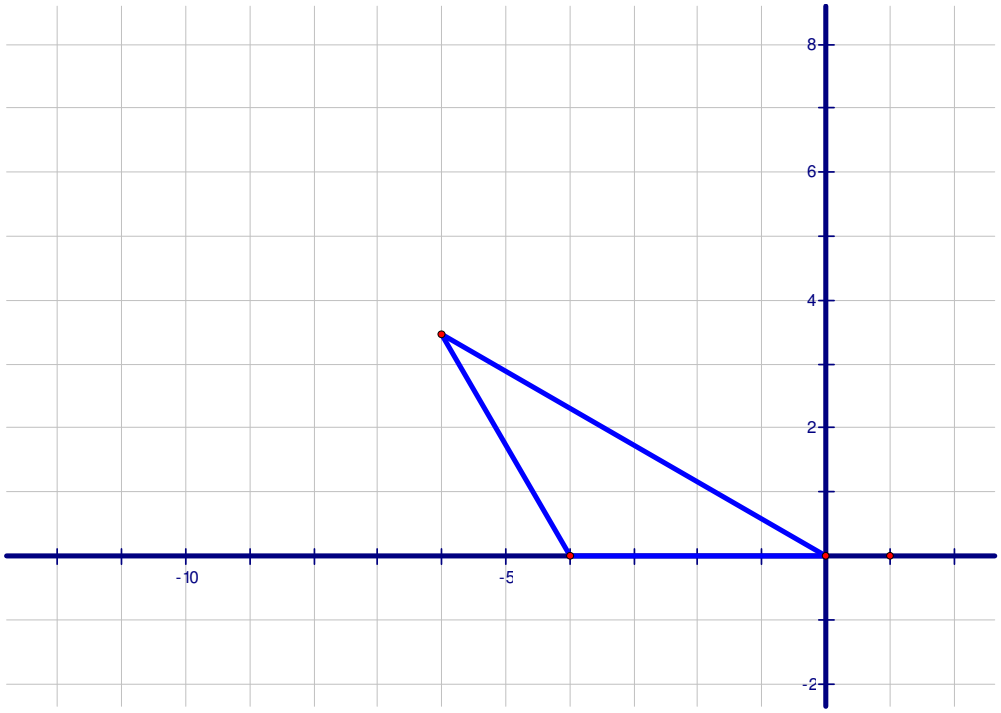
Using GSP construct a 4 cm segment on the x-axis where one endpoint is on the origin.



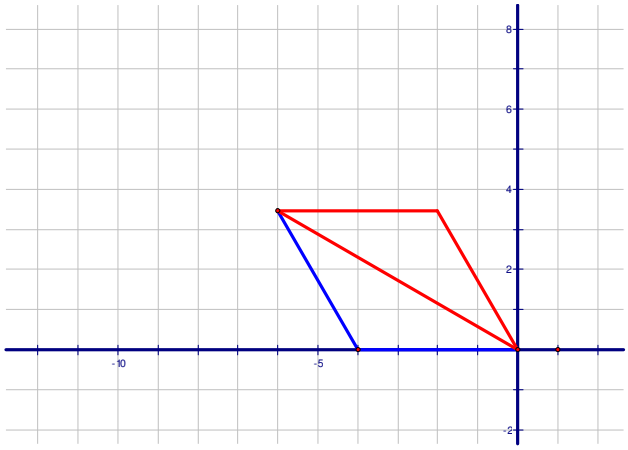
Next, rotate the segment 120 degrees using the left endpoint as the center of rotation.



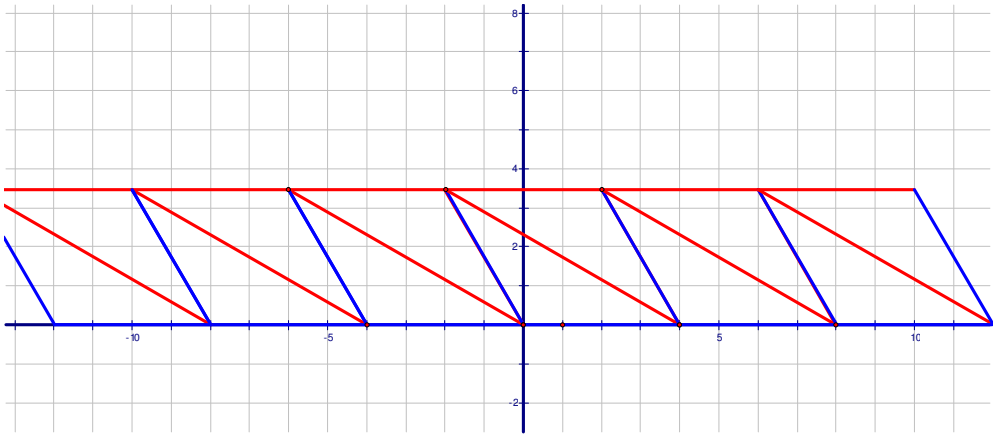
Now, construct an obtuse triangle by connecting endpoints as shown with the segment tool.



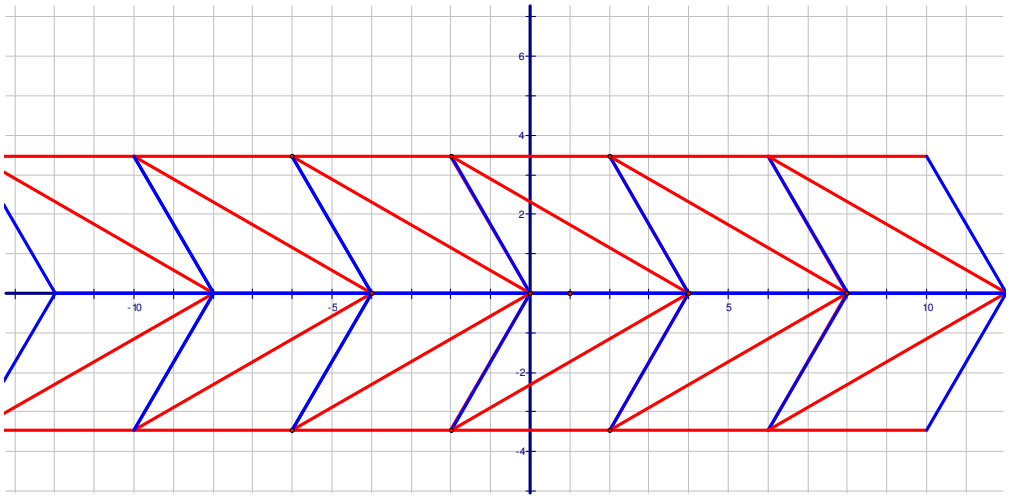
Next, reflect the triangle about the hypotenuse (longest side in this case).



Now you can make a series of parallelograms by translating the parallelogram to the right with the vector (4 cm, 0) and to the left with the vector (-4 cm, 0)



On the last step, reflect the above tessellation about the x-axis

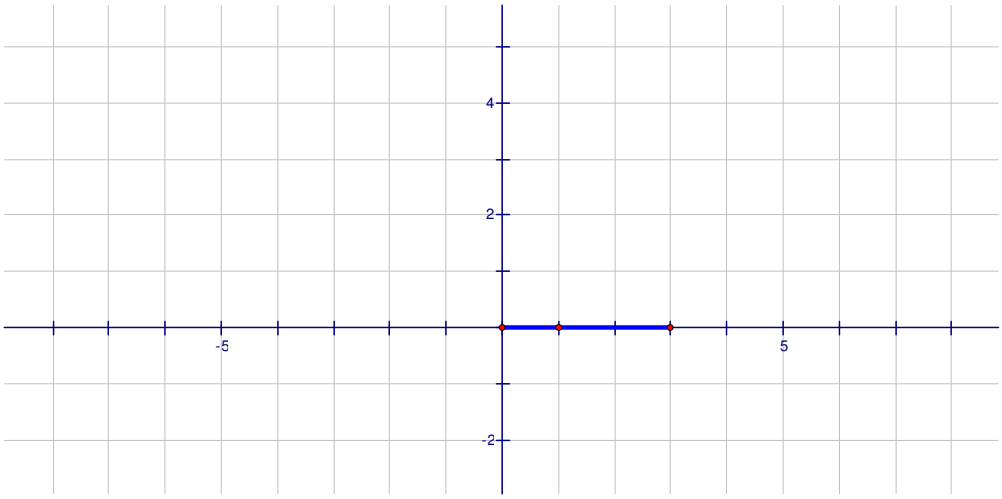


Constructing Regular Polygon with GSP

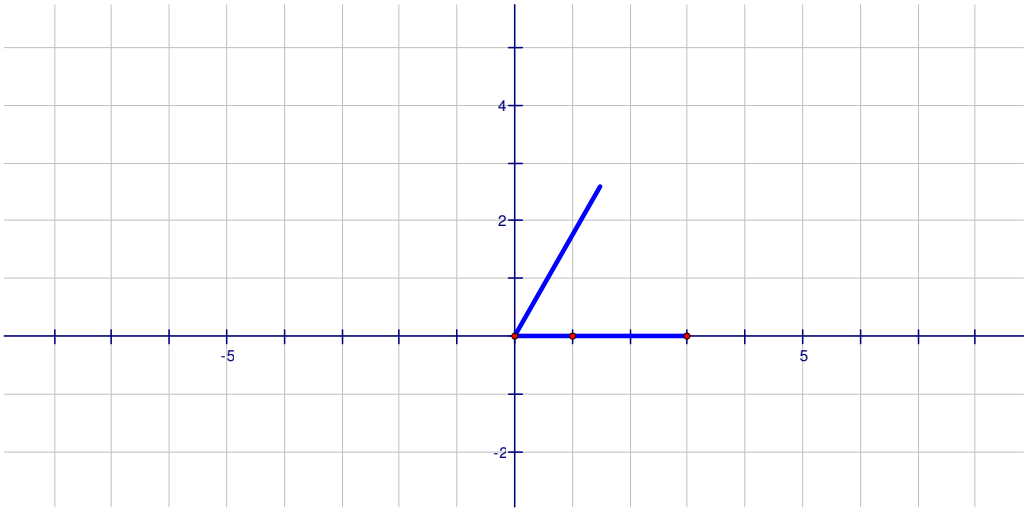
Example 4

How to construct a regular hexagon with GSP

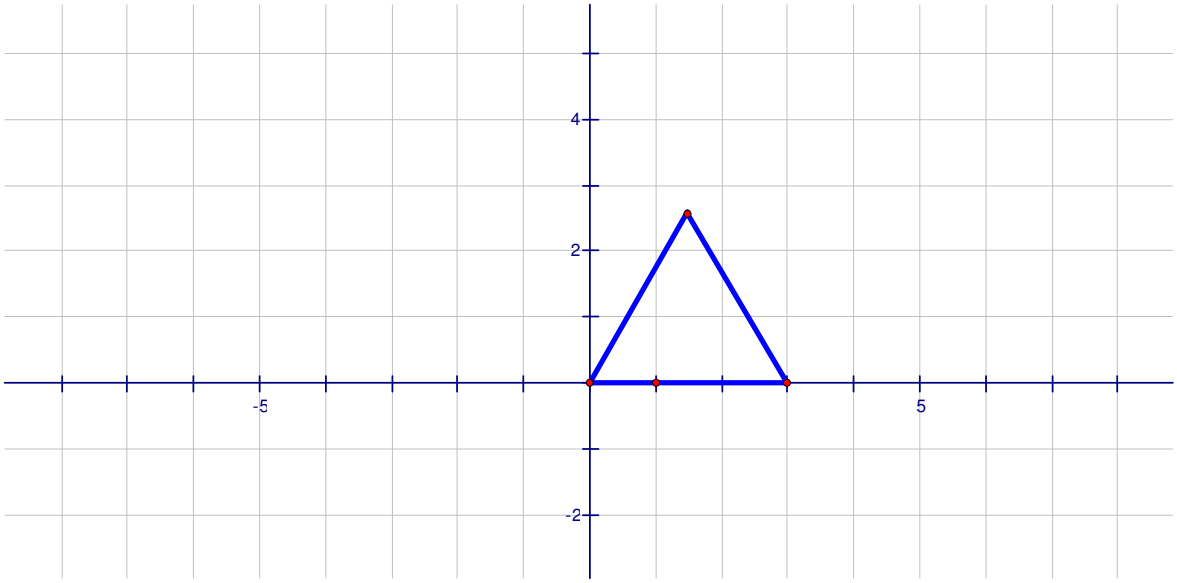
Step 1: Construct a 3 cm segment with one endpoint at the origin.



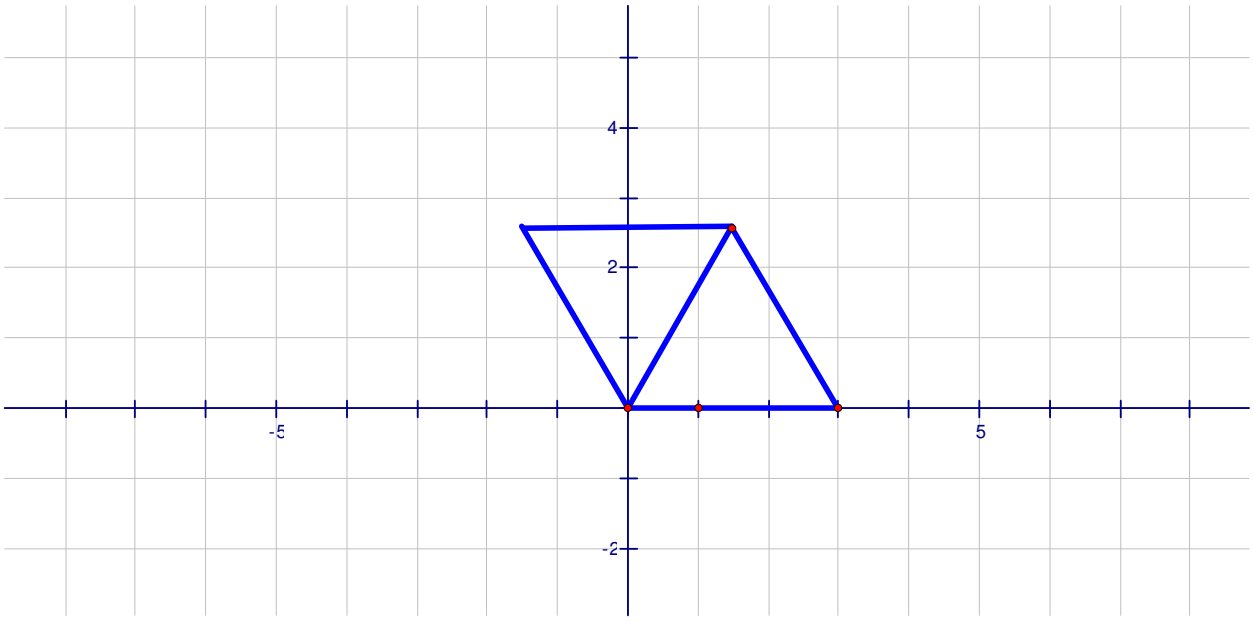
Step 2: Rotate the segment 60 counterclockwise about the origin using the rotate tools under the transform menu.



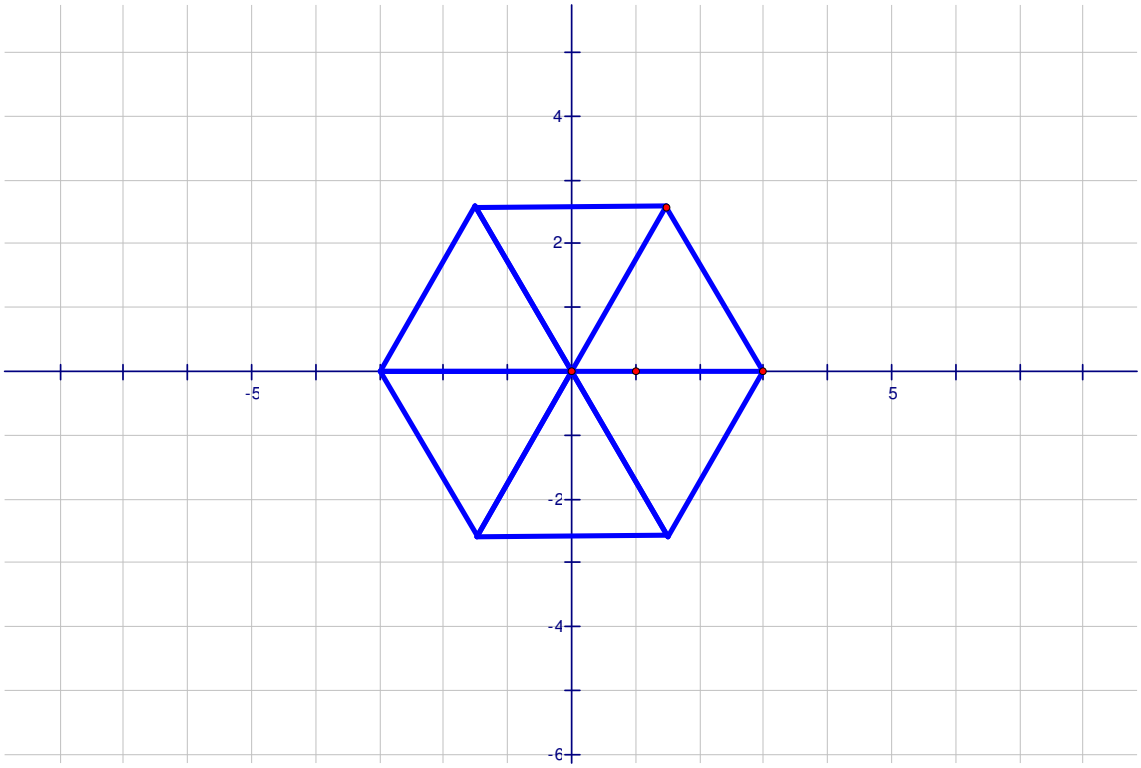
Step 3: Connect the endpoints to form an equilateral triangle.



Step 4: Using the rotate tool under the transform menu rotate the triangle 60 degrees about the origin.



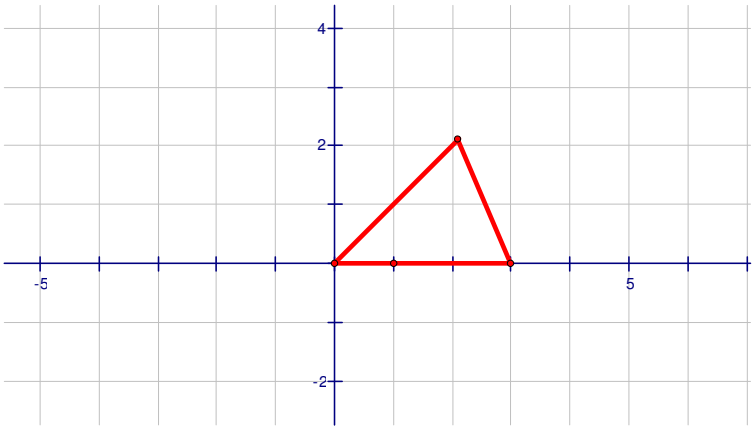
Step 5: Repeat the above 60 degree rotation 4 more times will result in a regular hexagon



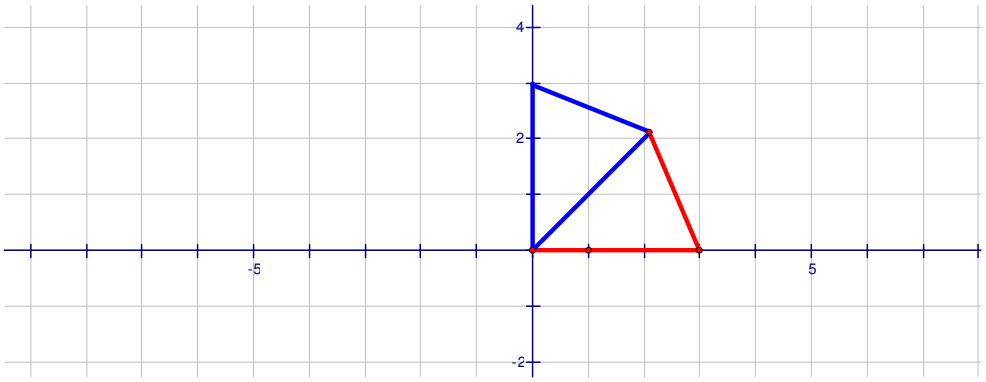
Example 5

If repeat the same procedures for constructing a hexagon using a side that measures 3 cm a rotation of 45 degrees, you construct a regular octagon.

First, construct a 3cm segment with one endpoint at the origin. Then, rotate the segment 45 degrees counterclockwise and connect endpoints to construct an isosceles triangle.



Next, rotate this triangle 45 degrees



Repeat the rotate above 6 six more times and you construct a regular octagon.