Math 135
Section 9.1

Transformations

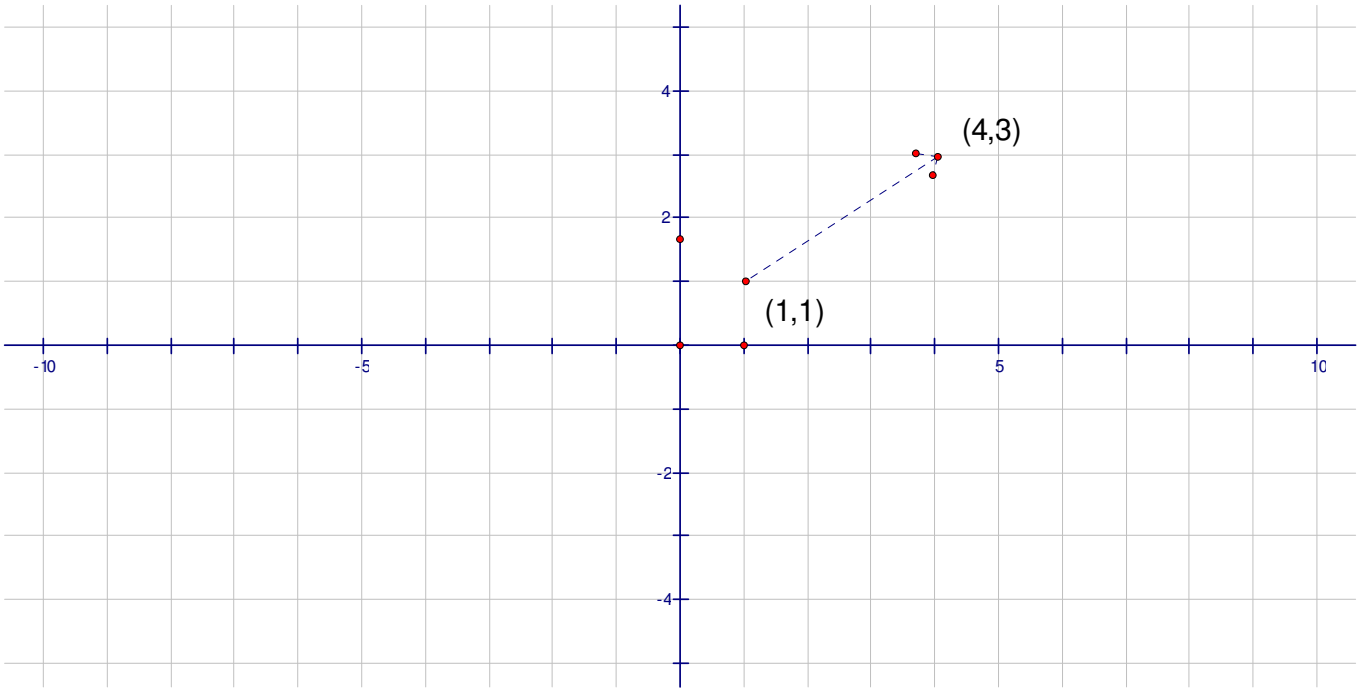
A transformations is a one-to-one correspondence that assigns every point A in the plane to another point in the plane A' where A' is the image of A and A is the preimage of A'

Types of transformations

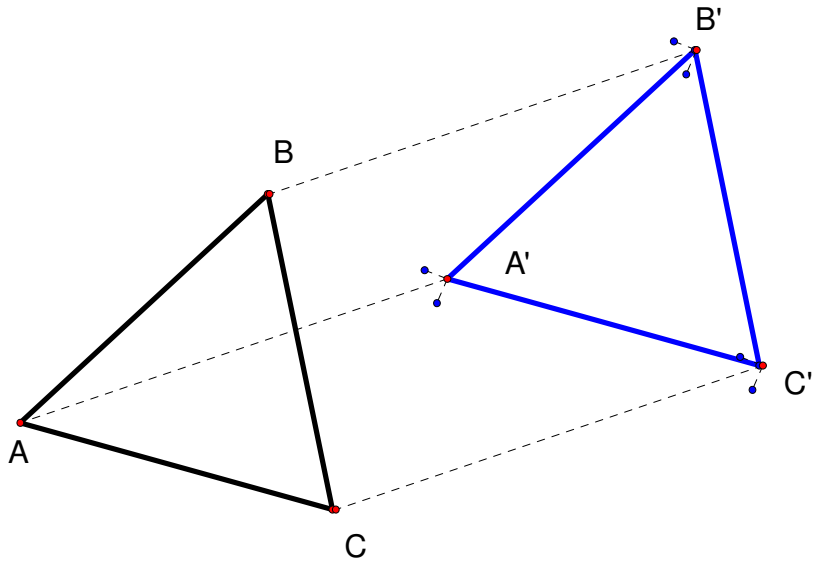
1) Translations
2) Rotations
3) Reflections
4) Glide Reflections

Translations

A translation or slide is directed line segment that maps one point in the plane to another point in the plane.

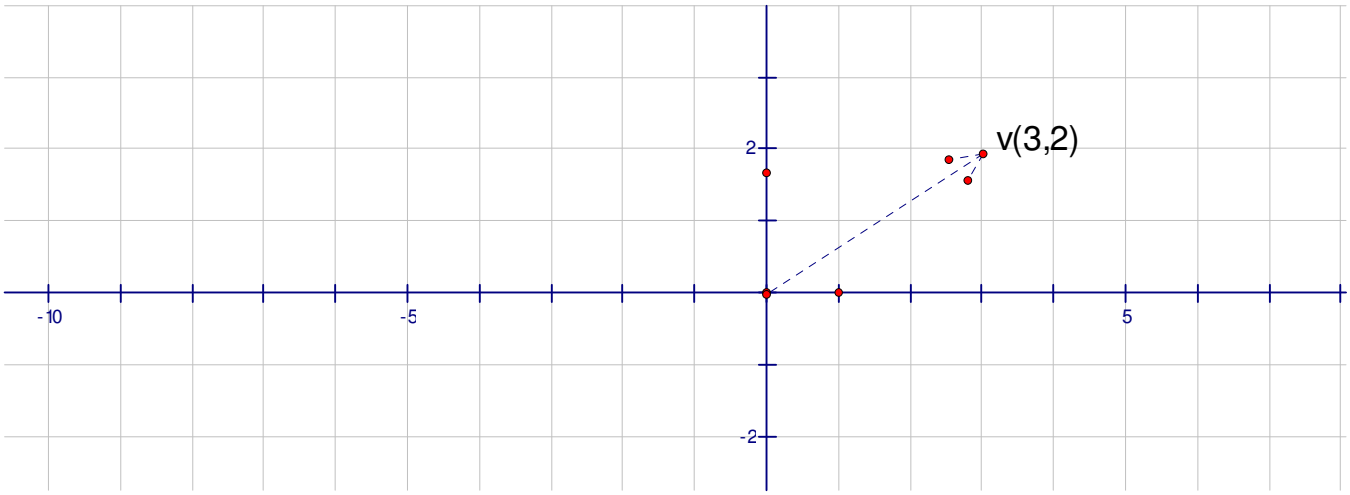


Here is an example of a triangle that has been translated.



Example 1

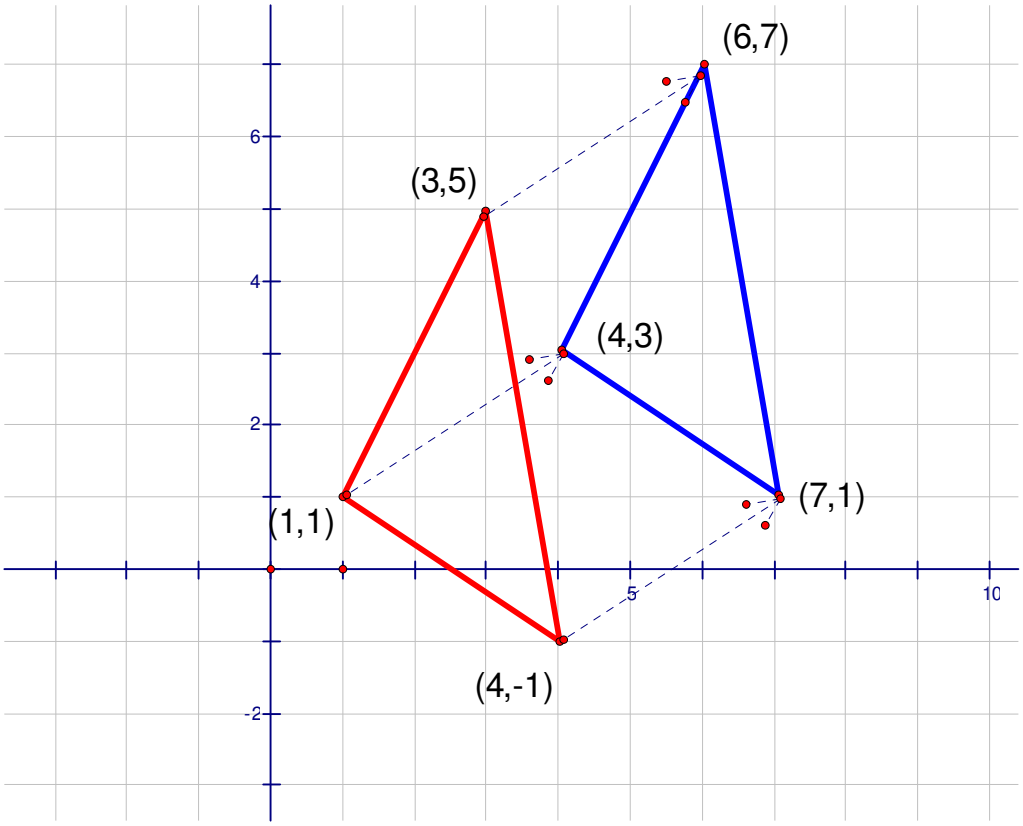
A vector v which is represents a translation is graphed on the coordinate system in the figure below:



A triangle has vertices $A(1,1)$, $B(3,5)$, and $C(4,-1)$. Find the preimage of ΔABC under the translation v. That is, find $\Delta A'B'C'$

Solution:

The point (1,1) translates to (4,3): (3,5) translates to (6,7): $(4,-1)$ translates to (7,1)



Properties of Translations

1) Translations take lines to lines, rays to rays, and line segments to line segments .
2) Translations preserve distance.
3) Translations preserve angle measure.
4) Translations preserve perpendicularity
5) Translations preserve parallelism.

Example 2

Find the coordinates of A' and B' that are image of A and B under translations shown in the figure 2.1

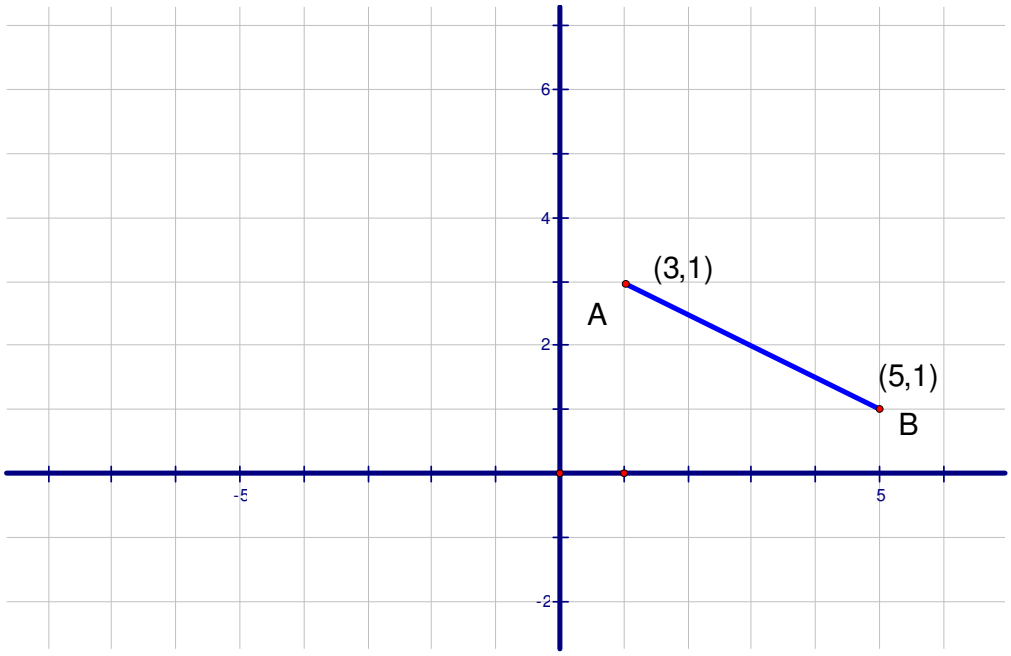
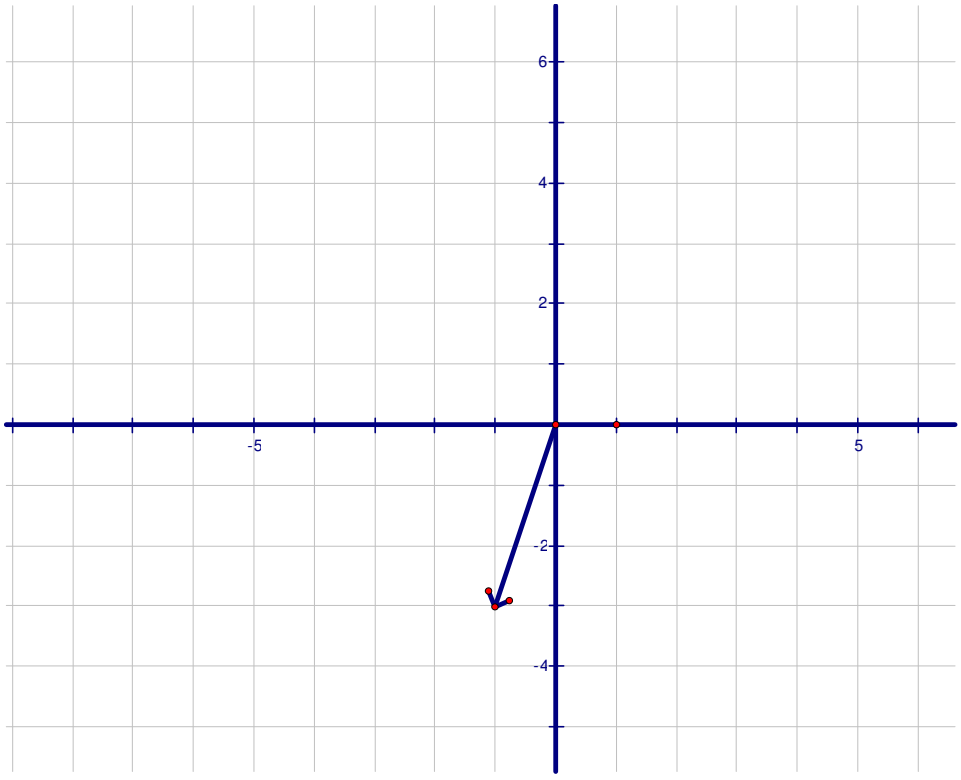
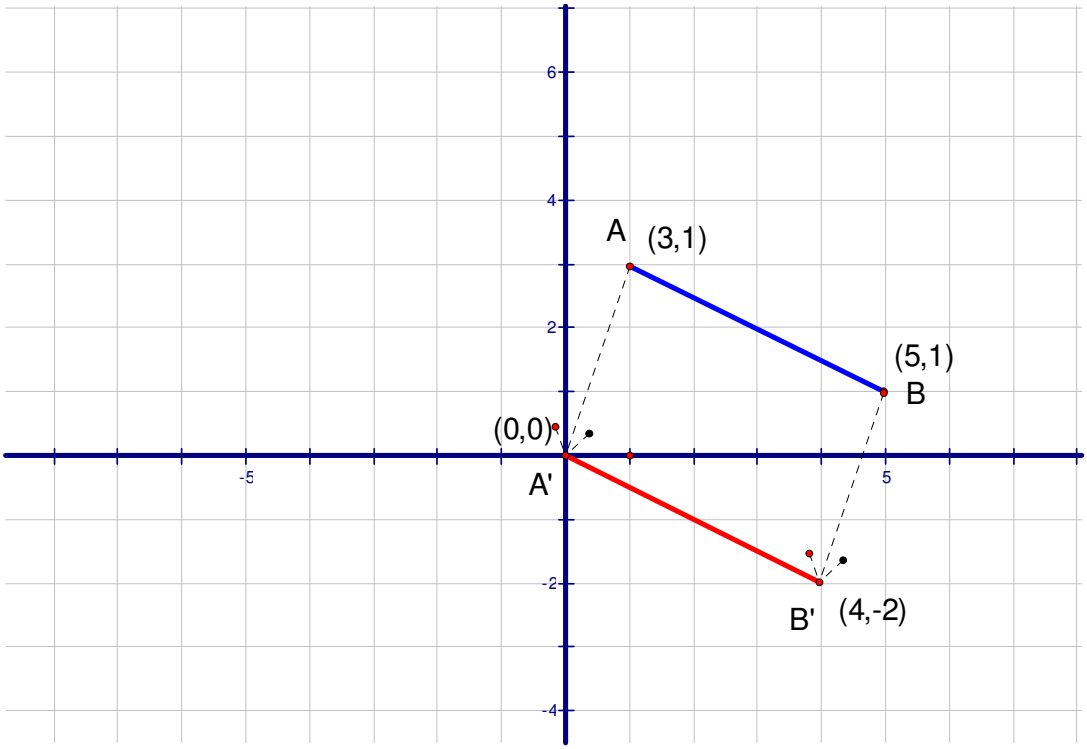


Figure 2.1

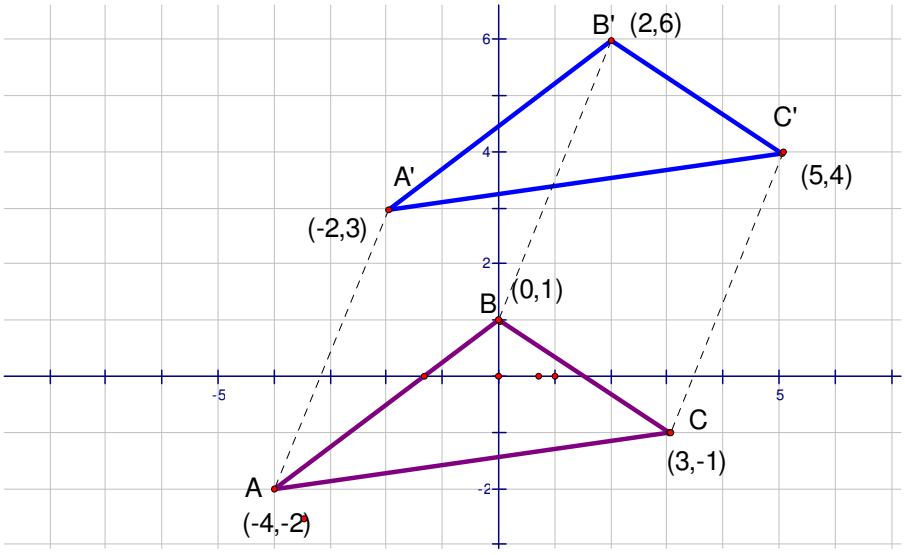


In this graph, the point $A\,(3,1)$ translates to the point $B\,(0,0)$ and the point $A'(5,1)$ translates to the point $B'\,(4,-2)$.



Example 3

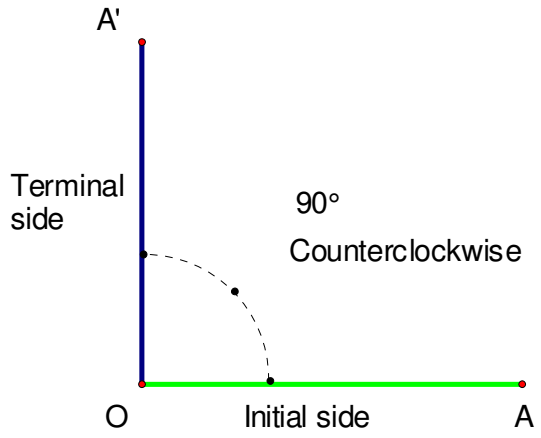
The coordinates of ΔABC are $A(-4,-2)$, $B(0,1)$, and $C(3,-1)$. Find the coordinates of A', B', and C' under translation that takes $P(0,0)$ to $Q(2,5)$. The translation would be represented by the following vector: $v = (2-0,5-0) = (2,5)$



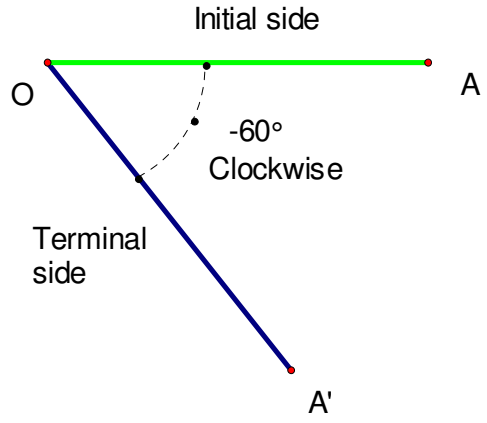
Rotations

Another type of transformation is a rotation, or a turn. A rotation is determined by a fixed point and directed angle. The directed angle is angle whose one side is the initial side and the other side is the terminal side. The angle can be directed in the clockwise direction or the counterclockwise direction.

Here is an example of a counterclockwise rotation labeled $\angle AOA'$ where O is the center of rotation. The directed measure of $\angle AOA'$ is $90°$



Here is an example of a clockwise rotation that is labeled $\angle BOB'$ where O is the center of rotation. The measure of $\angle BOB'$ is $-60°$



Example 4

Give the coordinates of the image of the following points under a $90°$ rotation around the origin.

a) $(4,2)$

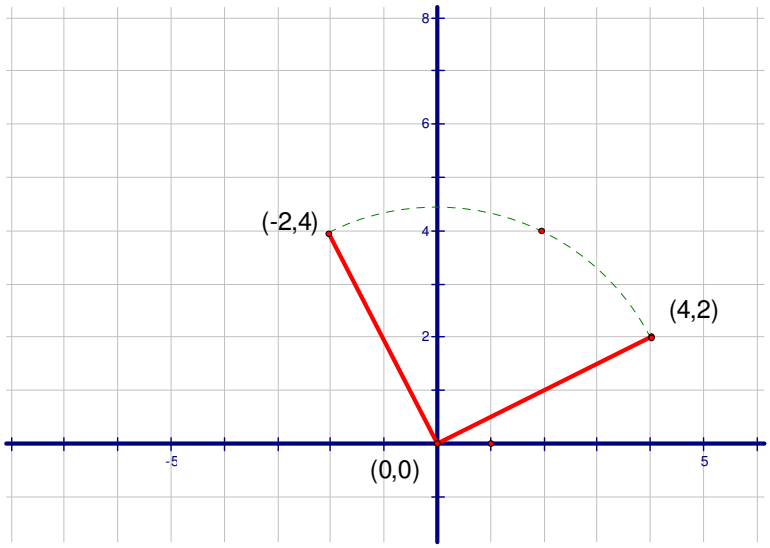


Image: $(-2,4)$

b) $(-4,3)$

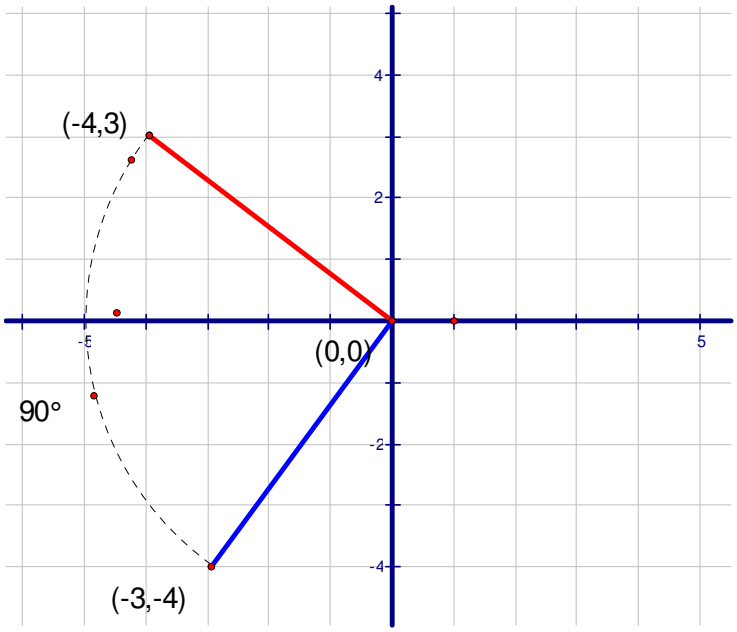
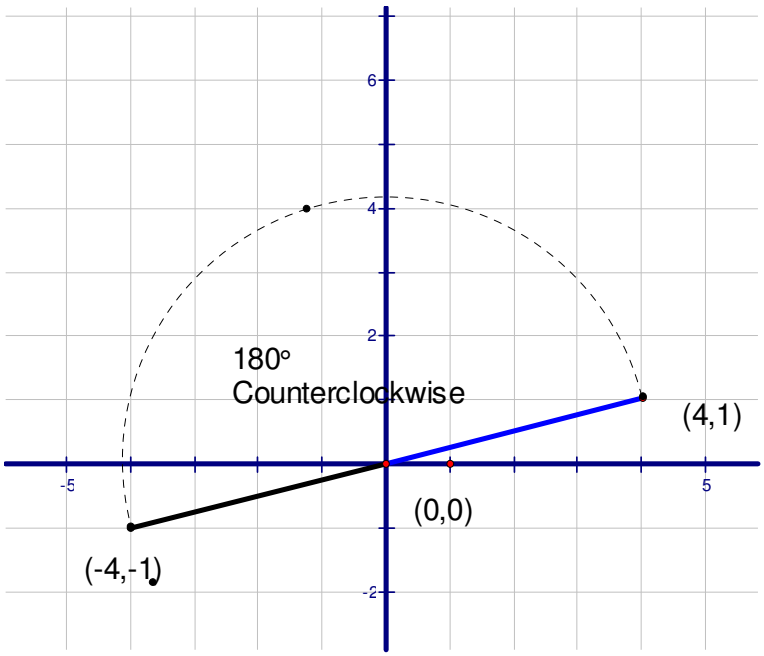


Image: $(-3,4)$

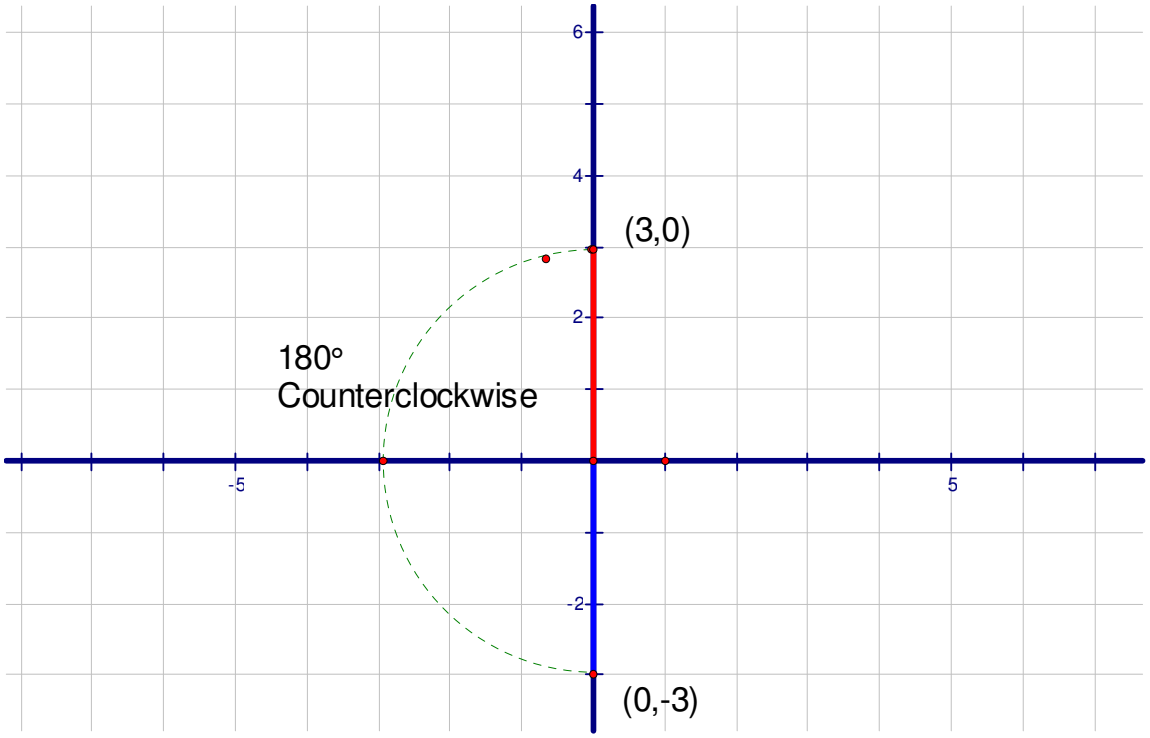
Example 5

Give the coordinates of the image of the following points under the rotation of 180°

a) (4,1)

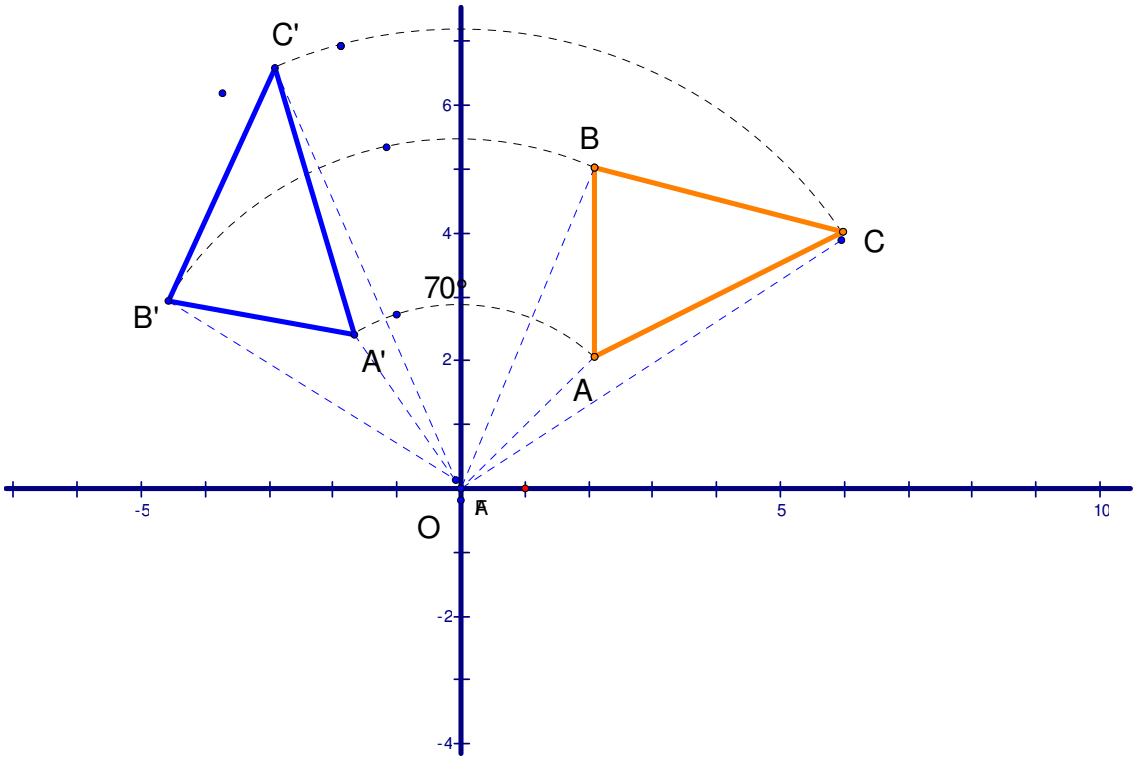


b) (0,3)



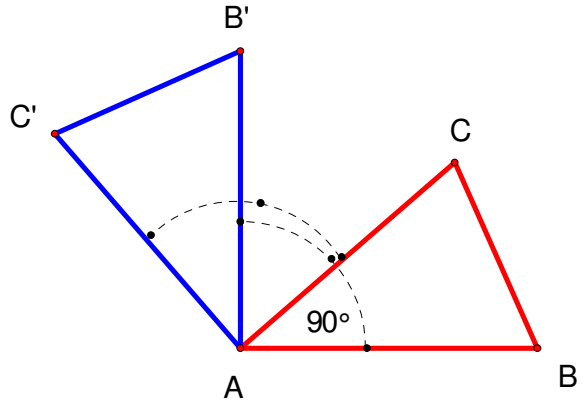
Example 6

Find the image $\Delta A'B'C'$, of ΔABC under a rotation of $70°$ about the point O



Example 7

Find the image $\Delta A'B'C'$, of ΔABC where ΔABC is rotated $90°$ about point A.



Properties of Rotations

1) Rotations take lines to lines, rays to rays, and line segments to line segments.
2) Rotations preserve distance.
3) Rotations preserve angle measure.
4) Rotations preserve perpendicularity
5) Rotations preserve parallelism.

Reflections

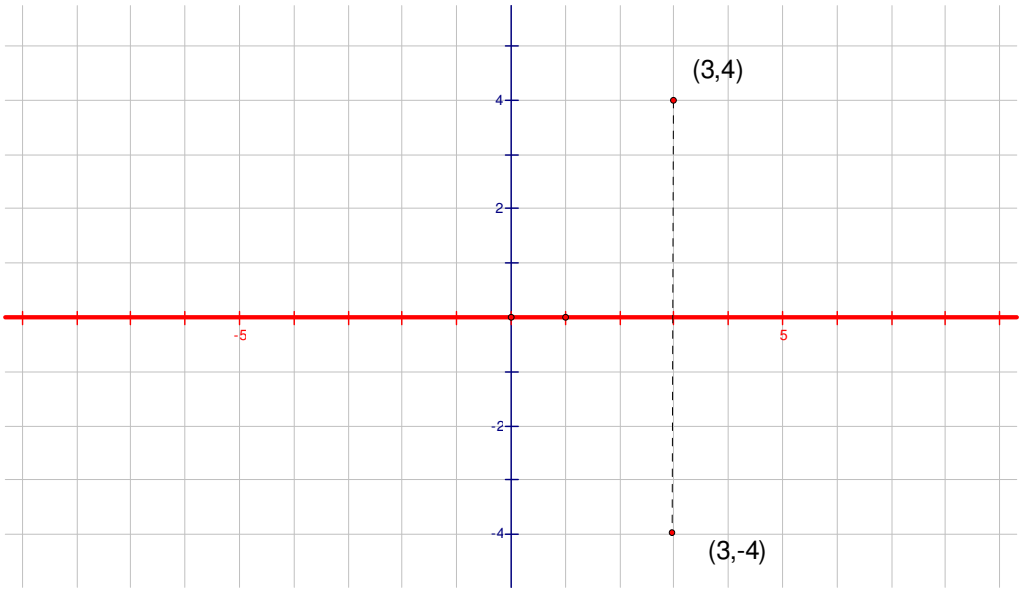
A reflection with respect to line l is defined by describing the location of the image of each point of the plane as follows:

1) If A is a point on l, then $A = A'$
2) If A is not on l, then l is perpendicular bisector of $\overline{AA'}$

Example 8

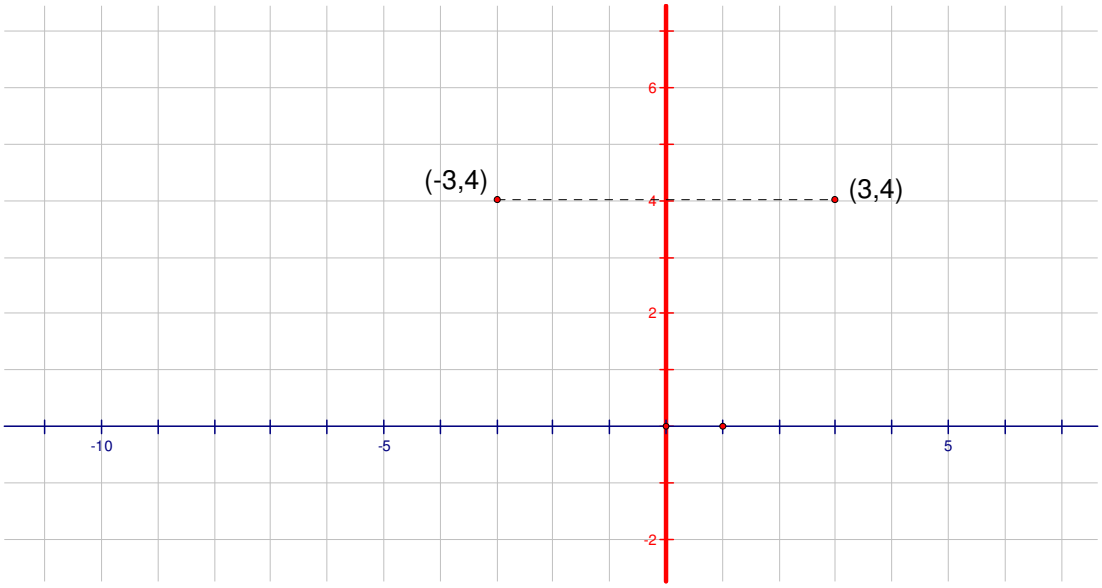
Reflect the point (3,4) in the coordinate plane about the a) x-axis b) the y-axis and c) about the line $y = x$

a) Reflect (3,4) about the x- axis.



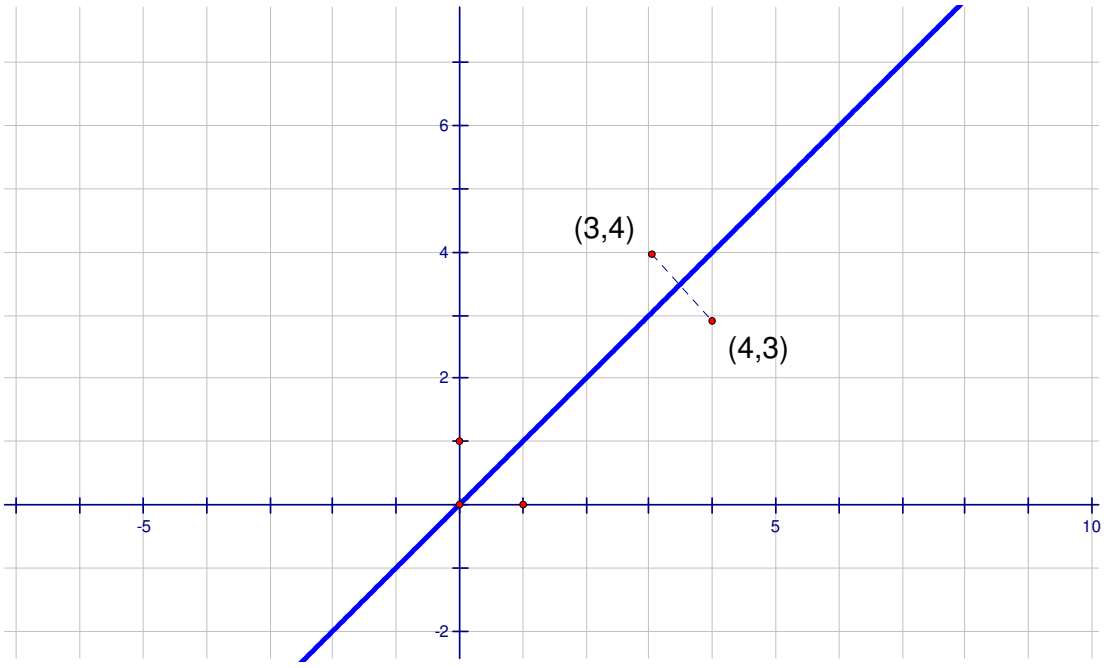
Notice that reflecting about the x-axis changes the sign of the y-coordinate.

b) Reflect (3,4) about the y-axis



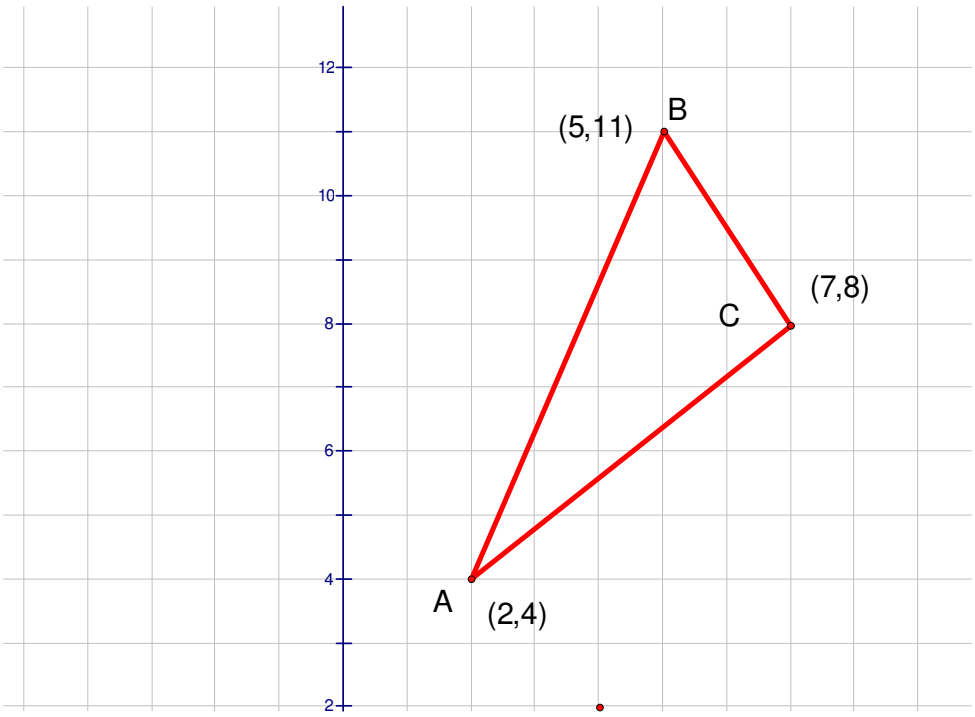
In this case, reflecting about the y-axis changes the sign of the x-coordinate.

c) Reflect (3,4) about $y = x$

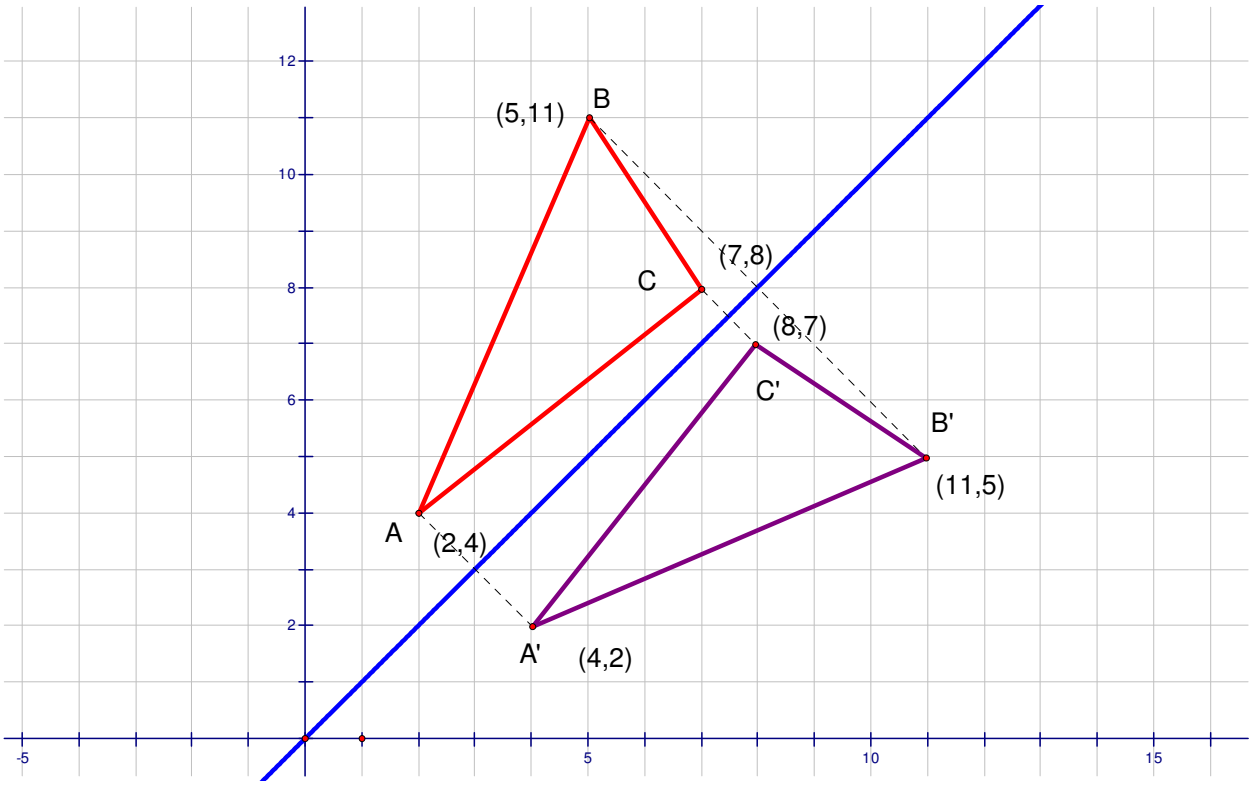


In this case, reflecting about the line $y = x$ inverts the values of the x and y coordinates.

Example 9



a) Graph ΔABC using the coordinates $A(2,4), B(5,11)$, and $C(7,8)$, and its image with respect to the line $y = x$.



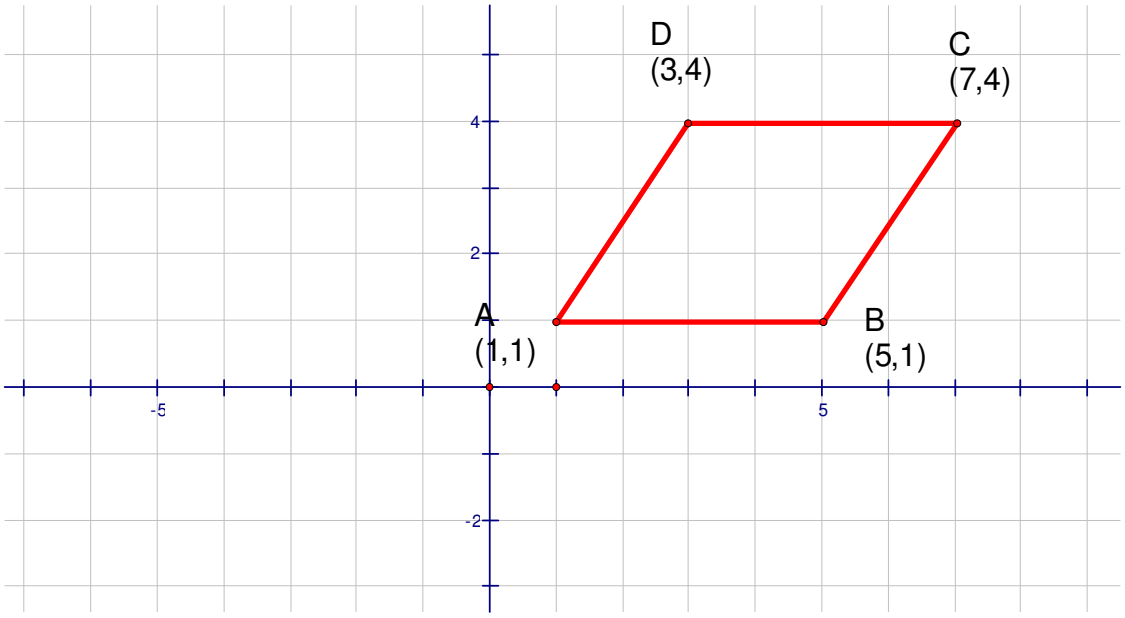
b) What are the coordinates of the images of points A,B, and C

$A'(4,2)$
$B'(11,5)$
$C'(8,7)$

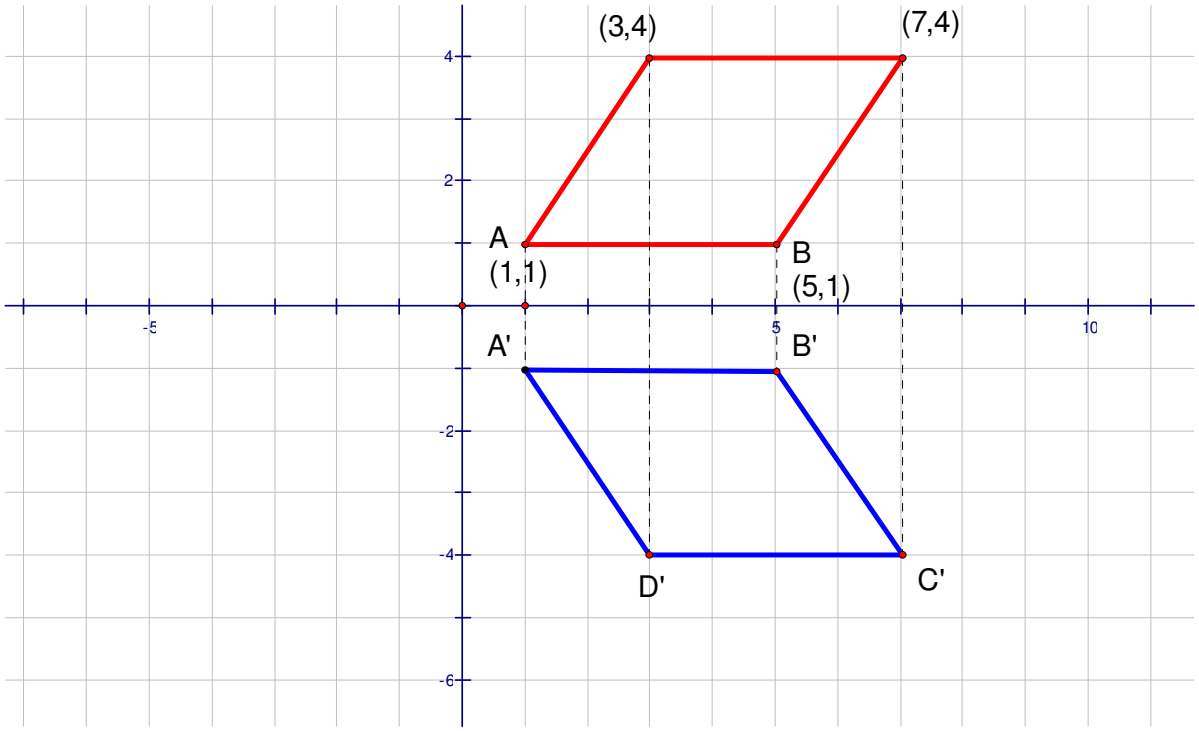
c) If the coordinates of point P is (a,b), what would be the coordinates of the image of the reflection about $y = x$

(b, a)

Example 10



a) Graph Quadrilateral ABCD using the coordinates $A(1,1), B(,51), C(7,4)$ and $D(3,4,)$, and its image with respect to x-axis.



b) What are the coordinates of the images of points A,B,C and D

$A'(1,-1)$
$B'(5,-1)$
$C'(7,-4)$
$D'(3,-4)$

c) If the coordinates of point P is (a,b), what would be the coordinates of the image of the reflection about $y = x$

$(b,-a)$