

Math 135

Section 5.1

Parallel and Perpendicular Lines

If two lines are intersected by a third line, the third line is called a **transversal**. In figure 1.1 below, there are lines l and m with transversal k .

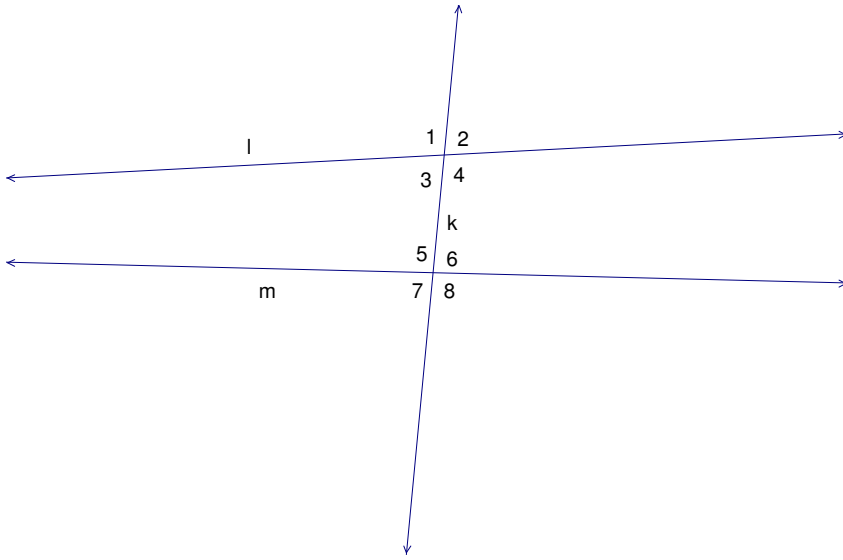


Figure 1.1

In the figure above, there are also some angles which are numbered 1 through 8. Angles $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are called **exterior angles**. Angles $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are **interior angles**. In Euclid Geometry there are some special angle pairs.

Exterior angles that lie on opposite sides of the transversal are called **alternate exterior angles**.

In figure 1.1, the pair of $\angle 1$ and $\angle 8$ are alternate exterior angles and the pair of $\angle 2$ and $\angle 7$ are alternate exterior angles.

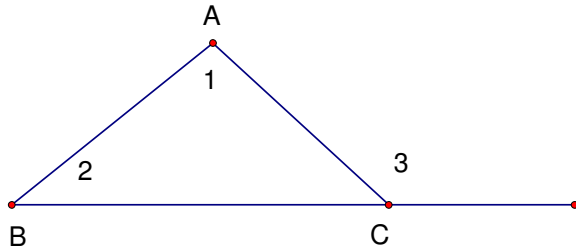
Interior angles that lie on opposite sides of the transversal are called **alternate interior angles**.

In figure 1.1, the pair of $\angle 3$ and $\angle 6$ are alternate interior angles and the pair of $\angle 4$ and $\angle 5$ are alternate interior angles.

Corresponding angles lie on the same side of the transversal where one angle is an interior angle and the other angle is an exterior angle.

In figure 1.1, the angle pairs of $\angle 1$ and $\angle 5$: $\angle 2$ and $\angle 6$: $\angle 3$ and $\angle 8$: $\angle 4$ and $\angle 7$ are corresponding angles.

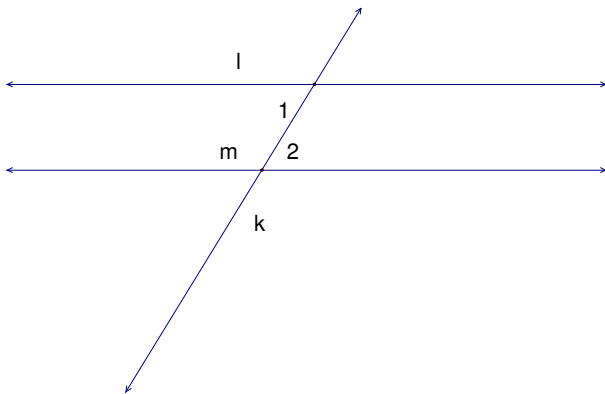
Recall: Theorem 4.11 (Exterior Angle Theorem)



The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle. **Note:** $\angle 3 > \angle 1$ and $\angle 3 > \angle 2$

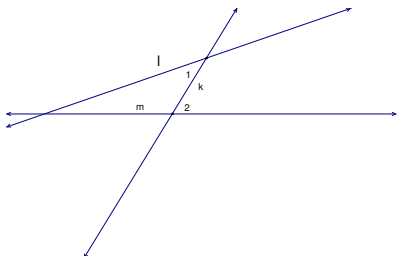
Theorem 5.1

If two lines are cut by a transversal to form a pair of congruent alternate interior angles, then the lines are parallel.



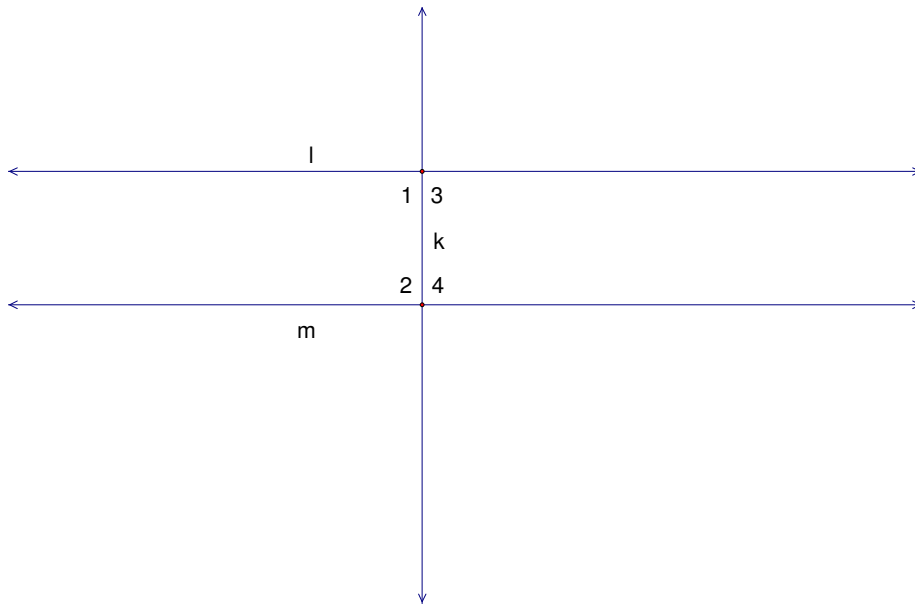
Proof:

Proof by contradiction: Assume that $\angle 1 \cong \angle 2$ and lines l and m are not parallel. If lines l and m are not parallel, then by theorem 4.11 (Exterior Angle Theorem) we can conclude that $\angle 1 > \angle 2$. This is a contradiction to the fact that $\angle 1 \cong \angle 2$. By indirect reasoning, the theorem 5.1 is true.



Corollary 5.2

If two lines are both perpendicular to a transversal, then the lines are parallel.



If $l \perp k$ and $m \perp k$, then $l \parallel m$

Proof:

Given $l \perp k$ and $m \perp k$, show that $l \parallel m$

Statement	Reason
$l \perp k$ and $m \perp k$	Given
$m\angle 1 = 90^\circ$ $m\angle 2 = 90^\circ$	Definition of perpendicular lines
$m\angle 1 + m\angle 3 = 180^\circ$	Supplementary angles have an angle sum of 180 degrees.
$90^\circ + m\angle 3 = 180^\circ$	Substitution property
$90^\circ - 90^\circ + m\angle 3 = 180^\circ - 90^\circ$ $m\angle 3 = 90^\circ$	Subtraction property of equality
$m\angle 1 = m\angle 3$	Transitive property of equality
$l \parallel m$	Theorem 5.1

Corollary 5.3

If two lines are cut by a transversal form a pair of congruent corresponding angles with the transversal, then the lines are parallel.

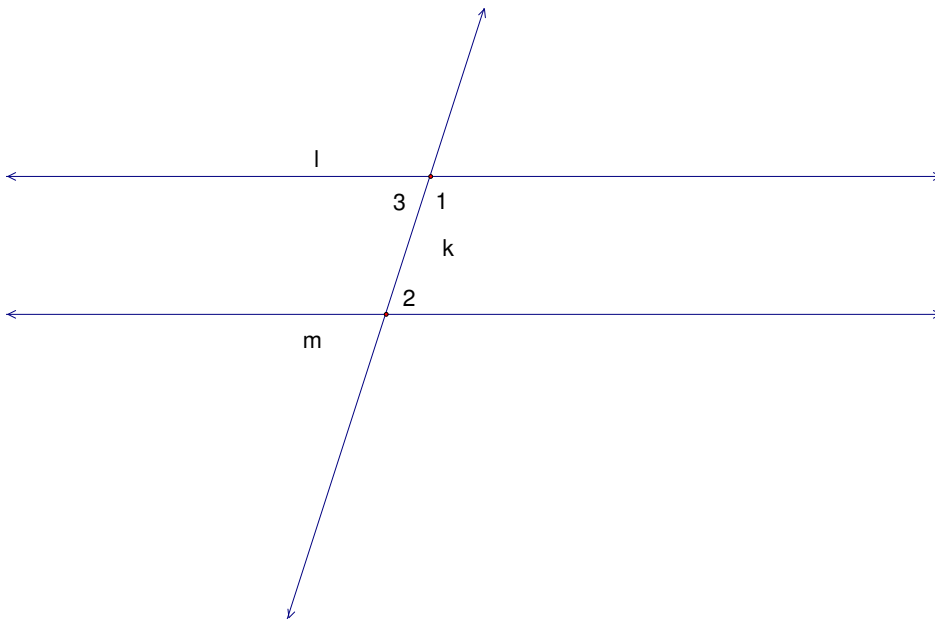
Proof:

Given $\angle 1 \cong \angle 2$, show that $l \parallel m$

Statement	Reason
$\angle 1 \cong \angle 2$	Given
$\angle 1 \cong \angle 3$	Vertical angles are congruent
$\angle 2 \cong \angle 3$	Transitive Property
$l \parallel m$	Theorem 5.1

Corollary 5.4

If two lines cut by a transversal form a pair of supplementary angles on the same side of the transversal, then the lines are parallel.



Proof: Given $m\angle 1 + m\angle 2 = 180^\circ$, show that $l \parallel m$

Let $m\angle 1 + m\angle 2 = 180^\circ$. Since $\angle 1$ and $\angle 3$ are supplementary angles, $m\angle 1 + m\angle 3 = 180^\circ$

By subtracting the two equations above we get the following result.

$$\begin{array}{r} m\angle 1 + m\angle 2 = 180^\circ \\ - (m\angle 1 + m\angle 3 = 180^\circ) \Rightarrow \\ \hline m\angle 2 - m\angle 3 = 0^\circ \end{array}$$

If $m\angle 2 - m\angle 3 = 0^\circ$, then $m\angle 2 = m\angle 3$

If two angles have the same angle measure they are congruent. Thus, $\angle 2 \cong \angle 3$

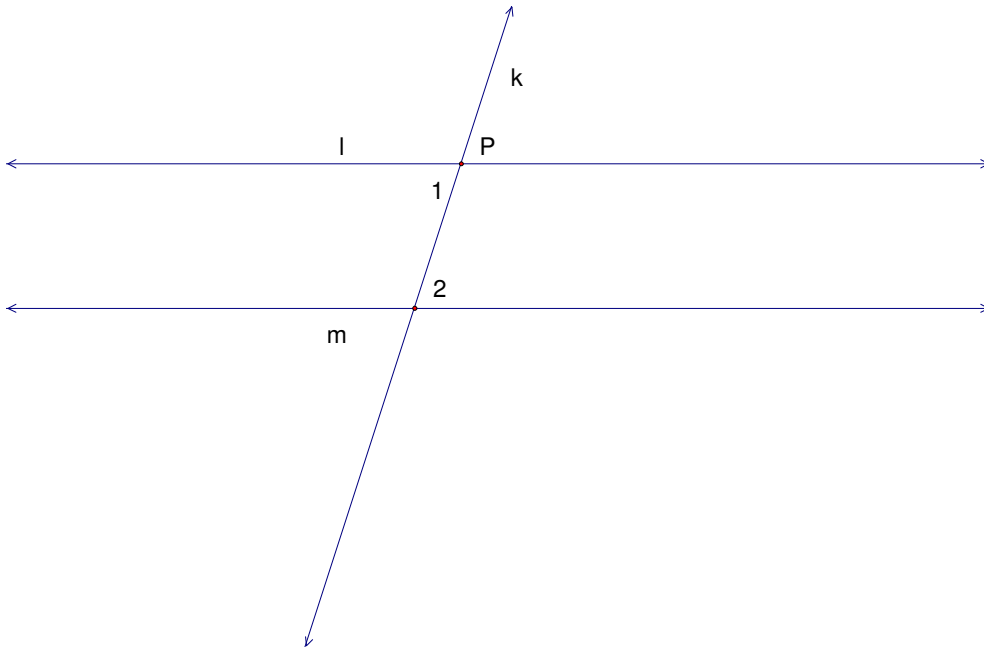
Therefore, $l \parallel m$ by theorem 5.1.

Postulate 5.1

Given a line l and point P not on l , there is only one line m containing P such that $l \parallel m$

Theorem 5.5

If a pair of parallel lines is cut by a transversal, then the alternate interior angles formed are congruent.



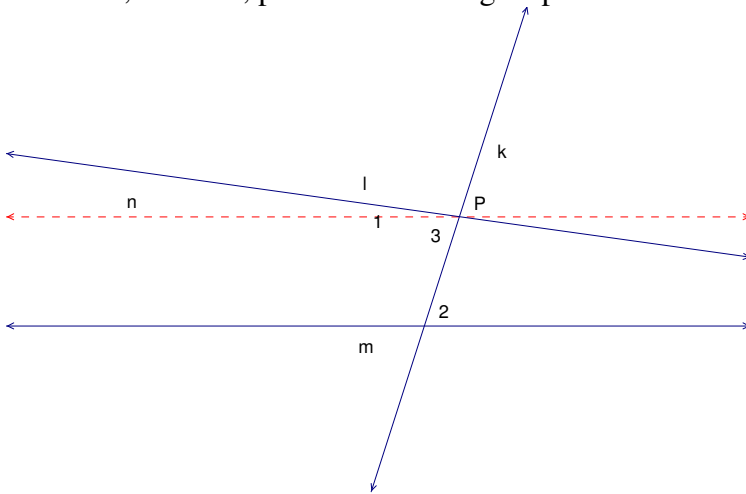
Proof:

Given $l \parallel m$, show that $\angle 1 \cong \angle 2$

Let $l \parallel m$, and assume $m\angle 1 \neq m\angle 2$. If $m\angle 1 \neq m\angle 2$, then there are two cases $m\angle 1 > m\angle 2$ and $m\angle 1 < m\angle 2$

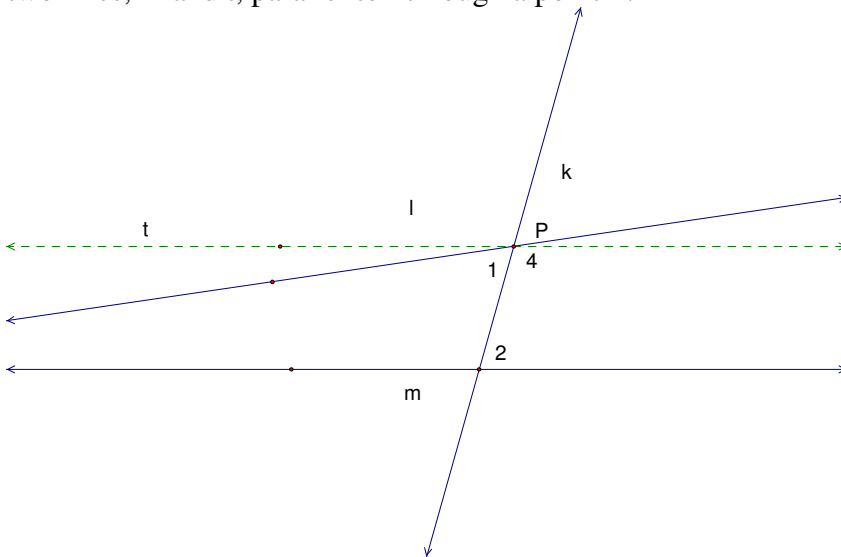
Case 1: $m\angle 1 > m\angle 2$

If $m\angle 1 > m\angle 2$, then exist a line n through P that form an angle $\angle 3$ where $\angle 3 \cong \angle 2$. This implies that $l \parallel n$. This is a contradiction to the parallel postulate because there are two lines, m and n , parallel to l through a point P .



Case 2:

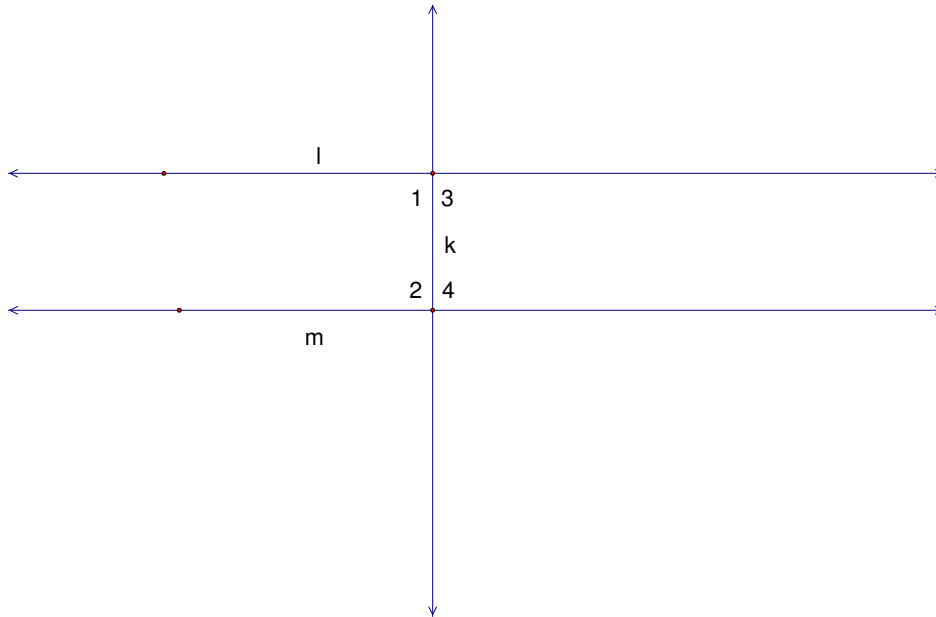
If $m\angle 1 < m\angle 2$, then exist a line t through P that form an angle $\angle 4$ where $\angle 4 \cong \angle 2$. This implies that $l \parallel t$. This is a contradiction to the parallel postulate because there are two lines, m and t , parallel to l through a point P .



Thus, by indirect reasoning, we have $\angle 1 \cong \angle 2$

Corollary 5.6

If two lines are parallel and a line is perpendicular to one of the two lines, then it is perpendicular to the other line.



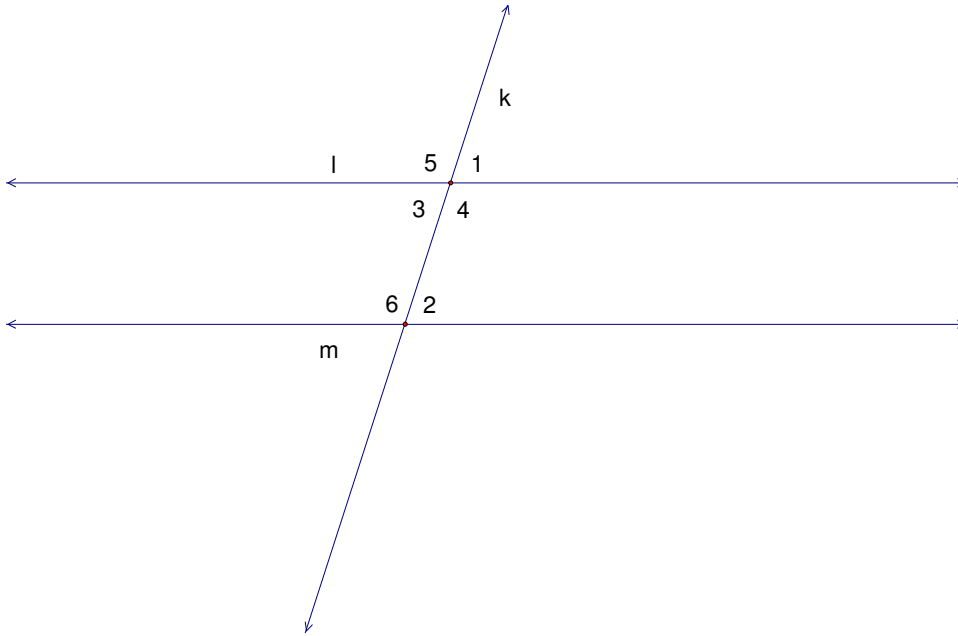
Proof:

Given $l \parallel m$ and $l \perp k$, show that $k \perp m$

Statement	Reason
$l \parallel m$ $k \perp l$	Given
$\angle 2 \cong \angle 3 \Rightarrow m\angle 2 = m\angle 3$	If two lines are parallel, then the alternate interior angles are congruent. Theorem 5.5
$m\angle 3 = 90^\circ$	Two lines are perpendicular iff the angles formed are right angles.
$m\angle 2 = 90^\circ$	Substitution property
$k \perp m$	Two lines are perpendicular iff the angles formed are right angles.

Corollary 5.7

If a pair of parallel lines is cut by a transversal, then each pair of corresponding angles formed is congruent.



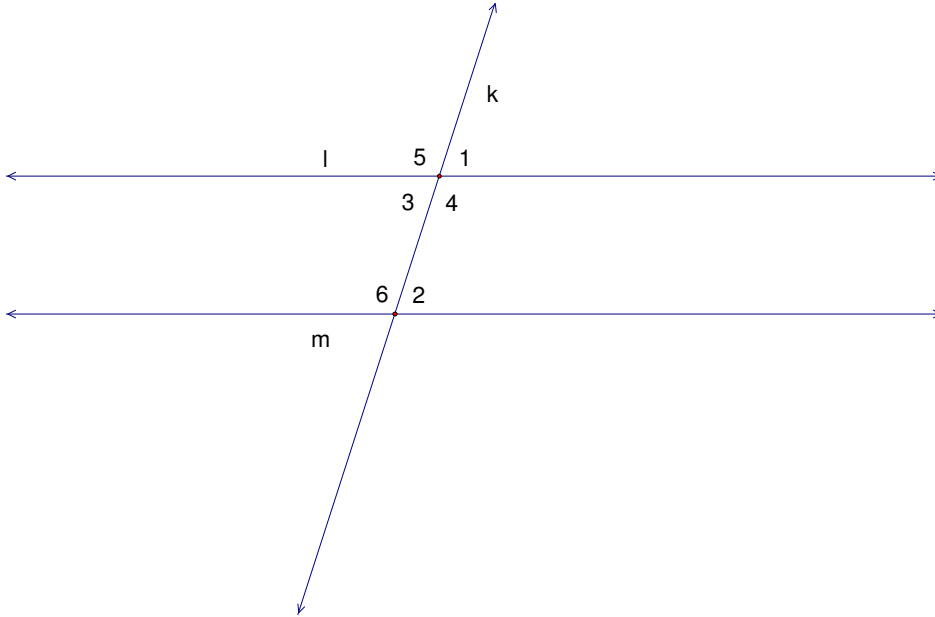
Proof:

Given $l \parallel m$, show that $\angle 1 \cong \angle 2$

Statement	Reason
$l \parallel m$	Given
$\angle 2 \cong \angle 3$	Theorem 5.5
$\angle 1 \cong \angle 3$	If two lines intersect, the vertical angles formed are congruent.
$\angle 1 \cong \angle 2$	Transitive property of congruence

Corollary 5.8

If two parallel lines are cut by a transversal, then both pairs of interior angles on the same side of the transversal are supplementary.



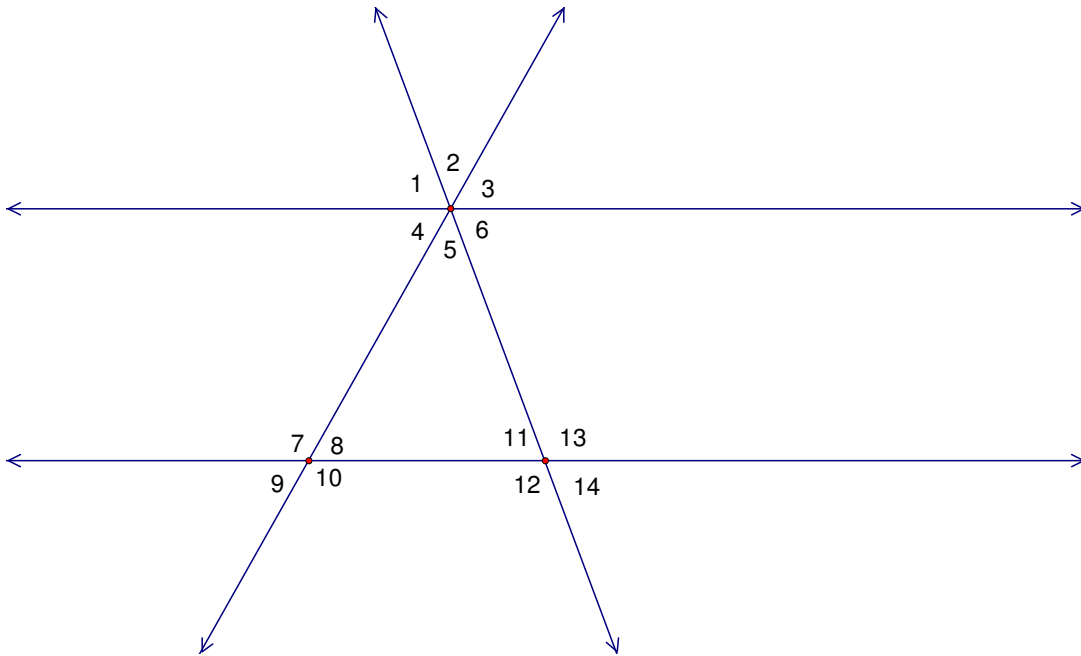
Proof:

Given $l \parallel m$, show that $m\angle 2 + \angle 4 = 180^\circ$

Statement	Reason
$l \parallel m$	Given
$\angle 2 \cong \angle 3$ $\Rightarrow m\angle 2 = m\angle 3$	Theorem 5.5
$\angle 3$ and $\angle 4$ are supplementary	
$m\angle 3 + \angle 4 = 180^\circ$	The sum of two supplementary angles is 180 degrees.
$m\angle 2 + \angle 4 = 180^\circ$	Substitution property

Example 1

Use the following figure to exercises 1-5



- 1) Identify one pair of alternate interior angles.

Solution: $\angle 4$ and $\angle 8$

- 2) Identify two pairs of corresponding angles.

Solution:

$\angle 4$ and $\angle 9$

$\angle 1$ and $\angle 11$

$\angle 6$ and $\angle 14$

$\angle 3$ and $\angle 8$

- 3) Find the measure of $\angle 1 + \angle 12$

$$m\angle 1 = m\angle 11$$

$$m\angle 11 + m\angle 12 = 180^\circ$$

$$m\angle 1 + m\angle 12 = 180^\circ$$

4) Find the measure of $\angle 3$ and $\angle 8$, if $m\angle 4 = 60^\circ$

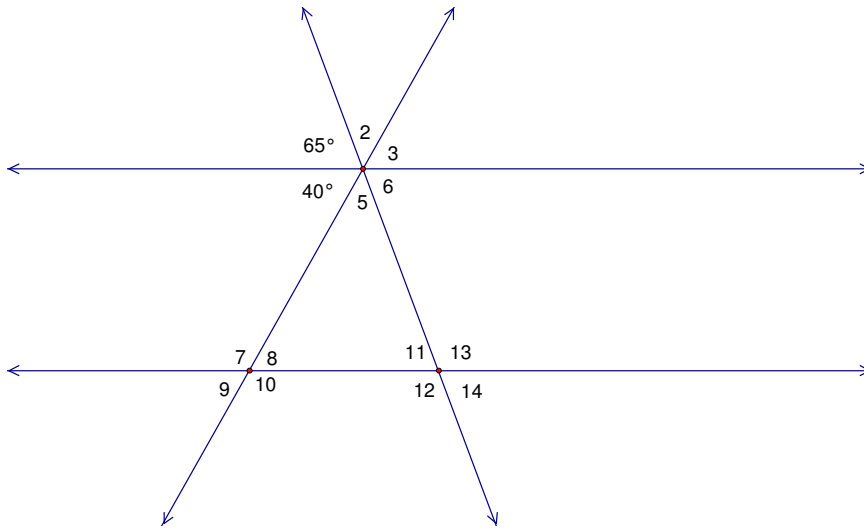
Solution:

$$m\angle 3 = m\angle 4 = 60^\circ$$

$$m\angle 8 = m\angle 3 = 60^\circ$$

5) Find the measure of all of the numbered angles given $m\angle 4 = 40^\circ$ and $m\angle 1 = 65^\circ$

Solution:



$$m\angle 2 = 180^\circ - (40^\circ + 65^\circ) = 180^\circ - 105^\circ = 75^\circ$$

$$m\angle 3 = m\angle 1 = 40^\circ$$

$$m\angle 6 = m\angle 4 = 65^\circ$$

$$m\angle 5 = m\angle 2 = 75^\circ$$

Corresponding Angles:

$$m\angle 9 = m\angle 4 = 40^\circ$$

$$m\angle 11 = m\angle 1 = 65^\circ$$

$$m\angle 14 = m\angle 6 = 65^\circ$$

$$m\angle 8 = m\angle 3 = 40^\circ$$

Supplementary angles give:

$$m\angle 7 = 140^\circ : m\angle 10 = 140^\circ : m\angle 12 = 115^\circ : m\angle 13 = 115^\circ$$