

Section 3.4

Volume and Surface Area

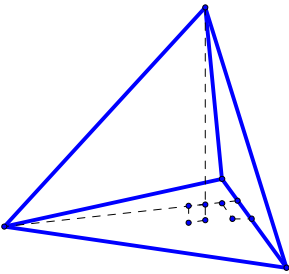
Definition: Volume is the amount of space occupied by an object.

Volume of a Pyramid

$$V = \frac{1}{3}B \cdot h$$

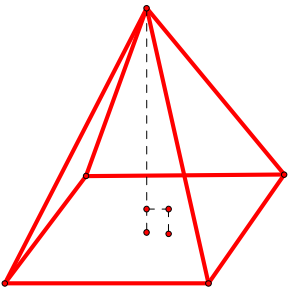
Examples of pyramids

A pyramid with a triangular base:



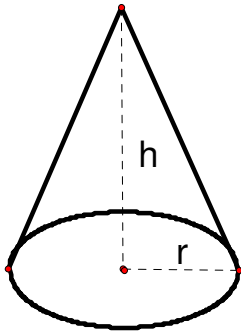
$$\text{Volume: } V = \frac{1}{3}Bh = \frac{1}{3} \cdot \frac{1}{2}b \cdot h_1 \cdot h_2 = \frac{1}{6}bh_1h_2$$

A pyramid with a rectangular base:



$$V = \frac{1}{3}Bh = \frac{1}{3} \cdot lwh$$

The Cone

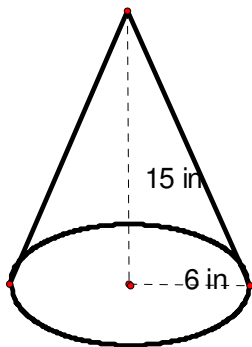


$$V = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)h = \frac{1}{3}\pi r^2 h$$

Example 1

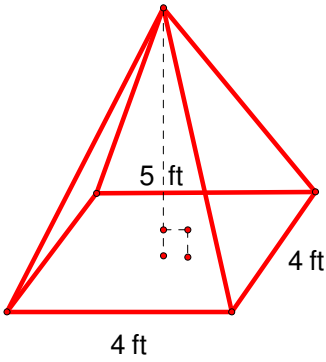
Find the volume of the following objects

a)



$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6 \text{ in})^2(15 \text{ in}) = \frac{1}{3}(540 \text{ in}^3) = 180\pi \text{ in}^3$$

b)



$$V = \frac{1}{3}lwh = \frac{1}{3}(4 \text{ ft})(4 \text{ ft})(5 \text{ ft}) = \frac{1}{3}(80 \text{ ft}^3) = 26.7 \text{ ft}^3$$

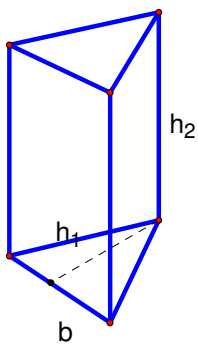
Prisms

The volume of a prism

$$V = Bh$$

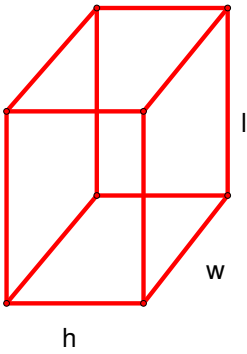
Types of Prisms

A prism with a triangular base



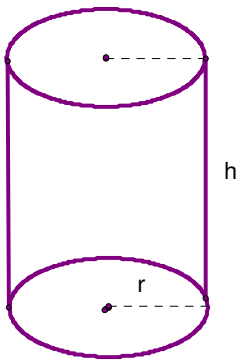
$$V = Bh = \frac{1}{2}bh_1h_2$$

A prism with a rectangular base



$$V = Bh = lwh$$

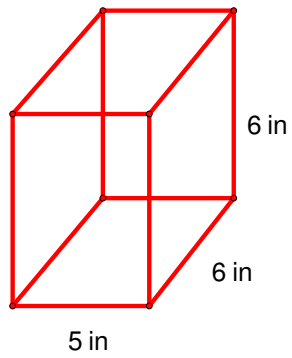
The Cylinder



$$V = Bh = \pi r^2 h$$

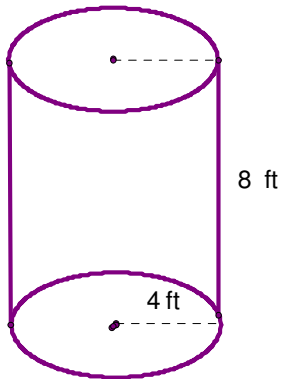
Example 2: Find the volume

a)



$$V = lwh = (5 \text{ in})(6 \text{ in})(6 \text{ in}) = 180 \text{ in}^3$$

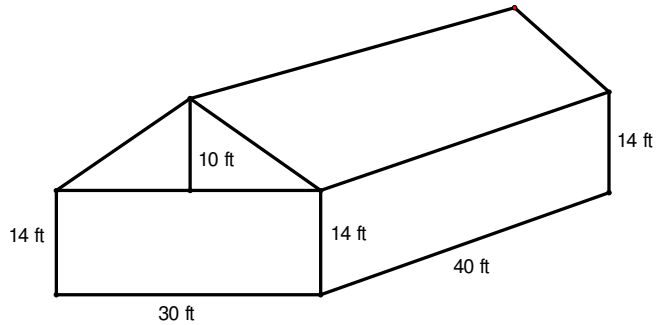
b)



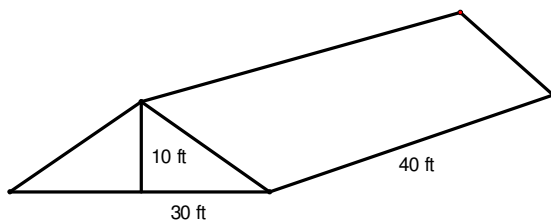
$$V = \pi(4 \text{ ft})^2(8 \text{ ft}) = 128\pi \text{ ft}^3$$

Example 3

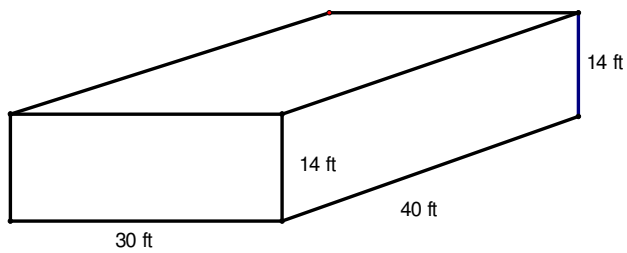
Find the volume and surface area



Volume:



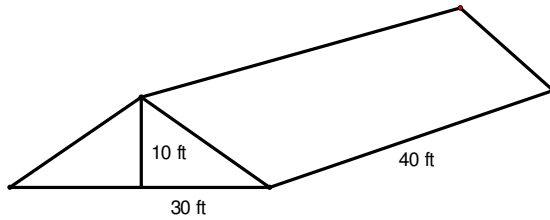
$$V_1 = \frac{1}{2}bh_1h_2 = \frac{1}{2}(10\text{ ft})(30\text{ ft})(40\text{ ft}) = \frac{1}{2}(12000\text{ ft}^3) = 6000\text{ ft}^3$$



$$V = lwh = (30\text{ ft})(40\text{ ft})(14\text{ ft}) = 16800\text{ ft}^3$$

$$\text{Total Volume: } V = 6000\text{ ft}^3 + 16800\text{ ft}^3 = 22800\text{ ft}^3$$

Surface Area:



Hypotenuse

$$c^2 = 10^2 + 15^2$$

$$c^2 = 100 + 225$$

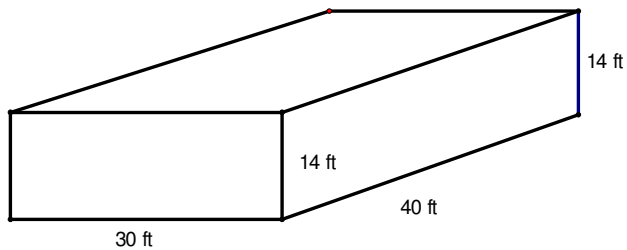
$$c^2 = 325$$

$$c = \sqrt{325} \approx 18 \text{ ft}$$

Triangular Prism:

$$A_1 = 2\left(\frac{1}{2}bh\right) + 2lw = (30 \text{ ft})(10 \text{ ft}) + 2(18 \text{ ft})(40 \text{ ft}) = 300 \text{ ft}^2 + 1440 \text{ ft}^2 = 1740 \text{ ft}^2$$

Rectangular Prism



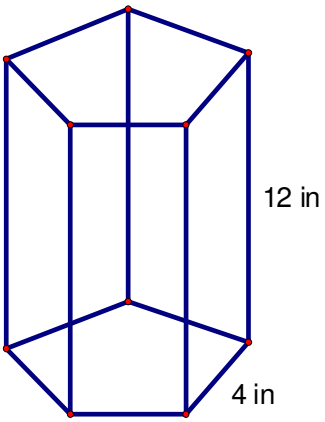
$$A_2 = 2lh + 2hw + lw$$

$$A_2 = 2(30 \text{ ft})(14 \text{ ft}) + 2(14 \text{ ft})(40 \text{ ft}) + (30 \text{ ft})(40 \text{ ft}) = 840 \text{ ft}^2 + 1120 \text{ ft}^2 + 1200 \text{ ft}^2 = 3160 \text{ ft}^2$$

$$A = 1740 \text{ ft}^2 + 3160 \text{ ft}^2 = 4900 \text{ ft}^2$$

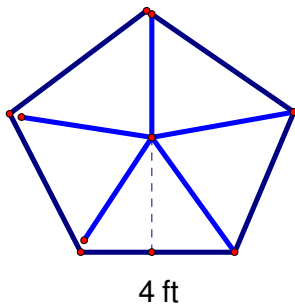
Example 4

Find the volume of a prism with a regular pentagon base



$$V = Bh$$

Area of regular pentagon



$$\text{Find the interior angle: } I = \frac{(5-2)180}{5} = \frac{3 \cdot 180}{5} = \frac{540}{5} = 108^\circ$$

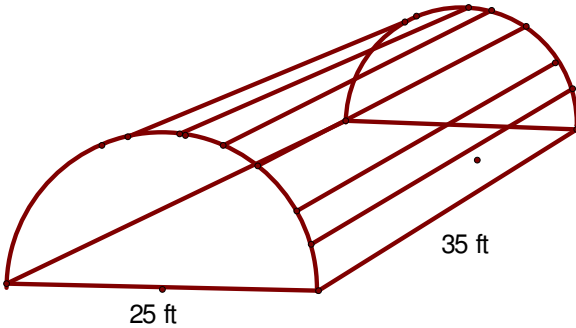
$$h = \frac{1}{2}(4) \tan 54^\circ = 2 \tan 54^\circ = 2.753 \text{ ft}$$

$$A = \frac{1}{2}Ph = \frac{1}{2}(5 \cdot 4 \text{ ft})(2.752 \text{ ft}) = 27.52 \text{ ft}^2$$

$$\text{Volume: } V = Bh = (27.52 \text{ ft}^2)(12 \text{ ft}) = 330.24 \text{ ft}^3$$

Example 5

Find the surface area and volume.



Surface Area

$$\text{Radius: } r = \frac{25 \text{ ft}}{2} = 12.5 \text{ ft}$$

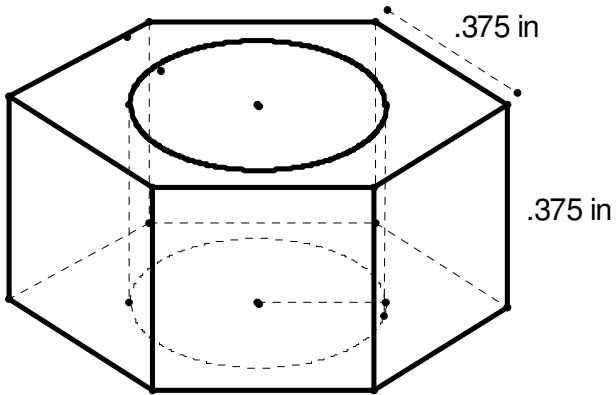
$$\begin{aligned} A &= 2\left(\frac{1}{2}\pi r^2\right) + \frac{1}{2}(\pi l)l + lw = \pi r^2 + \frac{1}{2}\pi dl = \pi(12.5)^2 + \frac{1}{2}\pi(25)(35) + 25 \cdot 35 \\ &= 490.625 + 1373.75 + 875 \\ &= 2739.375 \text{ ft}^2 \end{aligned}$$

Volume:

$$V = \frac{1}{2}Bh = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(12.5)^2(35) = 8585.94 \text{ ft}^3$$

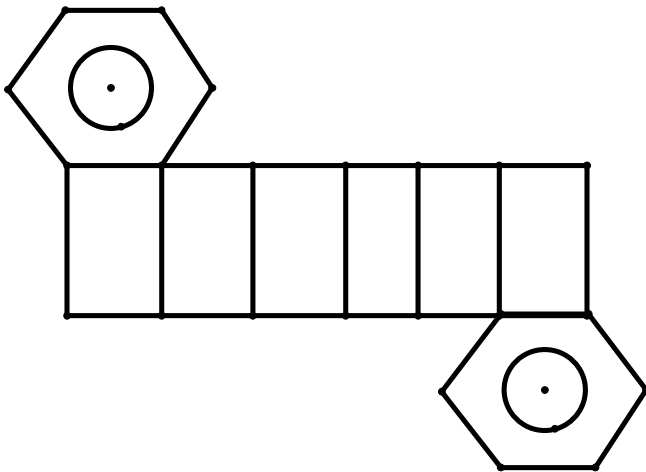
Example 6

The machined piece is a right regular hexagonal prism with a hole that is a half inch in diameter. Find the total surface area and volume of the machined piece to the nearest tenth of a square inch.



Surface Area:

Find the area of the hexagon.



$$h = \frac{1}{2} (.375 \text{ in}) \tan(60^\circ) = .5(.375)\sqrt{3} = .325$$

$$\text{Area of Hexagon} = \frac{1}{2} Ph = \frac{1}{2} (6(.375))(.325) = .366 \text{ in}^2$$

$$\text{Area of Circle} = \pi(.25)^2 = .196 \text{ in}^2$$

$$\text{Total Area} = .366 - .196 = .17 \text{ in}^2$$

Area of rectangular faces

$$\text{Area of one face} = (.375)(.375) = .141 \text{ in}^2$$

$$\text{Area of all faces} = 6(.141 \text{ in}^2) = .846 \text{ in}^2$$

$$\text{Total Area: } A = 2(.170) + .846 = 1.186 \approx 1.2 \text{ in}^2$$

Volume**Volume of Hexagonal Prism:**

$$V = Bh = (.366 \text{ in}^2)(.375 \text{ in}) = .137 \text{ in}^3$$

Volume of Cylinder:

$$V = Bh = \pi r^2 h = \pi (.25)^2 (.375) = .073 \text{ in}^3$$

Total volume = Hexagonal Prism – Cylinder

$$V = .137 - .073 = .064 \text{ in}^3$$