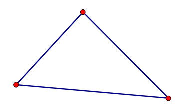
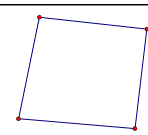
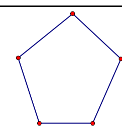
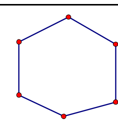
Math 135

Section 2.3
Polygons

A **polygon** is a geometric figure with more than two sides.

Types of polygons

Polygon	Shape
Triangle	
Quadrilateral	
Pentagon	
Hexagon	

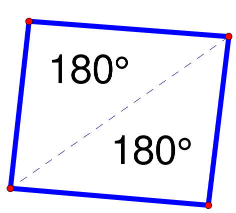
Theorem 2.1 Angle Measures in a Triangle

The sum of the measures of the angles in a triangle is 180°

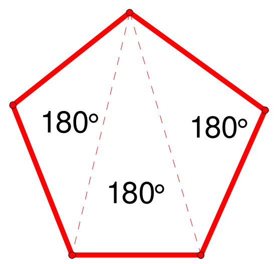
The sum of the angle measures of other polygons.

To find the angle measures of other polygons, you can simply divide each polygon up into triangles and multiple 180 times the number of triangles.

For example, a quadrilateral can be divided into two triangles. So two times 180 degrees will give an angle sum of 360 degrees. See below

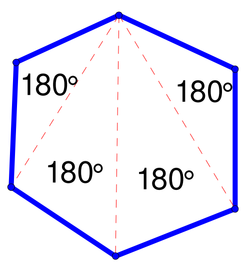


Next, look at a pentagon. A pentagon can be divided into three triangles



So, the angle sum for a pentagon would be: $3 \cdot 180° = 540°$

Likewise, the angle for a hexagon would be $4 \cdot 180° = 720°$ See Below:



If you study patterns for polygon with n sides, then the formula for finding the number of sides of a polygon is given by the formula $S = (n-2) \cdot 180°$

Theorem 2.2

The sum of the angle of a polygon with n sides is give by the formula $S = (n-2) \cdot 180°$

Example 1

Find the sum of interior angles of an octagon and a polygon with 15 sides

Part 1:

$$n = 8$$
$$S = (n-2) \cdot 180° = (8-2)180 = 6 \cdot 180° = 1080°$$

Part 2:

$$n = 15$$
$$S = (15-2)180 = 13 \cdot 180° = 2340°$$

A **regular polygon** is a polygon where all its sides are congruent.

Theorem 2.3

Vertex Angle Measure in a Regular Polygon

The measure of the vertex angle of a regular polygon is given by the equation: $S = \frac{(n-2)\cdot 180°}{n}$

Example 2

Find the measure of the vertex angle a regular hexagon.

$$n = 6$$
$$S = \frac{(n-2)\cdot 180°}{n} = \frac{(6-2)\cdot 180}{6} = \frac{4\cdot 180}{6} = \frac{720}{6} = 120°$$

Example 3

The sum of the interior angles of a regular polygon is $2160°$

$$S = (n-2)\cdot 180°$$
$$S = 2160°$$
$$2160 = (n-2)\cdot 180$$
$$\frac{2160}{180} = \frac{(n-2)\cdot 180}{180}$$
$$12 = n-2$$
$$\Rightarrow n = 12+2 = 14$$

Addition Facts of Polygons:

The sum of the exterior angle of a polygon is $360°$

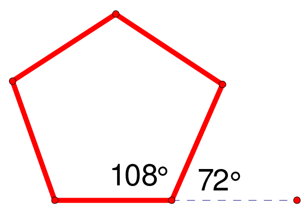
The measure of the interior angle of regular polygon is given by the formula: $S = \frac{360}{n}$

Example 5

Find the measure of the interior angle of a regular pentagon

$n = 5$

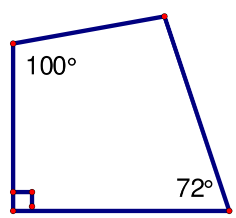
$$S = \frac{360}{n} = \frac{360}{5} = 72°$$



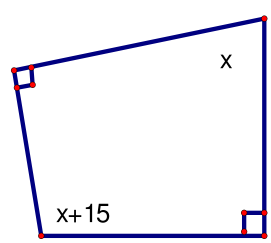
Example 6

Find the missing angle measures in the quadrilateral

A)



B)



Solutions:

Since both figures are quadrilaterals, their angle sum is $360°$

Recall: $S = (4-2)\cdot 180° = 2\cdot 180° = 360°$

In quadrilateral A, let x be the missing angle. Set up an equation by taking the sum of the four angles of the quadrilateral and setting them equal to $360°$. See below

$x + 100 + 72 + 90 = 360$

Now, simply solve the equation.

$$x + 100 + 72 + 90 = 360$$
$$x + 262 = 360$$
$$x = 98°$$

Similarly for quadrilateral B you will get the equation: $x + 90 + x + 15 + 90 = 360$

Now solve for x

$$x + 90 + x + 15 + 90 = 360$$
$$2x + 195 = 360$$
$$2x + 195 - 195 = 360 - 195$$
$$2x = 165$$
$$x = 82.5°$$

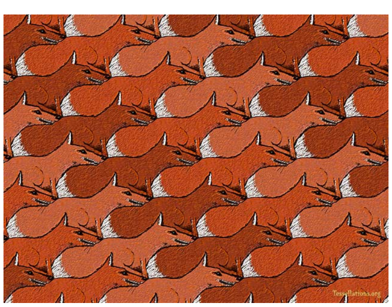
Substitute the value of x into the formula in B to get the missing angle.

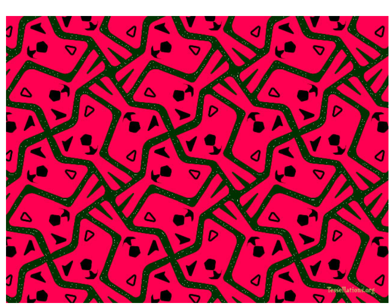
$x + 15 = 82.5 + 15 = 97.5°$

The missing angles are $82.5°$ and $97.5°$

Tessellations

A **polygon region** is a polygon together with the portion of the plane that is enclosed by the polygon. A group of polygon regions that can be arranged like ceramic tiles or carpet squares to completely cover a plane without any gaps are called a **tessellation** or tiling. Here are some examples of tessellations made with various objects courtesy tessellations.org.





Bluefield, WV native John Forbes Nash of Princeton University used a hexagon tessellation as board for his game called Hex which was an adaptation of a game called Go which used a regular square grid. The movie *A Beautiful Mind* portrays Nash life as a Mathematician at Princeton University.